Adaptive/Self-Tuning PID Control by Frequency Loop-Shaping

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Abstract

This paper addresses issues arising in the on-line adaptation of PID controller parameters. With frequency loop-shaping principles as the underlying controller design approach, the PID parameter adaptation can be performed directly by minimizing a suitable estimation error with standard least-squares algorithms. A variant of such an algorithm is also proposed in an effort to approximate the minimization of the H-infinity norm of the error operator. A numerical example is used to illustrate the implementation of the algorithm.

1 Extended Summary

Proportional-Integral-Derivative (PID) controllers are still the most common control algorithms used in the process industry. Many methods for PID tuning are found in the literature, and [1] provides an excellent review. These methods could employ low-order approximations of the plant (e.g., [2]), or optimize a frequency and/or time domain performance measure [3, 4, 5]. Yet, the tuning of PID loops remains a subject of great practical interest. The reasons for this are in the requirements of some user expertise for sucessful tuning and the large number of PID loops that need to be tuned (e.g., 3000 PID loops in a typical refinery, many of which are often poorly tuned ($\sim 30\%$, [6]).

Recently, considerable effort has been devoted to the development of methods that connect the identification properties with the controller design to produce more reliable control systems [7, 8]. These methods can be used to somewhat automate the PID tuning [9, 10, 11]. It goes without saying that many of the off-line PID tuning strategies can be converted to on-line tuners following the indirect adaptive control paradigm. Nevertheless, direct adaptation/tuning of the PID parameters is an important topic, with potential benefits in speed of execution as well as performance optimization.

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In this paper we consider the direct tuning/adaptation of PID parameters with a loop-shaping objective. The off-line version of this "frequency loop-shaping" (FLS) approach was developed in [10] and has produced reliable results in a variety of applications [12, 13]. Using similar ideas, [14] used a re-parametrization of the tuning objective which allows the direct estimation of the PID parameters from input-output data with a fairly standard least-squares scheme. Here, keeping the problem formulation largely unchanged, we design a different update law that approximates the constrained minimization of the operator norm of the error system rather than the energy of the error itself. Under persistent excitation, the tuning with this adaptive scheme is comparable (and ideally the same) with the off-line design. Its advantages lie in the interpretation of the minimization objective in terms of modeling mismatch/uncertainty bounds, that provides a simple indicator to assess tuning confidence. The main emphasis of this study is to construct a reliable tuner using the prior knowledge on achievable target loops, mismatch and uncertainty bounds, and constraints on the PID parameters, rather than to provide a "universal" PID tuner for arbitrary plants.

Problem Description: In the following, we consider PID controllers with transfer function $C = K_p + K_i/s + K_d s/(\tau s + 1)$, where τ is an *a priori* selected time constant. The FLS tuning objective is to determine the PID parameters (K_p, K_i, and K_d, denoted by θ) so that the compensated open-loop transfer function is close to the target L in a weighted $\mathcal{L}_\infty$ sense. This is computed as the solution of the following optimization problem [10]

$$\min_{\theta \in \mathcal{M}} \| S(CG - L) \|_\infty \tag{1}$$

where G denotes the LTI, SISO system to be controlled and $S = (1+L)^{-1}$ is the target sensitivity. $\mathcal{M}$ is a convex set of parameter constraints, e.g., positivity of all PID gains (ensuring no right-half plane cancellations), limitations on derivative action etc. This problem is motivated by the small gain theorem, that is, if L and CG have the same unstable poles then a sufficient condition for closed-loop stability is $\| S(CG - L) \|_\infty < 1$.

Clearly, the selection of the target loop L is an important component for the success of the tuning. The target should be reasonable in the sense that it can be adequately approximated with the limited degrees of freedom of the PID structure. Typical targets, covering most of the cases of interest, are first and second order, depending on the rolloff rate of the plant around the intended crossover frequency ω_c. For example, $L = \omega_c/s$ if the plant frequency response has approximately zero roll-off rate around ω_c, and $L = \omega_c(s+a\omega_c+\epsilon)/s(s+\epsilon)$ if the roll-off is -20dB/dec. Here $-\epsilon$ is the slow pole of the plant and a (typically $0.25-0.4$) defines the tradeoff between disturbance attenuation and step response overshoot. The target loop bandwidth should also satisfy fundamental limitations from right-half plane poles and zeros (for more details, see [10, 12]). In other words, the target loop selection requires some a priori knowledge about the plant; this, however, would normally be available from a preliminary PID tuning.

