Automatic Tuning for Classical Step-Response Specifications using Iterative Feedback Tuning

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Abstract

The objective of this contribution is to study how to tune PID controllers with respect to classical step-response specifications using iterative feedback tuning. Typically the closed-loop response is improved considerably using only six to nine closed-loop experiments.

1 Introduction

Many specifications for controller design can be expressed in terms of performance measures. Typically the most critical performance measure is minimized with respect to constraints on the other ones. For linear plants and a general controller structure this can be addressed with convex optimization as in [2]. A very attractive way of doing this is to use Iterative Feedback Tuning (IFT), e.g. [3, 6, 5]. An advantage of this method is that no model is needed. Instead a series of online closed loop experiments are performed to compute the gradient of the performance measure with respect to the controller parameters. IFT has recently been extended to consider also constrained optimization problems using Interior Point (IP) methods, [4]. So far only one constraint has been considered. The aim of this work is to show that it is only marginally more difficult to consider several constraints. In fact, we will consider constraints at each sampling instant for both the control signal and the measurement signal, i.e. envelope constraints. These specifications can easily be related to classical step-response specifications such as rise time, overshoot, and settling time. Hence the method can be used at different levels of sophistication.

2 Performance Specifications

Consider a plant described by the transfer function $G(q)$ and controlled by a controller $C(q;\rho)$ where $\rho \in \mathbb{R}^n$ is a design parameter and q is the forward shift operator. The objective is to control y_k, which is the measured output, with the control signal u_k. Define $y(\rho) = [y_k(\rho)]_{k=1,\dots,N} \in \mathbb{R}^N$ and similar for the control signal. Among the specifications that will be used in this paper are slight modifications of the classical step

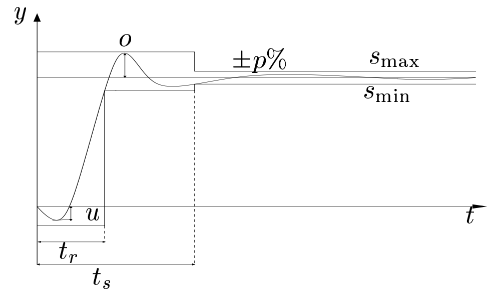

Figure 1: *Step-response specification.*

response specifications as defined in [2], see Figure 1. These specification are special cases of general envelope constraints on the step-response, with limits $s_{\max}$ and $s_{\min}$, as also can be seen in Figure 1. There might be many controllers, i.e parameter values ρ, that will meet the above specifications. Among all these controllers it could be desirable to choose one that has a steady state control error equal to zero. This could e.g. be accomplished by introducing a performance measure such as

$$J_e(y(\rho)) = \frac{1}{2N}\sum_{k=1}^{N}\alpha_k(r_k - y_k(\rho))^2 \qquad (1)$$

where r_k is the reference value and α_k is a weighting function used to trade off for what time values to make the control error small. Typically, one would like the control error to be small for $k > t_s$. Minimizing the performance measure with respect to the envelope specifications would now yield a controller meeting the envelope specifications with "minimal control error". Of course there are many more specifications that are relevant and which will fit into the above problem class. For example it might be desirable to keep the control signal and its rate of change below certain values because of limitations in the actuator. This can be formalized as

$$u_{\min} \le u \le u_{\max}; \qquad \Delta u_{\min} \le \Delta u \le \Delta u_{\max}$$

where $\Delta u_k = u_k - u_{k-1}$ and where $\le$ denotes component wise inequality. Also the performance measure can be extended in order to penalize, e.g. changes in u:

$$J(u(\rho), y(\rho)) = \frac{1}{2N}\sum_{k=1}^{N}\alpha_k(r_k - y_k(\rho))^2 + \beta_k \Delta u_k^2(\rho)$$

Here β_k is a weighting function, which define where a rapid change in the control signal is not desired. All the above specifications can be summarized in the optimization problem

$$\begin{aligned} &\text{minimize } J(u(\rho), y(\rho)) \\ &\text{subject to } s_{\min} \leq y(\rho) \leq s_{\max} \\ &\qquad u_{\min} \leq u(\rho) \leq u_{\max} \\ &\qquad \Delta u_{\min} \leq H u(\rho) \leq \Delta u_{\max} \end{aligned}$$

where H is a matrix of zeros and $\pm$ 1's.

