The Stable Orbit of the Small Satellite Flying-Around the Space Station and the Orbit Maintenance

Jianbo Hu, Hongye Su, Weimin Wu and Jian Chu
National Laboratory of Industrial Control Technology&
Institute of Advanced Process Control, Zhejiang University
Hangzhou, 310027, P.R. China
jbhu@iipc.zju.edu.cn
Tel: 86-571-7952369 Fax: 86-571-7951206

Abstract: The stable orbit of a small satellite flying-around the space station and the orbit maintenance are studied in this paper based on the Hill equation and a Sliding-Mode Control (SMC) approach. First, the problem of the stable orbit of the small satellite flying around the space station is transformed into the sliding-mode control problem. Then, the sliding-mode controllers are proposed to guarantee the existence of sliding-mode motion, and the controller are of relay-type controllers. Meanwhile, the motion of the small satellite will enter and maintain on the sliding-mode in finite time, and the stable orbit of the small satellite flying around the space station is obtained. In order to economize the energy and restrain the chatting of control signals, the sliding-mode with dead region is proposed, and the corresponding controllers are derived to make the motion of the closed-loop system enter a defined region. A mathematical derivation and simulation study are carried out to prove the following three main points:

1) the condition for the stable orbit of the small satellite flying around the space station can be guaranteed by some sliding-mode motion;
2) the constant term in the chosen sliding-mode can be used to adjust the position of the center of the stable orbit;
3) sling-mode with dead-region can also obtain the stable flying orbit.

The total influence of the parameters in our controllers to the working performance of the closed-loop system is discussed by simulation.

Key word: Flying orbit around the space station, stable flying around the space station, sliding-mode, sliding-mode with dead region, small satellite

1 Introduction

The orbit, which one spacecraft flies around another spacecraft, is called the orbit of one spacecraft flying around another spacecraft. This kind of orbit is used to make the small satellite fly around some space station or big spacecraft such that the small satellite can monitor continuously the state of the space station or big spacecraft, e.g., the operation state and airproof of the space station. Then, the activity of an astronaut leaving from the space station or big spacecraft, which is usually dangerous and expensive, may be decreased largely. This is a new concept of the application of the small satellite [1][2]. In order to get a stable orbit, some condition for the initial state proposed in [1] by Lai (1999). This is however not possible to use if there exist uncertainties in the Hill equations.

Sliding-mode control approach is a robust control strategy, which can avoid the influence of uncertainties matching with input matrix. If we can transform the condition on the initial state into the condition on the global condition, which is taken as sliding-mode and guaranteed by the sliding-mode controllers, the small satellite will enter and maintain on the sliding-mode in finite time. This is a new concept of the application of sliding-mode control into the orbit maintenance.

First, for the small satellite, we transformed the condition of stable flying around the space station into the existence of some chosen sliding-mode motions based on the Hill equations [2] and sliding-mode control approach [3][4]. Then, the corresponding sliding-mode controller, which can guarantee that the trajectories of the closed-loop system enter and maintain onto the sliding-mode in finite time, is derived. The obtained controllers are relay-type controller and may cause the chatting of the control signals, which is usually harmful to saving energy and the control performance. In order to avoid from the chatting and save energy, the sliding mode controllers with dead-region are derived such that the trajectories of the closed-loop system are entered and maintain into the field enclosed by some defined elliptical orbits in finite time. The total influence of the parameters of the chosen sliding-mode and dead-region is studied by mathematical proof and numerical simulation. Several simulations are carried out to illustrate the effectiveness of the proposed methods.