The development of an on-line adaptive algorithm that mimics the off-line FLS solution involves the transformation of the error operator minimization (1) into an error signal minimization. More specifically, with (u, y) denoting the plant input-output pair, we consider the estimation error signal $e_e = S_o(CG - L)u$. Since $y = Gu$, $e_e = SCy - Tu$ (T being the complementary sensitivity) which is a signal that can be constructed from input/output data, processed by stable filters. Then the minimization objective in (1) can be expressed as $\| S(CG-L) \|_\infty = \sup_{\| u \| \neq 0} \frac{\| e_e \|_2}{\| u \|_2}$ Due to the special PID structure, the estimation error has the familiar linear-in-the-parameters form $e_e = w^\top \theta - z$, modulo a swapping term. A recursive algorithm that minimizes the energy of such an error signal can be found in [15]. It was also used by [16] in combination with a suitable dead-zone to ensure that the contribution of the error system would eventually be bounded by the dead-zone, in an operator gain sense.

Here, however, we seek to minimize the error system gain, at least approximately. To achieve this, we employ a bank of bandpass filters $\{F_i\}$ to decompose the input and output into several frequency bands and then minimize the maximum cost functional. This approximation has been found to produce reasonably accurate results for the plants of interest with a small number of filters (its accuracy improves as the number of filters increases and their transitions become sharper). Thus, the optimization objective becomes approximately

$$\max_i \frac{\| S(CG-L)F_i u \|_2}{\| F_i u \|_2} \leq \max_i \frac{\| SCF_i y - TF_i u \|_{2,\delta}}{\| F_i u \|_{2,\delta}}$$

where $\| \cdot \|_{2,\delta}$, $\delta > 0$ denotes the exponentially weighted 2-norm, used here to allow for the recursive implementation of the resulting estimator (bounded cost functional for bounded signals). An algorithm performing this on-line minimization is summarized next.

An Adaptive PID Tuning Algorithm: The adaptation is based on the discrete-time implementation of a recursive least squares algorithm [15]. The optimization problem to be solved is

$$\min_{\theta \in \mathcal{M}} \max_i \frac{J_{i,k}}{m_{i,k}} \tag{2}$$

$$J_{i,k}(\theta_k) = \sum_{n=0}^{k} \lambda^{k-n} \left| z_{i,n} - w_{i,n}^T \theta_k \right|^2 \tag{3}$$

$$m_{i,k+1} = \lambda m_k + \left| [F_i u]_{k+1} \right|^2$$

where $\lambda = e^{-2\delta} \in (0, 1]$ is a constant forgetting factor. In each $J_{i,k}$ the signals z and w are generated with the filtered (by F_i) input-output pair, and $m_{i,k+1}$ is the $\{2, \delta\}$-norm of the corresponding filtered version of u.

In the interest of minimizing the on-line computations, we propose the following simple descent/line-search algorithm. For the computation of a descent direction, define the "prior" cost functionals (i.e., J_i at time $k+1$ but with parameters θ_k):

$$\hat{J}_{i,k+1} = \lambda J_{i,k}(\theta_k) + \left| z_{i,k+1} - w_{i,k+1}^T \theta_k \right|^2$$

Regardless of the update method for θ_k, the new cost functional can be computed as follows:

$$J_{i,k+1} = \hat{J}_{i,k+1} - S_{i,k+1}^T \Delta\theta_{k+1} + \frac{1}{2} \Delta\theta_{k+1}^T P_{i,k+1} \Delta\theta_{k+1} \tag{4}$$

where $\Delta\theta_{k+1} = \theta_{k+1} - \theta_k$ and $-S, P$ are the gradient and Hessian of the functional J:

$$\begin{aligned} R_{i,k+1} &= \lambda R_{i,k} + 2 z_{i,k} w_{i,k} \\ P_{i,k+1} &= \lambda P_{i,k} + 2 w_{i,k} w_{i,k}^T = \nabla_\theta^2 J_{i,k} \\ S_{i,k+1} &= R_{i,k+1} - P_{i,k+1}\theta_k = -\nabla_\theta J_{i,k} \end{aligned}$$