3 Example

In this section it will be shown how the proposed algorithm works on a experimental system, the double-tank process similar to the one described in [1]. The process is controlled with a PID controller given by

$$u(\rho) = K\left(br - y + \frac{K_i}{q-1}(r-y) - Kd\frac{q-1}{2q-1}y\right)$$

where $\rho = [K_p \quad b \quad K_i \quad K_d]^T$. The performance measure that is used in this example is (1) with $\alpha_k = k^2$. The constraints imposed can be seen in Figure 3.

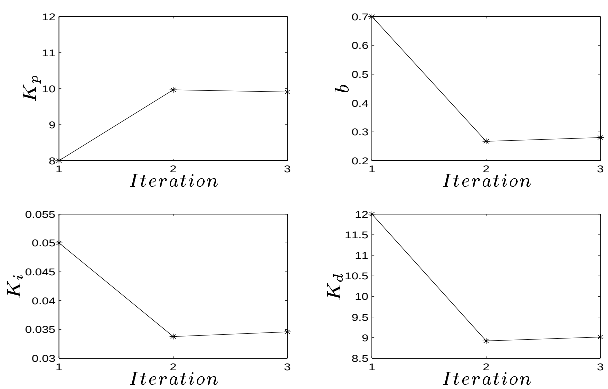

Figure 2: *Convergence of the controller parameters*

To produce the gradients for the optimization algorithm IFT is used. In the case of a two degree freedom controller three experiments must be performed to produce the gradient, see [5]. The first experiment is a step response with a load disturbance at 350 s. In the second and third experiment the reference and measurement signal from the first experiment is feed to the process as input load disturbances. The optimization was done with the builtin MATLAB optimization function `fmincon`. As can be seen in Figure 2 the parameters of the controller converge after only two iterations. In Figure 3 the closed-loop step response in shown for the initial and final tuning. As can be seen the initial parameters produce a step response that is outside both the constraints for the control and measurement signal, whereas the final response lies well within the constraints.

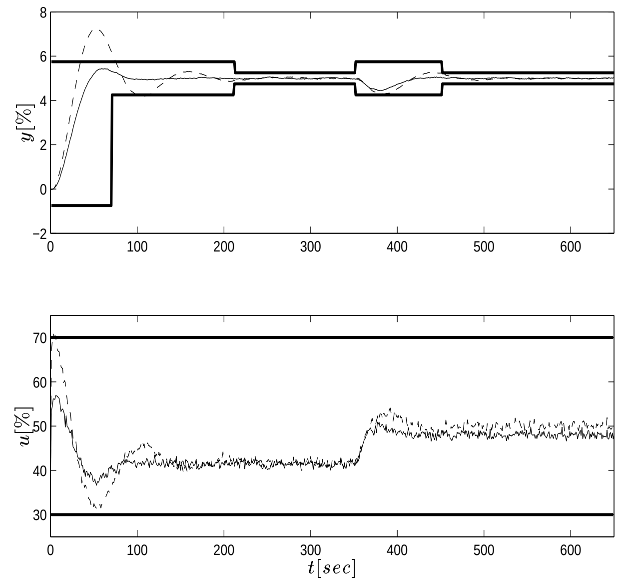

Figure 3: *The closed-loop step response and load disturbance with constraints. Dashed line: Initial tuning. Solid line: optimal tuning. Upper: Measurement signal. Lower: Control signal.*

4 Conclusions

In this contribution it has been shown that the classical step response specifications can be formalized as constraints in an optimization problem. The optimization problem is solved using IFT to produce the gradients of the performance measure and constraints. The improvement of the step-response is substantial in only a few iterations. Typically three experiments are performed in each iteration.

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