Bootstrap Filtering for the Position Location Using Wireless Communication on Highways

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Abstract

We propose a new position location algorithm based on the bootstrap filtering using the time difference of arrival (TDOA) measurements. The proposed algorithm imposes nonlinear kinematic constraints on the state estimates without destabilizing the algorithm. Such constraints can be most naturally incorporated in the Bayesian bootstrap filtering framework. The proposed algorithm is verified through simulation, and the result demonstrates that our algorithm is more robust than the extended Kalman filter and the bootstrap filter without constraints.

1 Introduction

The problem of position location is to determine the position of a radiating source by processing received signals using passive sensors. To determine the position relative to the sensors from the time differences of arrival (TDOA's) and/or the differential Doppler measurements, many researchers have suggested various approaches such as the maximum-likelihood estimation [1], the extended Kalman filter (EKF) [2] and the least squares estimation [3]. In these methods, insufficient measurements may force the system to be unobservable. Particularly, although the EKF provides a suboptimal solution of to the nonlinear estimation problem, lack of measurements cause the divergence problem due to the unobservability.

In this paper, we present a new position location algorithm using the measured TDOA's. The algorithm is based on the recursive Bayesian bootstrap filter [4] with nonlinear kinematic constraints on the state estimates. The specific problem being addressed in this paper is that of estimating the position of a mobile transmitter as a function of time. To check the performance of the proposed algorithm, we consider the unobservable situation due to missing measurements. Then the proposed algorithm is compared with the EKF and the bootstrap filter without constraints.

In Section 2, we describe the system dynamics, and the measurement model. The nonlinear kinemetic state constraint is introduced in Section 3, and simulation results are presented to demonstrate the accuracy of the proposed algorithm in Section 4.

2 System Dynamic and Measurement Models

The state vector that is estimated by the Bootstrap filter is chosen as $\mathbf{y}_k = [\mathbf{x}_k^T,\ \dot{\mathbf{x}}_k^T]^T$, where the superscript T denotes the transpose, and $\mathbf{x}_k = [x_k\ y_k]^T$ and $\dot{\mathbf{x}}_k = [\dot{x}_k\ \dot{y}_k]^T$ are the position and the velocity vectors of a moving mobile, respectively.

The system dynamic model can be described with the following state equation:

$$\mathbf{y}_{k+1} = F\mathbf{y}_k + G\mathbf{w}_k,$$

$$F = \begin{bmatrix} I & \Delta I \\ O & I \end{bmatrix}, \quad G = \begin{bmatrix} (\Delta^2/2)\cdot I \\ \Delta I \end{bmatrix}, \quad I \in \mathbb{R}^{2\times 2},$$

where $\mathbf{w}_k = [w_x\ w_y]_k^T$ is a zero-mean, white-noise sequence and Δ is the time interval between measurements.

We consider a simple highway map as shown Figure 1. Let $\mathbf{x}_l^s = [x_l^s\ y_l^s]^T$, $(l = 1, 2, 3)$ be the position vectors of stations. The station 1 is at the origin. Let s be the signal propagation speed, and $t_{lm} = t_l - t_m$ be TDOA at the pair of stations (l, m). From Figure 1, we have

$$\|\mathbf{x}_k - \mathbf{x}_l^s\| - \|\mathbf{x}_k - \mathbf{x}_m^s\| = s \cdot t_{lm} = r_{lm},$$

where r_{lm} is the range difference from the moving mobile to the stations, and is proportional to the measured TDOA. The measurement model is a nonlinear function of the moving mobile position and is given by

$$\mathbf{z}_k = \begin{bmatrix} \|\mathbf{x}_k - \mathbf{x}_l^s\| - \|\mathbf{x}_k - \mathbf{x}_n^s\| \\ \|\mathbf{x}_k - \mathbf{x}_m^s\| - \|\mathbf{x}_k - \mathbf{x}_n^s\| \end{bmatrix} + \mathbf{v}_k,$$

where $\mathbf{v}_k$ is a zero-mean, white-noise sequence. When a TDOA measurement is missed, the measurement equation is converted to a scalar equation, $z_k = \|\mathbf{x}_k - \mathbf{x}_l^s\| - \|\mathbf{x}_k - \mathbf{x}_n^s\| + v_k$. It is assumed that the effect of multipath fading is ignored.

Now we can apply the bootstrap filter using the above system and measurement models to get, respectively, the predicted samples $\{\hat{\mathbf{y}}_{k|k-1}^j\}_{j=1}^N$ and the filtered samples $\{\hat{\mathbf{y}}_{k|k}^j\}_{j=1}^N$.

