Compensation of input time delay for a class of nonlinear systems

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Abstract

A compensation strategy for nonlinear input time delay systems is considered in this work. The regulation problem is addressed by approximating a noncausal feedback that solves the problem by a non-anticipative one. This is done by considering a passivity based scheme proposed originally for nonlinear systems free of delays.

1 Introduction

Input time delays can produce severe limitations on the performance obtained with different control strategies[3], the search of solutions to this problem is important in the area of control systems. In this work, we propose a time delay compensation strategy for a class of nonlinear SISO systems with delay at the input. The work presented in this paper can be viewed as an extension of [2] where the case of systems without delays is considered.

2 Class of systems under consideration

We consider the class of nonlinear single-input single-output, input delay systems described by the following state space model,

$$\begin{aligned} \dot{x}(t) &= f(x(t)) + g(x(t))u(t-\tau) \\ y(t) &= h(x) \end{aligned} \tag{1}$$

where $x \in \mathcal{X} \subset \mathbf{R}^n$ is the state, $\mathcal{X}$ denotes the *operating region* of the system, $u \in \mathcal{U} \subset \mathbf{R}$, and $y \in \mathcal{Y} \subset \mathbf{R}$ is the output of the system defined over the open set $\mathcal{Y}$.

2.1 A canonical form for nonlinear systems

Consider system (1), and assume that a C^1 positive definite *storage function* $V : \mathbf{R}^n \to \mathbf{R}^+$ is given. We assume that the following assumption, known as the *transversality condition* (see [2]) holds,

$$L_g V(x) \neq 0, \quad \forall\, x \in \mathcal{X}. \tag{2}$$

We follow now, the results presented in [2]. Let us introduce the input coordinate transformation

$$u(t-\tau) = \Gamma(x(t), v(t)) \tag{3}$$

with v denoting a new external independent control input and $\Gamma(x,v) = \frac{1}{L_g V(x)}\left[h(x)v - \frac{\partial V}{\partial x^T} S_p(x) \frac{\partial V}{\partial x}\right]$, where $S_p(x)$ is defined in [2]. Now we fix the storage function as $V(x) = \frac{1}{2}\|x\|_2^2$.

2.2 Ideal Case: Feedback passivity of input delay systems

It is possible to show that the closed loop system (1)-(3) takes the form,

$$\dot{x} = \mathcal{I}(x)x + S_n(x)x + \frac{h(x)}{L_g V(x)} g(x) v \tag{4}$$

with $\mathcal{I}(x)$ being a skew-symmetric matrix and besides $S_n(x)$ are defined in [2]. Also, following [2] we can see that system (4) describes a passive map from the input $v(t)$ to the output $y = h(x)$. The following *auxiliary* dynamically controlled system,

$$\begin{aligned} \dot{x}_a &= \mathcal{I}(x)x_a + S_n(x)x_a + R\left[x - x_a\right] \\ &\quad + \frac{h(x)}{L_g V(x)} g(x) v \end{aligned} \tag{5}$$

where $R = diag\{R_1, R_2, \ldots, R_n\}$ and $R_i > 0$, $1 \le i \le n$, with suitable initial conditions can be used to show that the solution of

$$\dot{e}(t) = \mathcal{I}(x(t))e(t) + \left[-R + S_n(x(t))\right] e(t).$$

where $e(t) = x(t) - x_a(t)$, ideally converges exponentially to zero.

3 Time delay compensation

Assuming that the auxiliary input control $v(t)$ can be obtained from the auxiliary system (5) and that it is well defined for all t, we can rewrite (3) as

$$\begin{aligned} u(t) &= \Gamma(x(t+\tau), v(t+\tau)) \\ &= \varphi(x(t+\tau), x_a(t+\tau), \dot{x}_a(t+\tau)) \end{aligned}$$

here, the dependence of $\varphi(\cdot,\cdot,\cdot)$ on $x_a(t+\tau)$, and $\dot{x}_a(t+\tau)$ appears thanks to $v(t+\tau)$, this is, $v(t) = \rho(x(t), x_a(t), \dot{x}_a(t))$ for some function $\rho(\cdot,\cdot,\cdot)$ obtained from (5).

3.1 Approximate Control Law

Coming back to our developments, by shifting in time the arguments, $\varphi(x(t), x_a(t), \dot{x}_a(t)) = \Gamma(x(t), \rho(x(t), x_a(t), \dot{x}_a(t)))$. Supposing that φ is a continuously differentiable function of t, according to a well known Taylor theorem we can write

$$\begin{aligned} \varphi(x(s), x_a(s)) &= r(t,\tau) + \varphi(x(t), x_a(t), \dot{x}_a(t)) \\ &\quad + \tau\dot{\varphi}(x(t), x_a(t), \dot{x}_a(t)), \end{aligned}$$

where $s = t + \tau$, with the remainder $r(t, \tau)$ satisfying $\lim_{\tau \to 0} \frac{r(t,\tau)}{|\tau|} = 0$. Now, we propose to approximate the function $\varphi(x(t+\tau), x_a(t+\tau), \dot{x}_a(t+\tau))$ as

$$\tilde{\varphi}(\cdot) = \varphi(x(t), x_a(t), \dot{x}_a(t)) + \tau \dot{\varphi}(x(t), x_a(t), \dot{x}_a(t)) \tag{6}$$

equivalently, this approximate control law can be rewritten as

$$\tilde{u}(t) = \varphi(x(s), x_a(s), \dot{x}_a(s)) - r(t, \tau) \tag{7}$$

where $s = t + \tau$. Now, from equation (7) and (4), we obtain in closed-loop system the *approximate desired dynamics,*

$$\begin{aligned} \dot{x}(t) &= \mathcal{I}(x(t))x(t) + S_n(x(t))x(t) + \tfrac{h(x(t))}{L_g V(x)} g(x(t))v(t) \\ &\quad - g(x(t))r(t-\tau, \tau) \qquad (8) \\ y(t) &= h(x(t)) \end{aligned}$$

