A New Design Method of Plug-in Adaptive Controller via Root Locus Technique

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Abstract

This paper presents a new design method of Plug-in Adaptive Controller (P-in AC) that can reject periodic disturbance in adaptive manner at selected frequencies independently. Our proposed controller for rejecting disturbance is designed by evaluating the movement of the poles on imaginary axis, and does not need full information about the error system which derives adaptive law. So the design method becomes simpler than our conventional method.

1 Introduction

In many industrial fields, the periodic disturbance having known frequencies often occurs. P-in AC proposed in [1] and [2] can reject such periodic disturbance in adaptive manner, and its structure is similar the external model principle, i.e. the disturbance model is placed outside basic feedback loop. In our previous method [2], however, some procedures and some difficult calculations are needed to design P-in AC. So in this paper we propose simpler design method than [2]. This design method is carried out by evaluating the movement of the poles on imaginary axis, and does not need full information about the error system which drives adaptive law. Throughout this paper, we distinguish between $R(s)$ as real rational functions and $R[s]$ as real polynomials.

2 Problem formulation

Consider a scalar system shown in Figure 1, where $G(s)$ is the continuous-time Linear Time Invariant (LTI) controlled plant. And $v(t)$ is the output signal from P-in AC, which is introduced only to reject disturbance $d(t)$. For such system, we make some assumptions as follows: **(A1)** Disturbance $d(t)$ and input $r(t)$ are described by $a(t) = \boldsymbol{\theta}_a^T \boldsymbol{\zeta}_a(t)$. In this, $\boldsymbol{\theta}_a$ and $\boldsymbol{\zeta}_a(t)$ are defined as

$$\boldsymbol{\theta}_a \triangleq [\alpha_{a1}, \beta_{a1}, \cdots, \alpha_{aM_a}, \beta_{aM_a}]^T \in \boldsymbol{R}^{2M_a},$$

$$\boldsymbol{\zeta}_a(t) \triangleq [\sin\omega_{a1}t, \cos\omega_{a1}t, \cdots, \sin\omega_{aM_a}t, \cos\omega_{aM_a}t]^T,$$

where $a = d, r$. Then $\boldsymbol{\theta}_a$ is unknown and $\boldsymbol{\zeta}_a(t) \in \boldsymbol{R}^{2M_a}$ is composed of available signals. For all $a \in \{d, r\}$ the same frequency does not exist and upper bounds of M_a is known as $\bar{M}_a$. W. l. g. $M_a = \bar{M}_a$. **(A2)** $G(j\omega_{ai}) \neq 0, \forall i \in \{1, \cdots, M_a\}, a \in \{d, r\}$ is satisfied. **(A3)** The

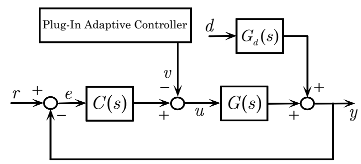

Figure 1: Schematic diagram of Plug-in AC system.

feedback controller $C(s)$ has already been designed so that plant $G(s)$ is stabilized. **(A4)** The disturbance transfer function $G_d(s)$ is stable and unknown.
Now tracking error $e(t)$ in Figure 1 can be described by

$$e(t) = W(s)\big[v(t) - \boldsymbol{\theta}_*^T \boldsymbol{\zeta}(t)\big],\ W(s) \triangleq S(s)G(s), \quad (1)$$

where $S(s) \triangleq \big(1 + G(s)C(s)\big)^{-1}$, $\boldsymbol{\zeta}(t) \triangleq \big[\zeta_r^T(t), \zeta_d^T(t)\big]^T$ and $\boldsymbol{\theta}_* \triangleq \big[-\boldsymbol{\theta}_{*^r}^T, \boldsymbol{\theta}_{*^d}^T\big]^T$ in which parameter vector $\boldsymbol{\theta}_{*^a}$ can be obtained from steady state property of sinusoidal signals. For the error system (1) we propose the structure of P-in AC as follows:

$$v(t) = \hat{\boldsymbol{\theta}}^T(t)\boldsymbol{\zeta}(t),\ \hat{\boldsymbol{\theta}}(t) = -\boldsymbol{\Gamma}(s)\big[\boldsymbol{\zeta}(t)e(t)\big], \quad (2)$$