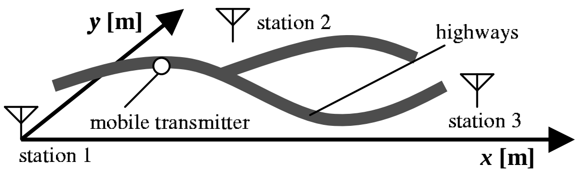


Figure 1: Geometry of the position location system.

3 Constrained Bootstrap Filter

Suppose that the sample $\hat{\mathbf{x}}^j_{k|k-1}$ is located outside highways, and let $d(\cdot)$ be the central line of the highways. We compute the projection error $\delta\hat{\mathbf{x}}$ by

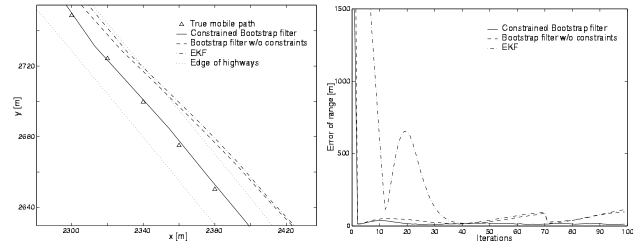
$$\min_{\delta\mathbf{x}} \|\delta\mathbf{x}\|_{P^{-1}_{k|k-1}},$$
$$\text{subject to } d(\hat{\mathbf{x}}^j_{k|k-1} + \delta\hat{\mathbf{x}}) = 0,$$

where $P_{k|k-1}$ is the predicted sample covariance matrix, and $\|\delta\mathbf{x}\|_{P^{-1}_{k|k-1}} \overset{\text{def}}{=} \delta\mathbf{x}^T P^{-1}_{k|k-1}\delta\mathbf{x}$. Then we replace the violating sample $\hat{\mathbf{x}}^j_{k|k-1}$ with the minimum 2-norm projection $\hat{\hat{\mathbf{x}}}^j_{k|k-1} \overset{\text{def}}{=} \hat{\mathbf{x}}^j_{k|k-1} + \delta\hat{\mathbf{x}}$ to the highway that gives the minimum value $\delta\hat{\mathbf{x}}$.

4 Simulation Results

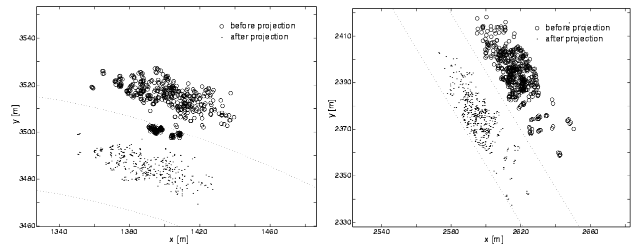
We assume that the station 2 and 3 are located at $(1500\text{m}, 4500\text{m})$ and $(4000\text{m}, 2000\text{m})$, respectively. The time interval between measurements, Δ is 1 second and the sample size N is 500. In our simulation, white Gaussian measurement noise with zero-mean and 30 m standard deviation is added to the measurement model. We postulate that the mobile communicates with the three stations during the first 40 seconds, with stations 1 and 2 during the next 30 seconds, and with station 1 and 3 during the last 30 seconds. The initial position of the mobile is $(800\text{m}, 3344.33\text{m})$. The x-directional velocity of the mobile is constant, 20m/s but the y-directional velocity is not. We start the iteration with the initial state $\mathbf{y}_0 = [2500\text{m}\ 2500\text{m}\ 0\text{m/s}\ 0\text{m/s}]^T$ in EKF. With the bootstrap filter, initial PDF $p(\mathbf{x}_0)$ is the uniform distribution over highways. The sample size N of the boostrap filter can be reduced from this fact. Fifty Monte Carlo runs are performed for each filter.

Figure 2(a) shows the moving mobile path and the estimated trajectories of the proposed constrained bootstrap filter, the bootstrap filter without constraints and the EKF. The triangles denote the true path of the mobile and the continuous line, the dashed line and the dot-dashed line give the trajectories of the proposed constrained bootstrap filter, the bootstrap filter without constraints and the EKF, respectively. Note that the trajectory of the constrained bootstrap filter does not get out of the highway even if the measurements are missed. Figure 2(b) shows range errors of each filter. As shown Figures 2(b), the bootstrap filters converge rapidly. Missing a measured data, the constrained bootstrap filter never diverge because of imposing the perfect measurement called constraint. In the constrained bootstrap filter, the (x, y) coordinates of the samples from the prior PDF at time step 30 and 90 are shown in Figures 3(a) and 3(b), respectively. The circles denote the time updated samples before projection onto the central lines of highways and the dots are for after projection. Note that the performance of the constrained bootstrap filter is superior to the others (if a map is available).



(a) (b)

Figure 2: (a) Mobile trajectory, and (b) range error.



(a) (b)

Figure 3: Samples (a) at k=30 and (b) at k=90.

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