Following the procedure shown for the ideal case in Section 2, the error dynamics between system (8) and the auxiliary system (5) will be given by,

$$\begin{aligned} \dot{z}(t) &= \mathcal{I}(x(t))z(t) + \left[-R + S_n(x(t))\right] z(t) \\ &\quad - g(x(t))r(t-\tau, \tau) \end{aligned} \tag{9}$$

where $z(t) = x(t) - x_a(t)$ and with R defined as before. Now we show that using (6), $z(t)$ is *uniformly ultimately bounded* (see [1]).

3.2 Uniform ultimate boundedness of the tracking error

Equation (9) can be rewritten as the perturbed systems

$$\dot{z}(t) = f(t, z) + p(t, z), \tag{10}$$

with $f(t, z) = \mathcal{I}(x(t))z(t) + \left[-R + S_n(x(t))\right] z(t)$, and $p(t, z) = g(x(t))r(t-\tau, \tau)$. Then, equation (10) can be associated with the nominal system

$$\dot{z} = f(t, z) \text{ with } f(t, 0) = 0. \tag{11}$$

Theorem 1 *Consider system (1) and assume that it is minimum phase. Suppose that the approximated control (6) is used. Then, if the time-delay τ is sufficiently small, the solutions of (9) are uniformly ultimately bounded.*

Proof: Consider the perturbed system (10). Since $z = 0$ is an exponentially stable equilibrium point of the nominal system (11), by the Lyapunov converse theorems [1], we know that there exists a Lyapunov function $V(t, z)$ which satisfies

$$\begin{aligned} c_1 \|z\|^2 &\le V(t, z) \le c_2 \|z\|^2, \\ \frac{\partial V}{\partial t} + \frac{\partial V}{\partial z} f(t, z) &\le -c_3 \|z\|^2, \\ \left\| \frac{\partial V}{\partial z} \right\| &\le c_4 \|z\| \end{aligned} \tag{12}$$

for every $(t, z) \in [0, \infty) \times D$ (D is defined in Lemma 3, appendix A) for some positive constants c_1, c_2, c_3, and c_4. We have also that $p(t, z)$ represents terms of order $o(\tau^2)$ and therefore, from the assumption that τ is sufficiently small and from the fact that the nominal systems is stable (passive and minimal phase) we have that for some $\delta > 0$ (depending on the value of τ) $\|p(t, z)\| \le \delta$ for all $t \ge 0$. We conclude the proof by noting that conditions of Lemma 3 in Appendix A are fulfilled and therefore $z(t)$ is uniformly ultimately bounded.■

Remark 2 *Using the boundedness of $|\dot{\varphi}(x(t), x_a(t), \dot{x}_a(t))|$, it can be shown that the error in the approximation $r(t, \tau)$ can be arbitrary small depending on the value of τ.*

4 Conclusions

The regulation problem is addressed in this work for a class of nonlinear input time delay systems. A compensation strategy of input time delay is proposed where the noncausal property of an ideal solution is tackled by considering its approximation. The proposed scheme can only be applied for minimum phase systems that satisfy a *transversality condition* and the result is valid only for a sufficient small input delay.

Appendix A: Stability of systems with nonvanishing perturbations

Lemma 3 *[1]Let $z = 0$ be an exponentially stable equilibrium point of the nominal system (11). Let $V(t, z)$ be a Lyapunov function of the nominal system that satisfies (12) in $[0, \infty) \times D$, where $D = \{ z \in \mathbf{R}^n | \ \|z\| < r \}$. Suppose the perturbation term in (10) satisfies $\|p(t, z)\| \le \delta < \frac{c_3}{c_4}\sqrt{\frac{c_1}{c_2}}\theta r$ for all $t \ge 0$, all $z \in D$, and some positive constant $\theta < 1$. Then, for all $\|z(t_0)\| < \sqrt{c_1/c_2}\, r$, the solution of the perturbed system $z(t)$ satisfies*

$$\begin{aligned} &\|z(t)\| \le k \exp\left[-\gamma(t - t_0)\right] \|z(t_0)\|, \\ &\quad \forall\, t_0 \le t < t_1 \text{ and } \|z(t)\| \le b,\ \forall\, t \ge t_1 \end{aligned}$$

for some finite time t_1, where $k = \sqrt{\frac{c_2}{c_1}}$, $\gamma = \frac{(1-\theta)c_3}{2c_2}$, $b = \frac{c_4}{c_3}\sqrt{\frac{c_2}{c_1}}\frac{\delta}{\theta}$. ■

References

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