Nonconservative Sliding Mode Control with the Feedback Linearization or Nonlinear System

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Abstract: An advantage of the feedback linearization technique is to make linear control theories can be used for nonlinear system. This advantage disappear when the SMC is used with the feedback linearization for uncertain nonlinear systems because the SMC can not be combined with a linear controller. In this paper, by defining a novel sliding surface, it is made possible that the feedback linearization technique, a linear controller and the SMC are used together for uncertain nonlinear system and the feedback linearization technique can have the robustness without loosing its advantage.

1. Introduction

The technique of feedback linearization can makes a linear control theories be used for nonlinear control system using the nonlinear coordinate transformation and the nonlinear feedback[1]. This is the advantage of the feedback linearization. For uncertain nonlinear system, the feedback linearization requires the robustness of the SMC . The important property of the SMC is to have the dynamics of the sliding surface[2][3][4]. When the states of the system are on the sliding surface, they are linearly dependent with each other. This meanns the dynamics of the SMC has the dynamics lower order than that of the original system. This makes the linear control theory can not be used with the SMC. This is called as the conservatism of the SMC. The conservatism make he feedback linearization loose its advantage when combined with the SMC. The combination of the linear controller and the SMC was made possible in the recent paper[5]. In that paper, a new sliding mode surface, which has the dynamics of the nominal dynamics of the original system, was proposed by defining a virtual state and constructing an augmented system. In this paper, using the result of the paper, the feedback linearization technique can be used with the linear controller and the SMC for the uncertain nonlinear system. In other word, the feedback linearization can have the robustness of the SMC without loosing its advantage which make the linear controller can be used for the nonlinear system. The input chattering problem of the SMC can be solved by selecting the virtual state appropriately.

2. Problem Formulation

Consider the n-th order system described by

$$\dot{\mathbf{x}}(\mathbf{t}) = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathrm{u}(\mathbf{t}) + \mathbf{h}(\mathbf{x}, \mathbf{t}) \quad (1)$$

where $\mathbf{x} \in \mathbf{R}^{\mathbf{n}}$, $u \in \mathbf{R}$, $\mathbf{h} \in \mathbf{R}^{\mathbf{r}}$ and the uncertainties are bounded by $\|\mathbf{h}(\mathbf{x}, \mathbf{t})\| \prec r(\mathbf{x}, t)$ and satisfies the following matching condition.

$$\mathbf{h}(\mathbf{x}, \mathbf{t}) = \mathbf{g}(\mathbf{x})\mathrm{h}_1(\mathbf{x}, \mathbf{t}) \quad (2)$$

The feedback linearization technique is to transform the above system to the following equation by using the new state variables $z = f(x)$ and a nonlinear feedback control law $u = a(x) + b(x)v$ where $a(x)$, $b(x)$ are appropriate terms which can remove the nonlinear terms and v is a linear control input.

$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{b}v + \mathbf{b}h_1(t) \quad (3)$$

where $\mathbf{A} = \begin{bmatrix} 0\,1\,0 \cdots 0\,0\,0 \\ 0\,0\,1 \cdots 0\,0\,0 \\ \vdots\ \vdots \cdots\ \vdots\ \vdots\ \vdots \\ 0\,0\,0 \cdots 0\,0\,1 \\ 0\,0\,0 \cdots 0\,0\,0 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$

The condition of feedback linearizability follows[1].

The SMC is robust for uncertainties. the feedback linearization technique can be made robust for the uncertain system by using the robustness of the SMC. The existing SMC use the following type of the sliding surface.

$$s(\mathbf{z}) = c_n z_n + c_{(n-1)} z_{(n-1)}(t) \cdots + c_1 z_1(t) = \mathbf{Cz(t)} = \mathbf{0} \quad (4)$$

In the above equation, the states of the SMC system are linearly independent and the order of its dynamics is lower than that of the original system. On the other hand, most system controlled by the other linear control method have more dynamic than the original system. This means that SMC system can not has the same performance as the other control methods. This results that the SMC can not be combined with other linear control method. It is called as the conservatism of the SMC. When the feedback linearization is used with the SMC, the one of the important property, which makes possible that the linear control theory can be used for nonlinear system without approximation, is disappear.

In order to use the linear control theory and the feedback linearization together with SMC for nonlinear system, the problem of conservatism of SMC has be solved.

