Forced Oscillations in Reset Control Systems[1]

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Abstract
In this paper we analyze oscillations forced by sinusoidal sensor noise.

1 Introduction

Reset controllers are linear systems that reset some or all of their states to zero based on a given reset law. Examples of reset controllers include the Clegg integrator [1] and first-order reset element (FORE) [2]. The motivating research in [2] introduced design guidelines for reset controllers and simulated their potential to improve on linear control: reset reduced the overshoot in a linear control system without sacrificing its disturbance-rejection or sensor-noise suppression performance. These properties were experimentally confirmed in [3] where a FORE was designed for a tape-speed control system. In [4] we established asymptotic stability results for reset control systems under constant inputs. Bounded-input bounded-output stability of reset systems is addressed in [5]. In this paper, we study their response to sinusoidal inputs. Our motivation is to establish reset control system response to (sinusoidal) sensor noise.

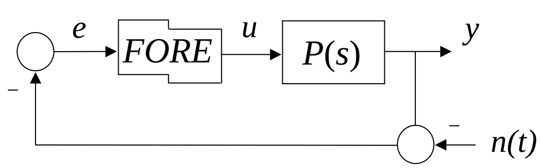

Figure 1: Block diagram of a reset system

The reset control system we consider is shown in Figure 1 where the FORE element is described by the impulsive differential equation

$$\begin{aligned} \dot{x}_r &= -bx_r + e; \quad e \neq 0 \\ x_r &= 0; \quad\quad\quad\quad e = 0 \\ u &= x_r; \end{aligned}$$

where x_r is the state, u is the reset element's output and $b > 0$ is the FORE's pole. The linear plant P has transfer function

$$P(s) = \frac{y(s)}{u(s)} = \frac{(s+b)\omega_n^2}{s(s+2\zeta\omega_n)}$$

where $\zeta > 0$ and $\omega_n > 0$ denote damping ratio and natural frequency, respectively. Since the FORE's pole b appears as a zero of $P(s)$, in the absence of resetting, the resulting *base-linear system* has complementary sensitivity transfer function $T(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$. We model the linear plant by the state equation

$$\dot{x}_p = A_p x_p + B_p u; \quad y = C_p x_p$$

where x_p is the plant state and

$$\begin{aligned} A_p &= \begin{bmatrix} -2\zeta\omega_n & 1 \\ 0 & 0 \end{bmatrix}; \quad B_p = \begin{bmatrix} 1 \\ b \end{bmatrix}; \\ C_p &= \begin{bmatrix} \omega_n^2 & 0 \end{bmatrix}. \end{aligned}$$

Let the sinusoidal sensor noise $n(t) = \sin(\omega_o t + \phi)$ be generated by the output of the state equation

$$\begin{aligned} \dot{x}_{osc} &= A_{osc} x_{osc}; \quad x_{osc}(0) = \begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix} \\ n &= C_{osc} x_{osc} \end{aligned}$$

where x_{osc} is the oscillator state and

$$A_{osc} = \begin{bmatrix} 0 & -\omega_o \\ \omega_o & 0 \end{bmatrix}; \quad C_{osc} = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

We can then describe the reset control system in Figure 1 by the impulsive differential equation

$$\begin{aligned} \dot{x} &= A_{c\ell} x; \quad x \notin \mathcal{M} \\ x &= A_R x; \quad x \in \mathcal{M} \end{aligned} \tag{1}$$

where $x = [x_{osc}' \;\; x_p' \;\; x_r]'$ is the closed-loop state,

$$A_{c\ell} \triangleq \begin{bmatrix} A_{osc} & 0 & 0 \\ 0 & A_p & B_p \\ C_{osc} & -C_p & -b \end{bmatrix}, \quad A_R \triangleq \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and where $\mathcal{M}$ is the *set of reset states*

$$\{\xi : \begin{bmatrix} C_{osc} & -C_p & 0 \end{bmatrix} \xi = 0\,; \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \xi \neq 0\}.$$

2 Limit Cycle Analysis

From simulations we have observed that the response of (1) exhibits limit-cycle behavior, of complexity depending on the sensor noise's frequency relative to the base-linear system's bandwidth. In this section we will give conditions under which these limit cycles are simple with period $\frac{\pi}{\omega_o}$.

[1]This material is based upon work supported by the National Science Foundation under Grant No. CMS-9800612.