2 The condition of the stable flying around the space station based on sliding-mode

2.1 The dynamics of the orbit flying around the space station

The dynamics of the small satellite flying around the space station can be described with Hill equations for the distance between the small satellite and the space station . This disturbance is very short compared with the flying altitude of them. In our coordinates, X is coincident with the flying direction, Y is toward the center of earth, and Z is vertical to the surface of the orbit. Hill equations can be described as follows.

$$\left.\begin{aligned} &\ddot{x}-2\omega\dot{y}=a_x+\eta_x \\ &\ddot{y}+2\omega\dot{x}-3\omega^2 y=a_y+\eta_y \\ &\ddot{z}+\omega^2 z=a_z+\eta_z \end{aligned}\right\} \quad (1)$$

where a_x, a_y, a_z are three accelerations in three directions. They are control inputs in our problems. In this paper, we only consider the two-value case and introduce the following sets to describe the allowable control inputs

$$a_x \in \{-\kappa_x, \kappa_x\}; a_y \in \{-\kappa_y, \kappa_y\}; a_z \in \{-\kappa_z, \kappa_z\}.$$
$$\kappa_x > 0, \kappa_y > 0, \kappa_z > 0$$

ω is the constant angular speed of the orbit .η_x, η_y, η_z are the disturbances in three directions. Our orbit is of plane $x-y$. Meanwhile, we can control z by properly a_z . Thus, we will consider only for axes x and y . The following assumption is introduced for the disturbances.

Assumption 1: There exist positive constants c_x, c_y, c_z such that

$$\left.\begin{aligned} &|\eta_x| \le c_x < \kappa_x \\ &|\eta_y| \le c_y < \kappa_y \\ &|\eta_z| \le c_z < \kappa_z \end{aligned}\right\} \quad (2)$$

2.2 The condition of the stable orbit flying around the space station based on sliding-mode

In Ref.[1], the condition of the stable orbit flying around the space station is described as the initial state of system (1), which is derived based on the special case when the right side of system (1) is equal to zero. However, this condition can not be directly used to analyze the stability of the orbit and derive the control strategy under the existence of disturbances. In our study, we transform the condition of the stable orbit flying around the space station into a sliding-mode control problem. This provides an effective way to the controller design and the stability analysis of the closed-loop system.

Theorem 1: If the system (1) with the proper controller can enter and maintain onto the following sliding modes

$$\left.\begin{aligned} s_1 &= \dot{x}-2\omega y=0 \\ s_2 &= \dot{y}+0.5\omega x=0 \end{aligned}\right\} \quad (3)$$

the surface orbit of the system (1) under the proper controller is a stable ellipse with center in $(0,0)$ major axis twice the minor.

Proof: Let the trajectories of system (1) with proper controller enter and maintain on the sliding modes (3) at time $t=t_0$ with state $(x_0, y_0, \dot{x}_0, \dot{y}_0)$. We have

$$\left.\begin{aligned} \dot{x}_0 &= 2\omega y_0 \\ \dot{y}_0 &= -0.5\omega x_0 \end{aligned}\right\} \quad (4)$$

From (3), we can obtain

$$\left.\begin{aligned} \ddot{x}+\omega^2 x=0 \\ \ddot{y}+\omega^2 y=0 \end{aligned}\right\}$$

Thus, we may assume that

$$x=c_1\cos(\omega(t-t_0))+c_2\sin(\omega(t-t_0))$$
$$y=d_1\cos(\omega(t-t_0))+d_2\sin(\omega(t-t_0))$$

It is easy to get $c_2 = 2d_1, d_2 = -0.5c_1$
Thus

$$x^2 + 4y^2 = c_1^2 + c_2^2 = x_0^2 + 4y_0^2$$

This completes our proof.

Theorem 2: If the system (1) with proper controller can enter and maintain onto the following sliding modes

$$\left.\begin{aligned} s'_1 &= \dot{x} - 2\omega y - A_1 = 0 \\ s'_2 &= \dot{y} + 0.5\omega x - A_2 = 0 \end{aligned}\right\} \quad (5)$$

where A_1, A_2 are any positive constants, the surface orbit of the system (1) under the proper controller is an stable ellipse with center in $(\frac{2A_2}{\omega}, -\frac{A_1}{2\omega})$ and long axes 2 times short one.

Proof: This theorem can be proved in the similar way with what is used in the proof of **Theorem 1**.

By now, we can transform the condition of the stable flying around the space station described in Ref.[1] into the existence of the sliding-mode motions described in Theorem 1 and Theorem 2. The controllers in section 3 can guarantee this.