3. Noble Nonlinear Sliding Surface

Various types of sliding surfaces have been proposed including time-varying sliding mode surface[6]. These existing sliding surfaces can not have the dynamics of the original system controlled by other type of controller. This makes the conventional SMC very conservative to be combined with the other types of control strategies. To overcome this conservatism completely, a new SMC with a novel sliding surface which has the dynamics of the nominal original system, is proposed. The novel sliding surface is designed based on the augmented system which has a virtual state. The virtual state is defined as

$$\dot{z}_V = K\left[z_2 z_3 \ldots z_V\right]^T \quad (5)$$

where K is the state feedback gain of the linear control input $v = Kz$.

The augmented system with virtual state is

$$\mathbf{x(t)} = \mathbf{f(x)} + \mathbf{g(x)u(t)} + \mathbf{h(x,t)}$$

$$\dot{z}_V = k_1 z_2 + k_2 z_3 \cdots + k_n z_v \quad (6)$$

Based on the above system, the novel sliding surface is constructed as

$$s_n(z, z_V) = z_V - k_1 z_1 - k_2 z_2 \cdots - k_n z_n = 0 \quad (7)$$

The initial value of the virtual state is selected as follows to remove the reaching phase.

$$z_V(t_0) = k_1 z_1(t_0) + k_2 z_2(t_0) \cdots + k_n z_n(t_0) \quad (8)$$

Now the following theorem is obtained.

Theorem 1. If the states of the system (6) are on the novel sliding surface (7), then the states of eqn.(1) have the dynamics of nominal system of eqn.(1).

Proof) Similar to the proof in [5]

From Theorem 1 mentioned above and SMC theory, the following result is obtained.

Theorem 2. If SMC input $u(t)$ is designed to force the states of the system onto the sliding surface (7), then the states $\mathbf{x}(t)$ follow the trajectories of the nominal system of eqn.(1).

Proof) It is obvious from the Theorem 1 and SMC theory.

To force the states onto the sliding surface (7), a SMC input, which make following condition satisfied, has to be derived.

$$s_n(\mathbf{z}, z_V)\dot{s}_n(\mathbf{z}, z_V) \prec 0 \quad (16)$$

$\dot{s}_n(\mathbf{z}, z_V)$ is calculated as follows. For simplicity, the uncertainty ΔB is assumed to be zero. It is generalized by using the existing method.

$$\dot{s}_n(\mathbf{z}, z_V) = \dot{z}_V + K\frac{\partial f(x)}{\partial x}\dot{\mathbf{x}}$$

$$= -\mathbf{k_n} z_V - \mathbf{C_0} f(\mathbf{x}) + \mathbf{C_1}(f(x) + g(x)u(t) + h(x,t)) \quad (17)$$

where $\mathbf{C_0} = \left[0 k_1 \ldots k_{n-1}\right]^T$, $\mathbf{C_1} = K\frac{\partial f(x)}{\partial x}$

There exist a positive function satisfying the following condition because the uncertainties is bounded.

$$\left\|\mathbf{C_1 h(x,t)}\right\| \prec r_1(x,t) \quad (18)$$

The following input guarantees the condition of eqn.(23).

$$u(t) = -(\mathbf{C_1 g(x)})^{-1}(-\mathbf{k_n} z_V - \mathbf{C_0} f(\mathbf{x}) + \mathbf{C_1} f(x)) - (\mathbf{C_1 g(x)})^{-1} r(x,t)\operatorname{sgn}(s_n) \quad (19)$$

Note that the nominal control input $\mathbf{u_o}(\mathbf{z}, t)$ can be any type of control input and this makes it possible that the SMC is used with the various types of controllers. This means that the conservatism of the SMC is removed.

4. Conclusions

In this paper, by defining a novel sliding surface, it can be made possible that the feedback linearization, linear control theory and SMC are used together for uncertain nonlinear system. This makae possible the feedback linearization can have the robustness of the SMC without loosing its advantage. The novel sliding surface of the SMC can have the dynamics of the nominal nonlinear system controlled by the feedback linearization technique. So the uncertain nonlinear system controlled by proposed controller can have the same dynamics as the nominal system. The reaching phase is removed by using an initial virtual state which makes the initial sliding function equal to zero

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