$$\boldsymbol{\Gamma}(s) \triangleq \text{block diag}\left\{ \begin{bmatrix} \Gamma_{1_{ai}}(s) & -\Gamma_{2_{ai}}(s) \\ \Gamma_{2_{ai}}(s) & \Gamma_{1_{ai}}(s) \end{bmatrix} \right\},$$

where $\hat{\boldsymbol{\theta}}(t)$ shows adaptive parameter and $\boldsymbol{\Gamma}(s)$ is called adaptive transfer function matrix. Our goal is to design $\boldsymbol{\Gamma}(s)$ for the error system (1) so that $e(t) \to 0$ as $t \to \infty$ and the stability of adaptation loop is guaranteed.

3 Design of Plug-in Adaptive Controller

3.1 LTI representation of adaptive structure [3]

Figure 2 shows the adaptive loop got from (1) and (2).
Lemma 1: *In Figure 2, the mapping H from $-e$ to v can be expressed by the linear time invariant operator, i.e. the transfer function described as follows:*

$$v(t) = H(s)\big[-e(t)\big], \quad (3)$$

$$H(s) \triangleq \sum_{i=1}^{M_r} H_{ri}(s) + \sum_{i=1}^{M_d} H_{di}(s), \quad (4)$$

$$H_{ai}(s) \triangleq \frac{\Gamma_{1_{ai}}(s - j\omega_{ai}) + \Gamma_{1_{ai}}(s + j\omega_{ai})}{2} + \frac{\Gamma_{2_{ai}}(s + j\omega_{ai}) - \Gamma_{2_{ai}}(s - j\omega_{ai})}{2j}, \quad (5)$$

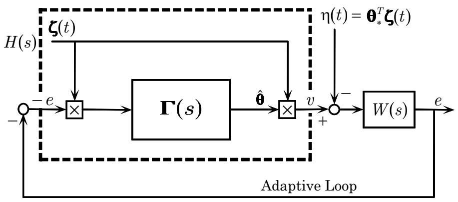

Figure 2: LTI representation of adaptive structure.

where $a = d, r$ in above equations.

Remark 1: If $\boldsymbol{\Gamma}(s)$ possesses integral property, then it is clear from Lemma 1 that LTI system $H(s)$ automatically has internal model of disturbance.

3.2 Design of adaptive transfer function matrix

The design of $\boldsymbol{\Gamma}(s)$ in [2] was attained through the following procedures: **(Step 1)** Design LTI system $H(s)$ for $W(s)$ in Figure 2 with some method. **(Step 2)** Decide the form of $\Gamma_{1_{ai}}(s)$ and $\Gamma_{2_{ai}}(s)$, and calculate $H(s)$ by using Lemma 1. **(Step 3)** Compare $H(s)$ of Step 1 with $H(s)$ of Step 2, and determine the parameters of $\Gamma_{1_{ai}}(s)$ and $\Gamma_{2_{ai}}(s)$. To avoid these troublesome procedures, we present new design method of $\boldsymbol{\Gamma}(s)$.

First, we express $\Gamma_{1_{ai}}(s)$ and $\Gamma_{2_{ai}}(s)$ as follows

$$\Gamma_{1_{ai}}(s) = \frac{\mu\Gamma_{1N_{ai}}[s]}{s\Gamma_{1D_{ai}}[s]} \quad , \quad \Gamma_{2_{ai}}(s) = \frac{\mu\Gamma_{2N_{ai}}[s]}{s\Gamma_{2D_{ai}}[s]}. \qquad (6)$$

They have integrator and μ is one of design parameters. Then the condition to ensure the stability of adaptive loop can be obtained as

$$\mathrm{Re}\left[\frac{1}{j\omega_{ai}}\frac{H_N[j\omega_{ai}]}{H_{mod}[j\omega_{ai}]\,H_D[j\omega_{ai}]}W(j\omega_{ai})\right] > 0, \qquad (7)$$

which is derived by evaluating the movement of poles at $j\omega_{ai}$-axis, that is, the poles at $j\omega_{ai}$-axis must move toward the left-half plane for small μ. In the above condition $H_{mod}[s]$ is polynomial excluding $s^2 + {\omega_{ai}}^2$ from $\prod_{i=1}^{M_r}\left(s^2 + {\omega_{ri}}^2\right)\cdot\prod_{i=1}^{M_d}\left(s^2 + {\omega_{di}}^2\right)$, $H_D[s]$ is a denominator of $H(s)$ excluding $\prod_{i=1}^{M_a}\left(s^2 + {\omega_{ai}}^2\right)$ and $H_N[s]$ is a numerator of $H(s)$ collected with respect to μ. The parameters included in $\Gamma_{kN_{ai}}(s)$ and $\Gamma_{kD_{ai}}(s)$ $(k = 1, 2)$ of (6) are determined by condition (7) and then the parameter μ is set small near zero.

