Dual composition control of a binary distillation columns based on nonlinear state observer

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Abstract

Dual composition control of binary distillation columns is proposed and solved based on observer design and an elementary state elimination procedure. A particular observer allows us to recover the unmeasured feed composition, which is used in the controller design in order to maintain desired compositions. The method is an alternative against the disturbance decoupling.

1 Binary Distillation Column

A binary system (two components) is assumed with constant relative volatility throughout the column. Any delay time will be neglected. The vapor rate $v(t)$ through all trays of the column is the same, the liquid flow rate leaving the trays is $l(t)$. The measured feed flow rate is $f(t)$, and the unmeasured composition is $z(t)$ at time t.

Thus, the nonlinear state equations are

$$\begin{aligned}
H_1\dot{x}_1 &= -v(t)x_1 + v(t)k(x_2),\\
H_i\dot{x}_i &= l(t)\left(x_{i-1} - x_i\right) +\\
&\quad +v(t)\left(k(x_{i+1}) - k(x_i)\right),\ i = 2,\ldots,m-1,\\
H_m\dot{x}_m &= l(t)\left(x_{m-1} - x_m\right) - f(t)x_m +\\
&\quad +v(t)\left(k(x_{m+1}) - k(x_m)\right) + f(t)z,\\
H_i\dot{x}_i &= l(t)\left(x_{i-1} - x_i\right) + f(t)(x_{i-1} - x_i) +\\
&\quad +v(t)\left(k(x_{i+1}) - k(x_i)\right),\\
&\quad i = m+1,\ldots,n-1,\\
H_n\dot{x}_n &= l(t)\left(x_{n-1} - x_n\right) + f(t)(x_{n-1} - x_n) +\\
&\quad +v(t)x_n - v(t)k(x_n),
\end{aligned}$$

where $y_1 = x_1$, $z_1 = x_n$ are the top and the bottom components, respectively. Our purpose is to design a nonlinear observer to recover the unmeasured disturbance $z(t)$, in order to control more robustly, the desired output compositions y_1 and z_1.

2 Observer design

We notice that from the last equation

$$\begin{aligned}
\left(l(t) + f(t)\right)x_{n-1} &= H_n\dot{z}_1 + \left(l(t) + f(t)\right)z_1 -\\
&\quad -v(t)z_1 + v(t)k(z_1), \qquad (1)
\end{aligned}$$

hence x_{n-1} can be expressed by z_1. Let us define new, computed outputs $z_2, z_3, \ldots, z_{n-m+2}$ and $y_2, y_3, \ldots, y_{m+1}$ in order to express the unmeasured states $x_2, x_3, \ldots, x_{n-1}$, similarly to expression (1):

$$\begin{aligned}
z_I &= \sum_{i=n-I+2}^{n} H_i\dot{x}_i +\\
&\quad +(l+f-v)x_n,\ 2 < I \le n-m+2,\\
y_2 &= -(H_1\dot{x}_1 + vx_1),\\
y_I &= l(x_1 - x_{I-1}) - vx_1 -\\
&\quad -\sum_{i=1}^{I-1} H_i\dot{x}_i,\ 2 < I \le m+1.
\end{aligned}$$

Then, recursively

$$\begin{aligned}
x_I &= \frac{1}{l+f}\left(z_{n-I+1} + vk(x_{I+1})\right),\\
&\quad I = n-1, n-2, \ldots, m,\\
x_{m-1} &= \frac{1}{l}\left(z_{n-m+2} + vk(x_m) - fz\right).
\end{aligned}$$

On the other hand,

$$x_I = -\frac{y_I}{\alpha v + (\alpha-1)y_I}, I = 2,\ldots,m.$$

Let us consider the sum of all equations of the dynamics using the expressions from y_{m+1} and z_{n-m+1}:

$$z_{n-m+1} - y_{m+1} - lx_m = fz \qquad (2)$$

Then, the observer is

$$H_1\dot{\hat{x}}_1 = -(v+1)\hat{x}_1 + vk\left(\frac{-y_2}{\alpha v + (\alpha-1)y_2}\right) + y_1$$

$$H_i\dot{\hat{x}}_i = -(l+1)\hat{x}_i + v\left(k\left(\frac{-y_{i+1}}{\alpha v+(\alpha-1)y_{i+1}}\right) - \right.$$
$$\left. -k\left(\frac{-y_i}{\alpha v+(\alpha-1)y_i}\right)\right) - \frac{l y_{i-1}}{\alpha v+(\alpha-1)y_{i-1}}$$
$$-\frac{y_i}{\alpha v+(\alpha-1)y_i},\ i=2,\dots,m-1,$$

$$H_m\dot{\hat{x}}_m = -(l+f+1)\hat{x}_m + v\Bigg(k(x_{m+1}) -$$
$$-k\left(\frac{-y_m}{\alpha v+(\alpha-1)y_m}\right)\Bigg) - \frac{l y_{m-1}}{\alpha v+(\alpha-1)y_{m-1}}$$
$$-\frac{(l-1)y_m}{\alpha v+(\alpha-1)y_m} + z$$

$$H_m\dot{\hat{x}}_m = -(l+f+1)\hat{x}_m + v\Bigg(k(x_{m+1}) -$$
$$-k\left(\frac{-y_m}{\alpha v+(\alpha-1)y_m}\right)\Bigg) - \frac{l y_{m-1}}{\alpha v+(\alpha-1)y_{m-1}}$$
$$-\frac{(l-1)y_m}{\alpha v+(\alpha-1)y_m} + z_{n-m+1}.$$