A descent direction is now computed by solving $\Delta\theta_{k+1}^* = \arg\min_{\theta \in \mathcal{M}} \sum_{i \in I} J_{i,k+1}/m_{i,k+1}$ where I is the set of indices with individual costs "close" to the maximum. This is done to prevent a zero step-size in the subsequent line search, caused by the non-smooth boundary of the constraints (other descent direction computations are also possible). Using (4), this computation is fairly simple (e.g., oblique projection of the unconstrained solution on the constraint set). Finally, given the descent direction $\Delta\theta_{k+1}^*$ the step size is computed via a simple line-search on (4):

$$\min_{\alpha \in [0,1]} \max_i \frac{J_{i,k+1}(\theta_k + \alpha \Delta\theta_{k+1}^*)}{m_{i,k}}$$

where α is the step-size parameter. The parameter update is then implemented as $\Delta\theta_{k+1} = \theta_k + \alpha_{opt} \Delta\theta_{k+1}^*$.

Remarks: In its present form, the algorithm requires u to have energy at all the frequency points (a sensible spectrum and amplitude) to avoid an ill-posed objective and ensure a reliable tuning. For an adaptive-mode operation, a dead-zone implementation is essential, together with a numerical fix to prevent division by zero. One interesting type of dead-zone condition is $S_{k+1}^T P_{k+1}^{-1} S_{k+1} > 2d_0^2 \ m_{k+1}$, reflecting a minimum decrease d_0 on the gain of the error operator.

For practical implementation, the possibly non-zero offsets for the input and output (e.g., linearization steady-state) should be taken into account. This can be achieved by augmenting the linear model with a constant term or, in a quick fix, high-pass filtering of the input-output pairs. Finally, a highly desirable property of this algorithm is that it is not affected by input saturations, as long as their level is known. Of course, in this case, the implemented PID should contain an integrator anti-windup mechanism [17].

Example: This adaptive algorithm was successfully simulated in a variety of problems. The results shown here are from its application to the system with transfer function $G = 1/(s+1)^3$ and with target loop $L = 1/s$. The simulation results illustrate the convergence properties of the algorithm (Fig.1) for a square wave reference input and excitation injected at the plant input for $t \leq 75$. The cost functional values can serve to assess the tuning confidence (Fig.2; the eventual maximum of 0.3 is approximately the same as the off-line FLS fitting error).

Conclusion: In this paper we presented an algorithm for the on-line tuning of PID controllers, based on frequency loop-shaping principles. The algorithm employs a filter bank and a convex optimization procedure to emulate the off-line tuning results. The results of simulation examples were very encouraging and an experimental evaluation is planned for the near future. Also in the future research plans is a more detailed theoretical analysis of the algorithm properties as well as the fine-tuning of the numerical algorithms.

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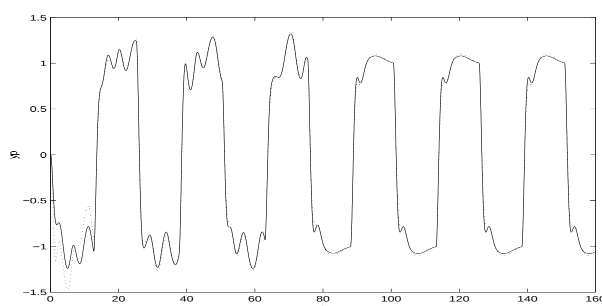

Figure 1: The output of the adaptive PID loop (...), compared with the off-line-tuned PID (—).

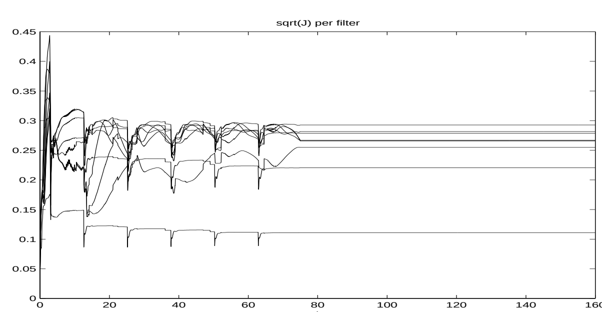

Figure 2: Evolution of the square-root of the cost functional for the different filters.