ON THE OUTPUT FEEDBACK CONTROL OF PASSIVE NONLINEAR SYSTEMS WITH INPUT PERTURBATIONS

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Abstract

Two problems concerned with output feedback control of passive nonlinear systems under input affine perturbations are considered. The first one is the problem of input-to-state stabilization of passive system with respect to "perturbation" input with semiglobal ISS gain assignment. The second problem is ultimate boundedness control of nominally passive nonlinear systems with structured uncertainties satisfying a matching condition. The key assumption is output-to-state stability of the system. It is shown that under this assumption both problems are solvable by almost smooth static output feedback.

1 Introduction

Let $|\cdot|$ and $\|\cdot\|_p$ be usual Euclidean norm and L_p-norm respectively. By $z_{[t_1,t_2]}$ we denote cut-off function z on interval $[t_1, t_2]$. In particular

$$\|z_{[0,t]}\|_\infty \equiv \underset{s\in[0,t]}{esssup}|z(s)|.$$

Denote $R^+ = [0, +\infty)$. A function $\gamma: R^+ \to R^+$ is *of class* $\mathcal{K}$ ($\gamma \in \mathcal{K}$) if it is continuous, strictly increasing and $f(0) = 0$; γ is *of class* $\mathcal{K}_\infty$ ($\gamma \in \mathcal{K}_\infty$) if $\gamma \in \mathcal{K}$ and $\gamma(s) \to \infty$ as $s \to \infty$. Also, a function $\beta: R^+ \times R^+ \to R^+$ is *of class* $\mathcal{KL}$ if for each fixed $t \geq 0$, $f(\cdot, t)$ is a klass $\mathcal{K}$ function and for any fixed s, $f(s,t)$ is decreasing and tends to zero as $t \to \infty$. A function $f: R^n \to R^m$ is said to be *almost smooth* if $f \in C^0(R^n) \cap C^1(R^n \setminus \{0\})$. Landau symbols O and o will be used in it's usual sence.

Consider an affine control system

$$\begin{array}{rcl} \dot{x} &=& f(x) + g(x)u, \\ y &=& h(x), \end{array} \qquad (1)$$

where $x \in X = R^n$ is a state, $u, y \in R^m$ are control input and measurable output respectively, f, h, and g_j, $j = 1, \ldots, m$ are smooth vector fields of appropriate dimensions. Assume $f(0) = 0$, and $h(0) = 0$. The set $\mathcal{U}$ of admissible control input functions consists of all piecewise continuous functions $u: R^+ \to R^m$. By $x(x_0, u, t)$ denote the value at time t of the solution of (1) with initial condition $x(0) = x_0$ and input function $u \in \mathcal{U}$.

Definition 1. The system (1) is called *passive* if there exists a C^0 storage function $V: X \to R^+$, $V(0) = 0$ such that for any initial condition $x(0) \in X$ and for any input function $u \in \mathcal{U}$ the inequality

$$V(x(t)) - V(x_0) \leq \int_0^t y^T(s)u(s)ds \qquad (2)$$

holds for all $t \in [0, t_*)$, where $[0, t_*)$ is maximal interval of existence of the solution $x(t) = x(x_0, u, t)$.

Definition 2. [1] The system

$$\dot{x} = f(x, u),$$

is said to be input-to-state stable (ISS), if there exist some $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$ such that

$$|x(t)| \leq \max\left\{\beta(|x(0)|, t), \gamma\left(\|u_{[0,t]}\|_\infty\right)\right\} \qquad (3)$$

for all $x(0) \in R^n$ and all $t \geq 0$. Function γ is called ISS gain.

The following definition introduced by Sontag and Wang describes an useful detectability-type property of the system.

Definition 3. [2]. The system (1) is output-to-state stable (OSS) if there exist some $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$ such that for all $u \in \mathcal{U}$

$$|x(t)| \leq \max\left\{\beta(|x(0)|, t), \gamma\left(\|y_{[0,t]}\|_\infty\right)\right\} \qquad (4)$$

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for all $x(0) \in R^n$ and all $t \geq 0$.