3 The sliding-mode controller

3.1 The sliding-mode controller for the stable orbit with the center in (0,0)

Theorem 3: For the system under the **Assumption1**, if the sliding modes are chosen as (3), the following controller

$$\left.\begin{aligned} a_x &= -\kappa_x \operatorname{sgn}(s_1) \\ a_y &= -\kappa_y \operatorname{sgn}(s_2) \end{aligned}\right\} \quad (6)$$

will guarantee that the trajectories of closed-loop system (1) enter and maintain onto some ellipse with center (0,0) and long axes 2 times short one.

Proof: From the sliding modes (3) and the system (1), it is easy to get

$$\begin{aligned} \dot{s}_1 &= \ddot{x} - 2\omega\dot{y} = a_x + \eta_x \\ \dot{s}_2 &= \ddot{y} + 0.5\omega\dot{x} = a_y + \eta_y + 3\omega^2 y - 1.5\omega\dot{x} \\ &= a_y + \eta_y - 1.5\omega s_1 \end{aligned}$$

Under the controller (6), we have $\dot{s}_1 s_1 \le -(\kappa_x - c_x)|s_1|$ from **Assumption 1**. Thus, there is finite time t_1 such that the trajectories of the closed-loop system enter and maintain on the first sliding-mode $s_1 = 0$ for $t > t_1$. Furthermore, we have $\dot{s}_2 s_2 \le -(\kappa_y - c_y)|s_2|$ for $t > t_1$, and after finite time t_2, the trajectories of the closed-loop system enter and maintain on sliding-modes $s_1 = 0, s_2 = 0$ $t > t_1 + t_2 = T_0$ [3][4]. Let the state in time T_0 be $(x(T_0), y(T_0))$, and from **Theorem 1** we have

$$x^2 + 4y^2 = x^2(T_0) + 4y^2(T_0) \text{ for } t \ge T_0.$$

This completes our proof.

3.2 The sliding-mode controller for the stable orbit with the center away from (0,0)

Theorem 4: For the system under the **Assumption1**, if the sliding modes are chosen as (5) and $|A_1| < \frac{2}{3\omega}(\kappa_y - c_y)$, the following controller

$$\left.\begin{aligned} a_x &= -\kappa_x \operatorname{sgn}(s'_1) \\ a_y &= -\kappa_y \operatorname{sgn}(s'_2) \end{aligned}\right\} \quad (7)$$

will guarantee that the trajectories of closed-loop system (1) enter and maintain onto some ellipse with center $(\frac{2A_2}{\omega}, -\frac{A_1}{2\omega})$ and long axes 2 times short one.

Proof: From sliding mode (5) and system (1), it is easy to get

$$\begin{aligned} \dot{s}'_1 &= \ddot{x} - 2\omega\dot{y} = a_x + \eta_x \\ \dot{s}'_2 &= \ddot{y} + 0.5\omega\dot{x} = a_y + \eta_y + 3\omega^2 y - 1.5\omega\dot{x} \\ &= a_y + \eta_y - 1.5\omega s'_1 - 1.5\omega A_1 \end{aligned}$$

For $|A_1| < \frac{2}{3\omega}(\kappa_y - c_y)$, we can prove this theorem based on **Theorem 2** in the similar way with what is used for the proof **Theorem 3**.

Remark 1: The controller (6) or (7) is two-values and relay-type, which belongs to the allowance set of the control signals.

Remark 2: The length of the long axes of the obtained orbit ellipse depends on the state of system (1) when the controller is acting, and the disturbances acting on the system (1) in the cause of reaching the sliding-modes (3) and (5). We can obtain the desired length of the long axes of the stable orbit ellipse by using the control strategy described in Ref.[1], and maintain the orbit by using our controller.

4 The sliding-mode with dead-region and its stable orbit flying around the space station

From theorems 3 and 4, although the perfect orbit with no errors can be obtained, the control signals will chat with high frequency. This will make the controllers work continuously in positive direction or opposite direction and be harmful to saving energy. If we introduce the dead regions for the sliding modes and make the controller stop working if the trajectories enter these regions, the chatting of the control signals will be largely decreased and the energy will be saved. Of course, this will lead to the departure of the center of the orbit. Now, we give the sliding mode controllers with dead regions in Theorems 5 and 6.