2.1 Limit Cycle Generators

Of the various limit cycles observed in simulations, we focus on so-called *simple limit cycles*, illustrated in Figure 2, which are symmetric and have only two resets per limit cycle. We denote one of these reset states z; due to symmetry, the other reset state is $-z$. The period of this simple limit cycle has been observed to be that of the sinusoidal sensor noise, $\frac{\pi}{\omega_o}$. Therefore, our first step in limit-cycle analysis is to seek a state $x \in \mathcal{M}$ satisfying

$$-z = e^{A_{c\ell}\frac{\pi}{\omega_o}} A_R z. \tag{2}$$

We call such z a *limit cycle generator*. Since we limit the oscillator states to unit norm, z is constrained by $\| \begin{bmatrix} z_1 & z_2 \end{bmatrix} \| = 1$. We define

$$\Theta \triangleq \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\omega_n^2}{\sqrt{1+\omega_n^4}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{1+\omega_n^4}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

whose columns span the closure of $\mathcal{M}$. Then, (2) and $z \in \mathcal{M}$ can be equivalently expressed as the generalized eigenvalue problem: find $x_z \in \mathbb{R}^4$ such that

$$-\Theta x_z = e^{A_{c\ell}\frac{\pi}{\omega_o}} A_R \Theta x_z, \tag{3}$$

where $z = \Theta x_z$. We can show that -1 is always an eigenvalue of the generalized eigenvalue problem (3) and that its associated eigenspace is one dimensional. Hence such x_z always exists; see [6]. We summarize in the following proposition.

Proposition 1: *The reset control system described in* (1) *always possesses a limit cycle generator.*

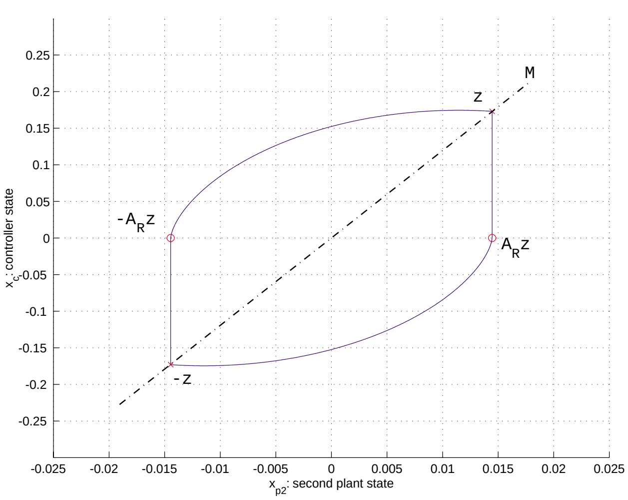

Figure 2: An example of a simple limit cycle

2.2 What Makes a Limit Cycle Simple?

We now develop a condition to guarantee that the limit cycle is simple. We must rule out the existence of intermediate resets, i.e., pairs $(t^*, x) \in \left(0, \frac{\pi}{\omega_o}\right) \times \mathcal{M}$ such that $e^{A_{c\ell}t^*} A_R z = x$. In [6] we construct a function f and show that no intermediate resets exist if $f(t) \neq 0$ for all $t \in \left(0, \frac{\pi}{\omega_o}\right)$. For brevity we omit the rather tedious expression of this function (see [6]). We note that $\lim_{\omega_o \to \infty} f(t) = \lim_{\omega_o \to \infty} -\frac{1}{\omega_n^2} \sin(\omega_o t)$, hence $f(t) \neq 0$ over $t \in \left(0, \frac{\pi}{\omega_o}\right)$ for sufficiently large ω_o implying that simple limit cycles exist for high-frequency sensor noise. We summarize in the following proposition.

Proposition 2: *The reset control system described in* (1) *possesses a simple limit cycle with period* $\frac{\pi}{\omega_o}$ *if* $f(t) \neq 0$ *for all* $t \in \left(0, \frac{\pi}{\omega_o}\right)$. *Furthermore, this condition is satisfied when the sensor noise is of sufficiently high frequency* ω_o.

3 Conclusion

In this paper we have analyzed the response of a class of reset control systems to sinusoidal sensor noise. We showed that limit cycles exist and gave a condition under which they are simple. Immediate future research directions include the stability analysis of limit cycles.

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