Below we restrict our consideration to a class of affine control systems with globally defined normal form. Namely, consider the following properties of the system (1) [3, 4]:

H1: the matrix $L_g h(x)$ is nonsingular for each $x \in X$,

H2: the vector fields $\tilde{g}_1(x), \ldots, \tilde{g}_m(x)$ defined by

$$[\tilde{g}_1(x), \ldots, \tilde{g}_m(x)] = g(x) \left[L_g h(x)\right]^{-1}$$

are complete.

H3: the vector fields $\tilde{g}_1(x), \ldots, \tilde{g}_m(x)$ commute.

The fulfillment of the properties H1 – H3 is equivalent to existence of globally defined diffeomorphism which transforms the system (1) into a system having normal form [3]:

$$\begin{array}{rcl} \dot{z} & = & q(z, y), \\ \dot{y} & = & b(z, y) + a(z, y)u, \end{array} \quad (5)$$

where the matrix $a(z, y)$ is nonsingular for all (z, y).

2 ISS gain assignment by static output feedback

Consuder an affine nonlinear control system

$$\begin{array}{rcl} \dot{x} & = & f(x) + g(x)\left[u + d\right], \\ y & = & h(x), \end{array} \quad (6)$$

which differs from (1) by additional exogenous input d. Suppose the following assumptions are valid.

Assumption 1. The system (6) with $d \equiv 0$ is passive with radially unbounded storage function $V \in C^1$.

Assumption 2. The system (6) is output-to-state-stable.

Assumption 3. H1 – H3 hold for (6).

Problem of input-to-state stabilization with semiglobal ISS gain assignment by almost smooth static output feedback for the system (6) is formulated as follows: for given function $\gamma^* \in K$ and positive constant d^* find a static output feedback $u = \phi(y)$ s.t. the closed loop system is input-to-state stable with respect to input d with ISS gain function $\gamma \in K_\infty$, satisfying the condition $\gamma(r) \leq \gamma^*(r)$ for all $r \in [0, d^*]$.

Theorem 1. Under the Assumptions 1 – 3 the problem of input-to-state stabilization with semiglobal ISS gain assignment by almost smooth static output feedback is solvable for the system (6). In particular, there exists a continuous function $\phi^*: R^+ \to R^+$, $\phi^*(0) = 0$ such that for arbitrary continuous function $\phi: R^+ \to R^+$, satisfying $\phi(0) = 0$, $\phi \in C^1(0, +\infty)$, and $\phi(r) \geq \phi^*(r)$ for all r, the control law

$$u = -\phi(|y|)\frac{y}{|y|} \quad (7)$$

solves the problem.

3 Stabilization under structured uncertainties

Now we address to a problem of output feedback stabilization of nominally passive nonlinear system in presence of structured uncertainties. Consider the system (6), and suppose d represents an uncertain part of the system. Namely, let

$$d(t) = \chi\left(t, x(\cdot)|_0^t, u(\cdot)|_0^t\right), \quad (8)$$

where the function χ satisfies the condition

$$\chi\left(t, x(\cdot)|_0^t, u(\cdot)|_0^t\right) \leq \alpha_0\left(|x(t)|\right) + D_1|u(t)| + D_2, \quad (9)$$

where α_0 is an arbitrary $\mathcal{K}$-class function, and D_1, D_2 are nonnegative constants, $D_1 < 1$.

Theorem 2. Under the Assumptions 1-3 there exists a continuous function $\phi^*: R^+ \to R^+$, $\phi^*(0) = 0$ such that for arbitrary continuous function $\phi: R^+ \to R^+$, satisfying $\phi(0) = 0$, $\phi \in C^1(0, +\infty)$, and $\phi(r) \geq \phi^*(r)$ for all r, the control law of the form (7) makes the trajectories of the closed loop system (6) – (9) ultimately bounded. If $D_2 = 0$, the closed loop system is globally asymptotically stable.

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