Design of Fault Detectors using $\mathcal{H}_\infty$ optimization

Henrik Niemann[1] Jakob Stoustrup[2]

Abstract

The problem of detecting and/or isolating faults in dynamical systems is assessed. In contrast to previous approaches, the residual vector is considered to be a design variable as a free transfer function in addition to the actual filter which is supposed to minize the residual. Some main directions are suggested, and a numerical algorithm implementing part of these is proposed.

1 Introduction

A number of papers has considered the problem of designing residual generators by using $\mathcal{H}_\infty$ optimization methods, see e.g. [1, 3, 4] to mention some of the papers. The two main concepts in these methods are either to use some residual generator to estimate the fault signals directly, [2], or to estimate the output of the system and then use the difference between the real output and the estimated as the residual signal, see e.g. [1]. $\mathcal{H}_\infty$ optimization methods are very useful in connection with robust control, where hard bounds are required to guarantee robust stability and robust performance. Also in connection with estimation, $\mathcal{H}_\infty$ design methods are applied with advantage to guarantee performance specifications. In contrast to the robust control problem and the robust estimation problem which the $\mathcal{H}_\infty$ design methods were derived for, the methods cannot be applied directly for an optimal design of residual generators for fault detection and fault identification as we shall argue below.

The main result in this paper is to formulate the fault detection problem as a direct optimization problem, where e.g. an $\mathcal{H}_\infty$ optimization can be applied.

2 System Setup

Let us consider the general linear feedback system given by

$$y = G_f f + G_d d = \begin{bmatrix} G_f & G_d \end{bmatrix} w \tag{1}$$

[1] Department of Automation, Technical University of Denmark, Building 326, DK-2800 Lyngby, Denmark. E-mail: hhn@iau.dtu.dk

[2] Dept. of Control Engineering, Fr. Bajers Vej 7C, Aalborg University, DK-9220 Aalborg, Denmark, E-mail: jakob@control.auc.dk, URL: www.control.auc.dk/~jakob

where $x \in \mathcal{R}^n$ is the state vector, $f \in \mathcal{R}^k$ is the additive fault vector, $d \in \mathcal{R}^r$ is the external disturbance input, and $y \in \mathcal{R}^m$ is the measurement output. Further, let a residual signal r be given by $r = Fy$ where F is a residual generator.

The design problem is to design a residual generator such that we will obtain fault detection, fault identification or fault estimation.

$$\sup_{\|w\|_2<1} \|r - Vf\|_2 = \sup_{\|w\|_2<1} \|Fy - Vf\|_2 < \gamma$$

where V is a weighting matrix, and γ is a suitably small, positive number.

The above design problem depend strongly on the selection of the weight matrix V. We get a fault detection problem for V selected as a non-zero transfer function of dimension $1 \times k$, a fault isolation problem for V selected as a non singular $k \times k$ matrix, static or dynamic, and a fault estimation problem for $V = I$.

If both F and V are considered as "controllers" , i.e. as free parameters that need to be designed in an $\mathcal{H}_\infty$ design, the optimal solution will be $F = 0$ and $V = 0$. It is therefore necessary to require that the residual generator and the weighting matrix are non zero. This condition cannot be included directly in a standard $\mathcal{H}_\infty$ design.

3 $\mathcal{H}_\infty$ optimization of V and F.

The problem with the optimization problem associated with infimizing the transfer function from w to e, T_{ew} is that no constraints are imposed by the condition

$$\|T_{ef}\|_\infty = \|V - FG_f\|_\infty < \gamma \tag{2}$$

In fact, since V is free, $V = FG_f$ can always be chosen. This, in effect, means that the presence of f does not play a role in the optimization, and the optimization problem reduces to a disturbance attenuation problem. Using the above calculation of V in the transfer function from input w to the estimation error e gives the following closed loop transfer function

$$e = [0 \;\; FG_d]w = T_{0,FG_d}w$$

It is quite clear that the $\mathcal{H}_\infty$ norm of this closed loop transfer function will be less than (or equal) to the obtained norm for the design of F. So using this weight matrix V in the next design. An upper bound for this design is given by the the $\mathcal{H}_\infty$ norm of T_{0,FG_d}, because we can obtain this by using the same residual generator from the previous step.

This method has also a number of drawbacks compared with the other design method. First of all, a scaling of either V or F need to be included to remove the trivial solution $F = 0$, $V = 0$. Another problem is the order of the derived residual generator and also the weight matrix V. The order will increase in every iteration. The increasing order of the residual generator F can be removed by instead of using a $V = FG_f$ directly for the calculation of V, a fixed order approximation of FG_f can be applied.

Based on the above description of the method, the following algorithm can be given.

Algorithm 1

1. *Fix V. Determine F by solving an $\mathcal{H}_\infty$ filtering problem*
2. *Fix F. Determine V as a fixed order approximation of FG_f*
3. *Scaling of V.*
4. *Iterate on the above scheme*

At last, it need to be pointed out that because the weight matrix V is a dynamic matrix, it will not necessary be optimal to apply a constant threshold. Instead a dynamic threshold should be applied.

4 Examples

The iterative design method described above is now illustrated on some examples. Let consider the system in (1) given in state space form. The design examples considered has 4 states, 3 measurement outputs, 3 fault signals and 6 disturbance input signals. A, B_f, C and D_f are generated as random matrices. The two disturbance input matrices B_d and D_d are given by

$$B_d = [diag(0.25\ 0.25\ 0.25\ 0)\ 0_{4\times 3}]$$
$$D_d = [0_{3\times 3}\ diag(0.25\ 0.25\ 0.25)]$$

Further, the output error e in Figure 1 is in the design setup weighted by the following weighting function $W_e = \frac{0.01}{s+0.01}$ such that $z = W_e e$.

In the design, the weighting matrix V is not selected directly as $V = FG_f$, but V is instead selected as a reduced order approximation of FG_f, such that the order of the fault detector is bounded. In the following, a first order approximation is selected. Further, in every iteration, the weighting matrix V is scaled such that the smallest gain from f to Vf is 1 at $\omega = 0$.

In Figure 1, the $\mathcal{H}_\infty$ norm γ of the transfer function from input (f and d) to the weighted estimation error output z is shown as function of the number of iterations for 3 different systems. As starting point, V is selected as a constant matrix given by $V = [1\ 3\ 5]$.

As it is shown in Figure 1, the γ value converge after a number of iterations for all 3 systems.

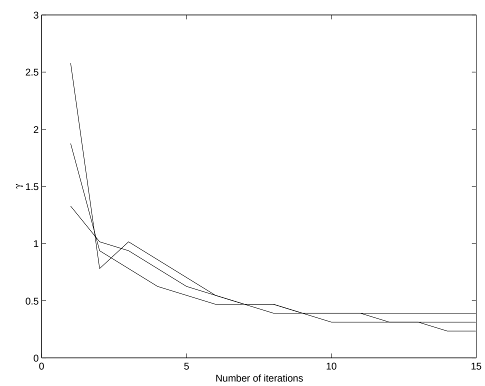

Figure 1: Iteration of the weight function for different random systems.

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