Theorem 5: Corresponding to the controller (6), the following sliding-mode controller with dead-region

$$\left.\begin{aligned} a_x &= -\frac{\kappa_x \operatorname{sgn}(s_1)(1+\operatorname{sgn}(-\delta_1+|s_1|)}{2} \\ a_y &= -\frac{\kappa_y \operatorname{sgn}(s_2)(1+\operatorname{sgn}(-\delta_2+|s_2|)}{2} \end{aligned}\right\} \quad (8)$$

,where $\delta_2 > 0; 0 < \delta_1 < \frac{2}{3\omega}(\kappa_y - c_y)$, will make the trajectories of the closed-loop system (1) and (8) enter and maintain into the field

$$\left.\begin{aligned} |s_1| \le \delta_1 \\ |s_2| \le \delta_2 \end{aligned}\right\} \quad (9)$$

in finite time.

Proof: Because of

$$\begin{aligned} \dot{s}_1 &= \ddot{x} - 2\omega\dot{y} = a_x + \eta_x \\ \dot{s}_2 &= \ddot{y} + 0.5\omega\dot{x} = a_y + \eta_y + 3\omega^2 y - 1.5\omega\dot{x} \\ &= a_y + \eta_y - 1.5\omega s_1 \end{aligned}$$

we have $\dot{s}_1 s_1 < -(\kappa_x - c_x)|s_1|$ for $|s_1| > \delta_1$. Thus, there exists finite time t_1 such that the trajectories of the closed-loop system (1) and (8) will enter the field $|s_1| \le \delta_1$ for $t > t_1$. Furthermore, because of $\dot{s}_2 s_2 < -(\kappa_y - c_y - 1.5\omega\delta_1)|s_2|$ for $t > t_1$ and $|s_2| > \delta_2$, there exists finite time t_2 such that $|s_2| \le \delta_2$ and $|s_1| \le \delta_1$ for $t > t_1 + t_2$.

This completes our proof for **Theorem 5**.

Theorem 6: Corresponding to the controller (7), the following sliding-mode controller with dead region

$$\left.\begin{aligned} a_x &= -\frac{\kappa_x \operatorname{sgn}(s'_1)(1+\operatorname{sgn}(-\delta'_1+|s'_1|)}{2} \\ a_y &= -\frac{\kappa_y \operatorname{sgn}(s'_2)(1+\operatorname{sgn}(-\delta'_2+|s'_2|)}{2} \end{aligned}\right\} \quad (10)$$

,where $\delta_2 > 0; 0 < \delta_1 < \frac{2}{3\omega}(\kappa_y - c_y - 1.5\omega|A_1|)$, will make the trajectories of the closed-loop system (1) and (8) enter and maintain into the field

$$\left.\begin{aligned} |s'_1| \le \delta'_1 \\ |s'_2| \le \delta'_2 \end{aligned}\right\} \quad (11)$$

in finite time.

Proof: This theorem can be proved in the similar way with what is used for the proof of **Theorem 5** and by using the fact $0 < \delta_1 < \frac{2}{3\omega}(\kappa_y - c_y - 1.5\omega|A_1|)$.

5 Simulation studies

For the system (1) with the parameters

$\omega = 0.0011324, \kappa_x = \kappa_y = 0.04 m/s^2$,

$$\eta_x = \eta_y = 0.0000001, \quad \eta_z = 0,$$

and with initial condition

$$\begin{aligned} &x_0 = 500m, y_0 = 0, \dot{x}_0 = 0m/s, \dot{y}_0 = -0.25m, \\ &z_0 = 0m, \dot{z}_0 = 0m/s \end{aligned},$$

we take the controller as follows

$$\left.\begin{aligned} a_x &= -\frac{\kappa_x \operatorname{sgn}(s_1)(1+\operatorname{sgn}(-\delta_1+|s_1|)}{2} \\ a_y &= -\frac{\kappa_y \operatorname{sgn}(s_2)(1+\operatorname{sgn}(-\delta_2+|s_2|)}{2} \end{aligned}\right\};$$

$$\left.\begin{aligned} s_1 &= \dot{x} - 2\omega y - A_1 = 0 \\ s_2 &= \dot{y} + 0.5\varpi x - A_2 = 0 \end{aligned}\right\}$$

A lot of numerical simulations are carried out to illustrate the effectiveness of the proposed methods. Every simulation test runs 24 hours (86400s).

