Iterative learning control using adjoint systems for nonlinear non-minimum phase systems

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Abstract

Most of iterative learning control (ILC) using causal updating law obtains the input given by Silverman's or Hirshorn's causal inversion. When the objective system is that of a non-minimum phase, we cannot use those methods because the input is exponentially increasing. To overcome this difficulty, an approach called stable inversion was proposed to give a non-causal but bounded input instead. However, no simple iterative method to obtain this non-causal input was proposed. In this paper, from a viewpoint of minimization, we develop a simple iterative method for the stable inversion toward ILC for non-minimum phase systems.

1 Introduction

Iterative learning control (ILC) is a trial-based approach to obtain an input which makes uncertain systems achieve exact output tracking to given trajectories. In most of the ILC problem settings, it was often assumed that the desired input is the one given by Silverman's or Hirshorn's causal inversion[5, 2]. However, when the system is that of a non-minimum phase, this input diverges exponentially. Conventional ILC using causal updating law can no more be applied to non-minimum phase systems than these causal inversions. In order to avoid the shortcomings of causal inversions, an approach called stable inversion [1, 3] was proposed to give a non-causal but bounded input which achieves exact output tracking. Much work has been published on the theory and applications of stable inversion, while no simple and direct iterative method for the stable inversion was proposed. In this paper, we formalize the stable inversion problem as a minimization problem and develop a simple iterative method toward ILC for non-minimum phase systems.

2 Stable inversion and a minimization problem

Consider a nonlinear system

$$\dot{x}(t) = f[x(t)] + g[x(t)]u(t) \quad (1)$$

$$y(t) = h[x(t)] \quad (2)$$

where $x(\cdot) \in R^n$, $u(\cdot) \in R^q$ and $y(\cdot) \in R^q$; $f_i(x)$, $g_{ij}(x)$ and $h_j(x)$ ($i = 1, \cdots, n$, $j = 1, \cdots, q$) are smooth functions. Assume that the relative degree of the system (1) and (2) is well-defined at $x = 0$ and a desired output trajectory $y_d(\cdot)$ is sufficiently smooth. Then, Hirshorn's causal inversion gives the unbounded input that achieves exact output tracking if (1) and (2) are non-minimum phase. However, the stable inversion technique gives a non-causal but bounded input [1]. It was shown that if $\frac{d^r}{dt^r} y_d(\cdot) \in L_1 \cap L_\infty$ (r: relative degree) and some conditions on the internal dynamics are met, there exists a solution for the stable inversion problem, i.e. a bounded control input $u_d(\cdot)$ and a bounded state trajectory $x_d(\cdot)$ which satisfy (1) and (2) where $y(t) = y_d(t)$ with boundary conditions

$$u_d(t) \to 0, x_d(t) \to 0 \qquad \text{as} \ \ t \to \pm\infty. \quad (3)$$

In the following discussion, we formalize the stable inversion problem as a minimization problem to develop a simple iterative method to find the stable inversion solution. Let's consider a functional

$$J(u) = \int_{-\infty}^{+\infty} (y(t) - y_d(t))^{\mathrm{T}} (y(t) - y_d(t)) dt \quad (4)$$

which gives the minimum value 0 when the exact output tracking is achieved. Then, the stable inversion problem can be solved by searching the minimizer of $J(u)$ subject to $\dot{x} = f(x) + g(x)u$ with $x(-\infty) = 0$. Since the Hamiltonian function for this minimization problem is $H(x, u, \xi, t) = \xi^{\mathrm{T}}\{f(x) + g(x)u(t)\} + (y(t) - y_d(t))^{\mathrm{T}}(y(t) - y_d(t))$ the gradient function of $J(u)$ with respect to $u(\cdot)$ is $\eta = g(x)^{\mathrm{T}}\xi$ where ξ satisfies the adjoint system

$$-\dot{\xi} = \left(f_x(x) + \sum_{i=1}^{q} g_{ix}(x) u_i \right)^{\mathrm{T}} \xi + h_x^{\mathrm{T}}(x)(h(x) - y_d) \quad (5)$$

with $\xi(+\infty) = 0$.

3 ILC using adjoint systems

Based on the discussion in the previous section, we present a simple iterative method to obtain the stable inversion solution:*0) Let $k = 0$ and $u_0 \equiv 0$. 1)*

Integrate the system (1) in forward-time direction with $u(t) = u_k(t)$ and $x(-\infty) = 0$. 2) Integrate the adjoint system (5) in backward-time direction with $\xi(+\infty) = 0$ by using the output error $h(x) - y_d$ and the state trajectory x. 3) Update the input as $u_{k+1} = u_k + \alpha g(x)^T \xi$. 4) Let $k := k+1$ and go to 1.