Here formula (2) is expressed in terms of the outputs, x_{m+1} is also computed recursively from $z_1,\dots,z_{n-m}$. Analogously, for $i=m+1,\dots,n-1$, the corresponding x_i, x_{i+1} can be computed from outputs $z_1, z_2, \dots$

The observer must be stable, which means that the error $e(t) = x(t) - \hat{x}(t)$ must tend to zero at infinity. In our case, the vapor rate $v(t)$, liquid flow rate $l(t)$, feed flow rate $f(t)$, and the physical constants $H_i,\ i=1,\dots,n$ are positive, hence the error equation is asymptotically (exponentially) stable.

3 Controller design

The dual composition problem supposes the measurement (modelling) of the quality of the top or bottom product. The quality may depend on the states, and the disturbance terms. That dependence can be identified, and it is supposed to be functions of all variables

$$y_3(t) = g_{\mathrm{Top}}(x(t), v(t), l(t), f(t), z(t)),$$
$$y_4(t) = g_{\mathrm{Bottom}}(x(t), v(t), l(t), f(t), z(t)).$$

The purpose is, to control the distillation column such that $y_3(t) = \mathcal{C}_1$, $y_4(t) = \mathcal{C}_2$, using the computed disturbance term. First, let us show the case $n=5$, as an example. In this case, the computed outputs are

$$\begin{aligned}
y_2 &= -(H_1\dot{x}_1 + v x_1) = -(H_1\dot{y}_1 + v y_1),\\
x_2 &= -\frac{y_2}{\alpha v+(\alpha-1)y_2},\\
y_3 &= l(x_1-x_2) - v x_1 - H_1\dot{x}_1 - H_2\dot{x}_2,\\
y_4 &= l(x_1-x_3) - f x_3 - v x_1 - H_1\dot{x}_1 - H_2\dot{x}_2 - H_3\dot{x}_3,\\
z_2 &= H_5\dot{x}_5 + (l+f-v)x_5,\\
z_3 &= H_4\dot{x}_4 + H_5\dot{x}_5 + (l+f-v)x_5,\\
z_4 &= H_3\dot{x}_3 + H_4\dot{x}_4 + H_5\dot{x}_5 + (l+f-v)x_5.
\end{aligned}$$

Hence,

$$fz = z_3 - y_4 - l x_3.$$

Now, let us suppose that the desired output compositions $y_1 = y_c$, $z_1 = z_c$ are constant. Then, $\dot{x}_1 = 0$, $\dot{x}_5 = 0$, and from $v \neq 0$ and $0 = H_1 0 = -v y_c + v k(x_2)$, $x_2 = \frac{y_c}{\alpha v-(\alpha-1)y_c}$, obviously $y_c \neq \frac{\alpha}{\alpha-1}$ is supposed. x_2 is also a constant. Therefore the system dynamics is reduced to

$$0 = l\left(y_c - \frac{y_c}{\alpha-(\alpha-1)y_c}\right) + v(k(x_3) - y_c), \qquad (3)$$

$$0 = (l+f)(x_4 - z_c) + v z_c - v(k(z_c)) \qquad (4)$$

By substituting the unmeasured term

$$fz = H_3\dot{x}_3 + H_4\dot{x}_4 + (l+f-v)z_c + (v-l)y_c$$

$$\begin{aligned}
H_4\dot{x}_4 &= (l+f)x_3 - l\frac{y_c}{\alpha-(\alpha-1)y_c} + (l-v)y_c -\\
&\quad -v(k(x_4) - k(x_3)) + (v-l-f)z_c. \qquad (5)
\end{aligned}$$

Considering the coupled system (3), (4), (5), x_3, and x_4 can be expressed algebraically in terms of l, v, f. For example,

$$x_4 = \left(1 - \frac{v}{l+f}\right) z_c + \frac{v}{l+f} k(z_c). \qquad (6)$$

and

$$\dot{x}_4 = \frac{d}{dt}\left(\frac{v}{l+f}\right)(k(z_c) - z_c). \qquad (7)$$

Similarly for x_3, $k(x_3)$, and $k(x_4)$.

The obtained equation is the implicit form of the control law with one free control term. For example choosing $l(t)$, $v(t)$ is computed.

4 Conclusions

Dual composition control of a binary distillation columns based on the unmeasured feed composition estimation has been proposed. Unmeasured composition is estimated by using elementary state elimination procedure. This procedure simpler than the disturbance decoupling.

References

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