5.1 The motion trajectories of the small satellite relative to the space station without control action

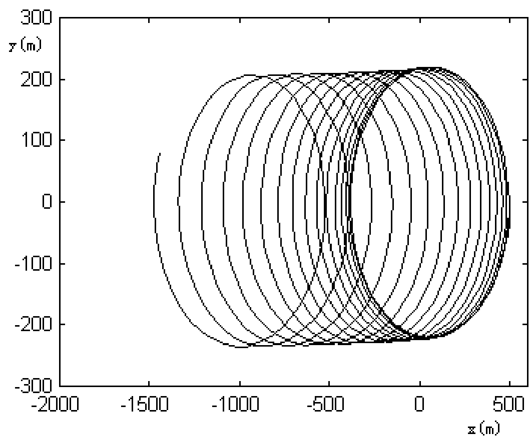


Fig 1 The trajectories of system (1) without control inputs

It can be seen from Fig 1 that the motion trajectories move in one direction under the constant disturbances.

(relative tolerance in calculation is 0.001)

5.2 The Sliding mode control

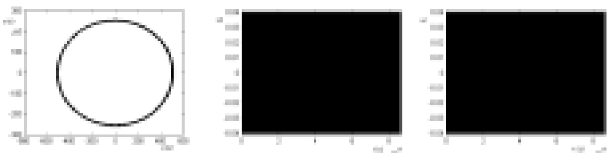
Choose

$A_1 = A_2 = 0(m)$

$\delta_1 = \delta_2 = 0(m/s),$.

The orbit is an ellipse with center in $(0,0)$ as shown in Fig 2. The control signals have largely chatting. This is harmful to saving energy. The error in orbit is caused by calculation error.

(relative tolerance in calculation is 0.001)



(a) (b) (c)

Fig 2 The results in sliding-mode control case

Remark: For figs 2~7, we denote

(a) Orbit in surface; (b) Controller $a_x(m/s^2)$ and (c) Controller $a_y(m/s^2)$, respectively. The unit for time is second.

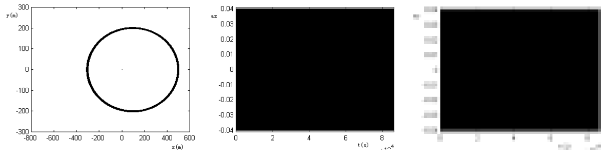
5.3 The sliding mode with constant term

Choose

$A_1 = 0, A_2 = 0.11/2(m)$

$\delta_1 = \delta_2 = 0(m/s),$.

The orbit is an ellipse with center in (97.14,0) as shown in Fig 3. The control signals have largely chatting. This is harmful to saving energy. The error in orbit is caused by calculation error. (relative tolerance in calculation is 0.001)



(a) (b) (c)

Fig 3 The results for sliding-mode with constant term

5.4 The sliding-mode with dead region

Choose

$A_1 = A_2 = 0(m)$

$\delta_1 = 0.1(m/s), \delta_2 = 0.11/2(m/s)$.

The orbit is a ellipse with center apart from (0,0) , about at point (-100,0) as shown in Fig 4. There is no chatting in control signals. The cost energy is largely decreased.

(relative tolerance in calculation is 0.00001).

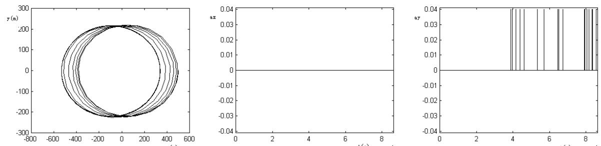


Fig 4 The results for the sliding-mode with dead region

5.5 The sliding-mode control with constant term and dead region

The results shown as Fig 4 are coincident to the **Corollary 1**, and the results shown as Fig 3 are coincident to the **Theorem 3**. They lead to the departure of the center of the ellipse orbit in two opposite directions. Thus, if we choose the parameters as follows

$$A_1 = 0, A_2 = 0.11/2(m)$$

$$\delta_1 = 0.1(m/s), \delta_2 = 0.11/2(m/s)$$

, and the obtained orbit will have the center in a small neighborhood of (0,0). The controller signals have no chatting. The Simulation results are shown in Fig 5.