Remark 2: Since we do not need strictly positive real condition to prove the stability, our method can be extended to nonminimum phase plant and/or controller.

Remark 3: In our P-in AC integrator in $\Gamma_{k_{ai}}(s)$ and the summation of $\Gamma_{k_{ai}}(s - j\omega_{ai})$ and $\Gamma_{k_{ai}}(s + j\omega_{ai})$ in (5) make the design problem easier than direct use of $H(s)$ which must include the multiplication of $s^2 + {\omega_{ai}}^2$ in its denominator to reject the disturbance. Furthermore, the condition to determine the parameter of P-in AC depends on frequency response $W(j\omega_{ai})$ only.

4 Numerical examples

Let us consider the case that $G(s) = G_o(s)\big(1 + \Delta(s)\big)$, where $G_o(s)$ is nominal system and $\Delta(s)$ is unknown multiplicative unmodeled dynamics of the plant. Upper bound of $\Delta(s)$ is known as $W_m(s)$: $|\Delta(s)| \le \big|W_m(s)\big|$. These $G_o(s)$, $W_m(s)$ and $C(s)$ which stabilizes $G(s)$ robustly are given by

$$G_o(s) = \frac{s-2}{(s-1)(s+1)} \;,\; W_m(s) = \frac{0.42s+0.14}{s+1}, \qquad (8)$$

$$C(s) = \frac{-1.047\times10^6 s^2 - 2.095\times10^6 s - 1.047\times10^6}{s^3 + 3285s^2 + 1.579\times10^6 s + 1.748\times10^6}.$$

Disturbance $d(t)$ is set by $d(t) = 3\sin 0.3t - 3\cos 0.3t$ where $\omega_{d1} = 0.3$ [rad/sec] is used to design P-in AC, and unknown $G_d(s)$ is expressed as

$$G_d(s) = \begin{cases} \frac{15(s-1)}{s+7.5} \;, & 0 \le t \le 500 \text{ [sec]}. \\ 1 \;, & 500 \le t \le 1000 \text{ [sec]}. \end{cases} \qquad (9)$$

We decide the forms of $\Gamma_{k_{d1}}(s)$ as $\Gamma_{kN_{d1}}(s) = \beta_k$ and $\Gamma_{kD_{d1}}(s) = 1$ where β_k $(k = 1, 2)$ and μ are the design parameters. And the condition (7) can be calculated as $\mathrm{Re}\big[(\beta_1 + j\beta_2)\,W(j\omega_{d1})\big] > 0$. We choose the parameter μ as $\mu = 0.0178$, then we set $\beta_1 = 1.5$ and $\beta_2 = -1.5$. On the simulation, we consider the case that

$$\Delta(s) = \frac{0.32s + 0.04}{1.42s + 1}. \qquad (10)$$

Figure 3 shows the simulation results for both $G_o(s)$ and $G(s)$, and our desired specification is achieved.

5 Conclusions

This paper presented a new design method of plug-in adaptive controller which is based on evaluating the movement of poles on imaginary axis. Our new method is easier to design than the conventional one.

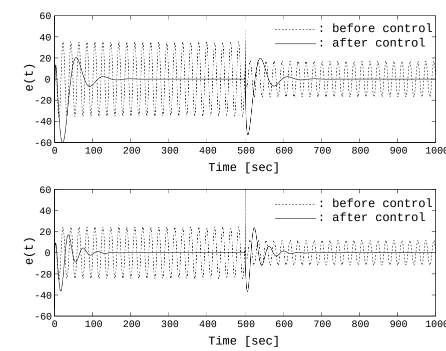

Figure 3: Simulation results for $G_o(s)$ (Top) and for $G(s)$ (Bottom) in the case that $r(t) = 0$.

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