Although the exact gradient ξ is not available when there exists some modeling error, we can still expect convergence of the iteration when the modeling error is not very large. We clarify a margin of the modeling error for linear cases. Let $q = 1$ (i.e. SISO) and $(f(x), g(x), h(x)) = (Ax, b, cx)$ where A, b and c are appropriate matrices and vectors. Then, the input-output mapping of (1) and (2) with $x(-\infty) = 0$ is defined as $y = Su = \int_{-\infty}^{t} c \exp(A(t-\tau)) b u(\tau) d\tau$. Moreover, the input-output mapping of the adjoint system (5) and $\eta = g^{\mathrm{T}} \xi$ with modeling error is defined as

$$\eta = \hat{S}^* e = \int_{t}^{+\infty} \hat{b}^{\mathrm{T}} \exp(\hat{A}^{\mathrm{T}}(t-\tau)) \hat{c}^{\mathrm{T}} e(\tau) d\tau \qquad (6)$$

Let $S(s) = c(sI - A)b$ and $\hat{S}(s) = \hat{c}(sI - \hat{A})\hat{b}$. Then, by straightforward calculations, we can see that Fourier transformation of the system S and the non-causal system $\hat{S}^*$ are $S(j\omega)$ and $\hat{S}(-j\omega)$, respectively

Theorem 1 *Assume that $\frac{d^r}{dt^r} y_d \in L_1 \cap L_\infty$ where r is the relative degree of $S(s)$ and that $Re\{\hat{S}(-j\omega) S(j\omega)\} > 0$ for all $\omega \in (-\infty, +\infty)$. Then, the sequence of input functions $\{u_k; k = 0, 1, \cdots\}$ generated by the ILC $u_{k+1} = u_k - \alpha \hat{S}^*(Su_k - y_d)$ with $u_0 \equiv 0$ satisfies*

$$\|u_k - u_d\|_2 \to 0 \quad (as \;\; k \to \infty) \qquad (7)$$

where the constant α is chosen as $0 < \alpha < \min_\omega 2Re\{\hat{S}(-j\omega)S(j\omega)\} / |\hat{S}(-j\omega)S(j\omega)|^2$ and $u_d \in L_1 \cap L_\infty$ such that $y_d = Su_d$.

4 Numerical examples

Consider a nonlinear non-minimum phase system[1]:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ -3x_2 + x_1^3 \\ x_1 - 2x_3 \\ -x_4 + x_3^2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 + \sin^2 x_4 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = x_1 - 3x_3 \qquad (8)$$

with a desired trajectory

$$y_d = \begin{cases} 0.2(1 - \cos(t - 10)) & t \in (10, 10 + 2\pi) \\ 0 & \text{otherwise} \end{cases} \qquad (9)$$

Then, the systems for input updating are $-\dot{\xi} = F(x,u)\xi + \begin{bmatrix} 1 & 0 & -3 & 0 \end{bmatrix}^{\mathrm{T}} (y - y_d)$ and $\eta = \begin{bmatrix} 0 & 2 + (1+e_2)\sin^2 x_4 & 0 & 0 \end{bmatrix} \xi$ where $F(x,u) =$

$$\begin{bmatrix} -(1+e_1) & 3(1+e_0)x_1^2 & 1 & 0 \\ 1 & -3 & 0 & 0 \\ 0 & 0 & -2 & 2x_3 \\ 0 & u(1+e_2)(2\sin x_4 \cos x_4) & 0 & -1 \end{bmatrix}$$

e_0, e_1 and e_2 represent the modeling error if they are nonzero. Figure 1 shows the simulation result when the adjoint system with the error $(e_0, e_1, e_2) = (2.0, 1.0, 2.0)$ is used for updating the input function with $\alpha = 2.0$; the input and output are plotted for $k = 5, 10, 20, 30$.

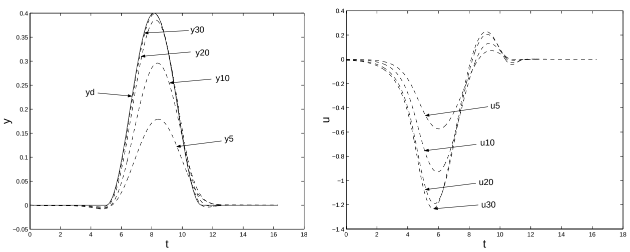

Figure 1: Simulation results with the modeling error

5 Concluding remarks

In this paper, we developed a simple iteration to solve the stable inversion problem from a viewpoint of minimization. We presented a margin of modeling error for convergence and demonstrated that the iterative method can be used as a ILC scheme for non-minimum phase systems. Effect of truncation of the time horizon was discussed in another paper [4].

References

[1] S. Devasia, D. Chen, and B. Paden. Nonlinear inversion-based output tracking. *IEEE Transactions on Automatic Control*, 41(7):930–942, 1996.

[2] R. M. Hirschorn. Invertibility of nonlinear control systems. *SIAM J. Control and Optimization*, 17(2):289–297, 1979.

[3] L. R. Hunt and G. Meyer. Stable inversion for nonlinear systems. *Automatica*, 33(8):1549–1554, 1997.

[4] K. Kinoshita, T. Sogo, and N. Adachi. Iterative learning control using noncausal updating for non-minimum phase systems. *Proceedings of the 3rd Asian Control Conference*, pages 1548–1553, 2000.

[5] L. M. Silverman. Inversion of multivariable linear systems. *IEEE transactions on automatic control*, AC-14(3):270–276, 1969.