(relative tolerance in calculation is 0.00001).

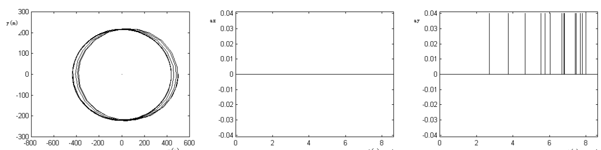


Fig 5 The results of the sliding mode with constant term and dead

5.6 The change of the long axes by switching controller

In our test, we change the length of the long axes by switching controller. At first, we choose

$A_1 = A_2 = 0(m)$, $\delta_1 = 0.1(m/s), \delta_2 = 0.11(m/s)$

and switch to

$$A_1 = 0(m), A_2 = -0.11(m)$$
$$\delta_1 = 0.1(m/s), \delta_2 = 0.11(m/s)$$

when the trajectory arrives at some point in the left side of the orbit. This leads to the longer axes as shown in Fig 6. On the other test, we carry out the same switching at some point in the right side of the orbit. This leads to the shorter axes as shown in Fig 7.

(relative tolerance in calculation is 0.00001).

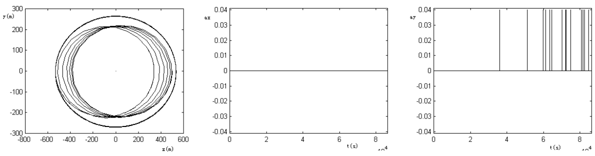


Fig 6 Test for longer axes

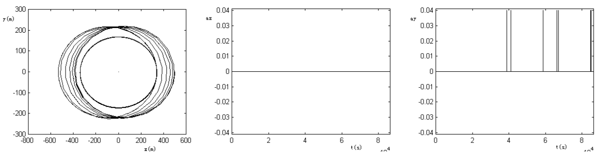


Fig 7 Test for shorter axes

6 Conclusion

In this paper, the stable orbit of the small satellite flying around the space station and its maintenance are studied based on the Hill equation and a sliding mode control approach. Two kinds of sliding modes are proposed to guarantee the stable orbit flying around the space station. One of them can obtain an ellipse orbit with the center at the point (0,0) and the long axes 2 times short one, and another of them can get an ellipse orbit with the center apart from the point (0,0) and the long axes 2 times short one. The sliding mode controller is derived for each sliding-mode, respectively, such that the corresponding closed system can enter and maintain onto the sliding mode in finite time. In view of saving energy and restraining the chatting of the control signals, the sliding mode controllers with dead-region are proposed. For the case of the small satellite flying around the space station under constant disturbances, a lot of simulations are carried out to illustrate the effectiveness of our approaches. It is shown from our simulation results that the center of the orbit may lie to a small neighborhood around the points (0,0) and the length of the long axes may be changed by the properly choosing for the parameters of the controller.

Acknowledgement:

This work is supported by National Natural Science Foundation of China (69934030) and Zhejiang Province Natural Science (ZD9905). The author would like to express his gratitude to the reviewers for their constructive comments and suggestion, which have greatly improved the quality of this paper.

Reference

[1] Lin laixin, Dynamics and control of the small satellite flying around the space station, aerospace, 1999(3), pp26-33.

[2] Lin laixin, Dynamics of the closed-quarters spacecraft, Space Science of China, 1998, 18(2), pp40-47.

[3] U Itkis, 1976, Control system of variable structure, Jeruslam.

[4] Hu jianbo, Su hongye and Chu jian, Nonlinear robust adaptive and variable structure control for trajectory maintenance of satellite, Aerospace, 1999(3), pp1-5.