Adaptive Integral Sliding Mode Control for Active Vibration Absorber Design

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Abstract: A new tuning method for active vibration absorber design is presented in this paper. A robust, adaptive control scheme based on a variable structure with an adaptive discontinuity surface is designed and simulated. Robust synthesis of an adaptive discontinuity surface based on an augmented state-space is discussed. The proposed tuning scheme has three superior features compared with the existing counterparts in that: (i) it is completely insensitive to changes in the stiffness and damping of the absorber, (ii) it is capable of suppressing cyclic vibrations over a wide range of frequencies, (iii) its real-time operation requires only one adjustable gain.

Key words: Sliding mode control, variable structure, adaptive discontinuity surface, vibration absorber.

I. Introduction

Many engineering structures are disturbed by cyclic load and undergo unwanted vibration. In order to avoid large oscillatory amplitude when the frequency of the disturbance force coincides with the natural frequency of the structure, a vibration absorber is employed. The vibration absorber technique refers to the use of a mass-spring-damper system that is attached to a primary vibrating structure in order to suppress its vibration [1]. Vibration absorbers can be passive, active, or passive-active in nature. A variety of active vibration absorber designs, such as the dual-frequency fixed delayed resonator [2], have been reported in the literature. A newly proposed band-pass vibration absorber has been discussed in [3], however, the proposed absorber itself is not stable even though the whole system is stable. Moreover, in these works, modelling errors due to changes in stiffness and damping of the absorber are not fully addressed. Recently, we have proposed a robust control scheme for the design of active vibration absorbers that are completely insensitive to their parametric uncertainties using the conventional sliding mode control law [4]. This paper continues our work to develop a simple adaptive algorithm combined with an integral sliding mode control law to enhance the attenuation level when the system is subjected to simple harmonic load.

II. Development of Adaptive Integral Sliding Mode (AISM) Absorber

Consider the construction of a vibration absorber device which consists of an absorber mass-spring-damper trio [m_a, c_a, k_a] attached to a primary mass-spring-damper trio [m, c, k]. The control objective is to minimise the displacement x of the primary mass while keeping the displacement x_a of the absorber mass bounded. The equations of motion for the combined system are:

$$\begin{cases} m\ddot{x}+c\dot{x}+kx-c_a(\dot{x}_a-\dot{x})-k_a(x_a-x)=-u+f, \\ m_a\ddot{x}_a+c_a(\dot{x}_a-\dot{x})+k_a(x_a-x)=u. \end{cases} \quad (1a,b)$$

The system is augmented to a 3rd order equation by introducing two integral state variables:

$$y_a(t)=\int_0^t x_a(\xi)d\xi \quad , \quad y(t)=\int_0^t x(\xi)d\xi . \quad (2a,b)$$

(1b) then becomes:

$$m_a\ddot{y}_a+c_a(\ddot{y}_a-\ddot{y})+k_a(\dot{y}_a-\dot{y})=u. \quad (3)$$

Assume a linear sliding surface of the form:

$$S=\ddot{y}_a+p\ddot{y}+q\dot{y}+r(y_a-y). \quad (4)$$

Then during sliding mode, i.e., when S=0, the system dynamics are described by:

$$\ddot{y}_a+ry_a+p\ddot{y}+q\dot{y}-ry=0. \quad (5)$$

The transfer function from x to x_a is obtained by applying the Laplace transform to (5), resulting in:

$$\frac{X(s)}{X_a(s)}=\frac{Y(s)}{Y_a(s)}=-\frac{s^2+r}{ps^2+qs-r}. \quad (6)$$

The principle involved in designing an active absorber is to tune the numerator of (6) such that it has a zero at the disturbance frequency ω, thus producing zero displacement at the point of attachment to the primary structure. This can be achieved by simply tuning the gain r of the sliding surface S to ω^2. Since the system has been augmented to 3rd order and the corresponding sliding surface is of 2nd order, (6) now functions as a notch filter. Note that due to the feedback of the integral state defined in (2a,b), a saturation problem may arise if the disturbance has a DC component. However, this problem, if it exists, can be effectively removed by simply using the *relative* integral displacement state (y - y_a). As seen from (6), $y = y_a$ at the DC level. The presence of a sliding mode controller enables such an absorber to be completely insensitive to changes in the stiffness k_a and damping c_a of the absorber.

III. Sliding Mode Controller Design

In state-space representation, equations (1a,b) can be expressed in the form:

$$\dot{\mathbf{Z}}=\mathbf{AZ}+\mathbf{B_1}u+\mathbf{B_2}f, \quad (7)$$

where: $\mathbf{Z}=[y_a \quad y \quad \dot{y}_a \quad \dot{y} \quad \ddot{y}_a \quad \ddot{y}]^T$.

The design of the sliding mode controller consists of two phases. A stable discontinuity surface is first designed to describe the dynamics of the system in sliding mode. A switching control law is then formed to guarantee that all states can converge to this surface. In the design of this sliding surface, only the ideal regulator is considered. Matched uncertainties such as unknown excitation force can be handled by proper selection of the control function [5]. The discontinuity surface, S, is a linear combination of the state variables:

$$S=\mathbf{CZ}. \quad (8)$$

System stability during sliding mode, i.e., when S=0, can be achieved by proper selection of $\mathbf{C}$. Given that the matrix $\mathbf{CB_1}$ is nonsingular, the equivalent control during sliding mode is then described by:

$$u_{eq}=-(\mathbf{CB}_1)^{-1}\mathbf{CAZ}. \quad (9)$$

Finally, the control law is expressed as:

$$u=u_{eq}+K\text{sign}(S), \quad (10)$$

where K is chosen to be large enough to account for uncertainty in the magnitude of the disturbance f. Asymptotic stability of the controlled system is guaranteed once the following reaching condition is satisfied [5]:

$$S\dot{S} \le -\eta|S|, \quad \eta > 0. \tag{11}$$

IV. Discontinuity Surface Synthesis

Combining (1a,b) and (2a,b) gives:

$$\begin{cases} m\ddot{y} + c\dot{y} + k\dot{y} + m_a\ddot{y}_a = f, \\ m_a\ddot{y}_a + c_a(\dot{y}_a - \dot{y}) + k_a(\dot{y}_a - \dot{y}) = u. \end{cases} \tag{12a,b}$$

Assuming a sliding surface as in (4), the system dynamics during sliding mode are then described by:

$$\begin{cases} m\ddot{y} + c\dot{y} + k\dot{y} + m_a\ddot{y}_a = f, \\ \ddot{y}_a + ry_a + p\ddot{y} + q\dot{y} - ry = 0. \end{cases} \tag{13a,b}$$

Taking the Laplace transform of (13a,b) and then substituting (13b) into (13a) yields:

$$\frac{X(s)}{F(s)} = \frac{s^2 + r}{D(s)}, \quad \frac{X_a(s)}{F(s)} = -\frac{ps^2 + qs - r}{D(s)}, \tag{14a,b}$$

where: $D(s) = M(s)(s^2 + r) - m_a s^2 (ps^2 + qs - r)$,

$M(s) = ms^2 + cs + k$.

For closed-loop stability, the discontinuity surface gains are selected such that $D(s)$ is Hurwitz.

V. Adaptation Law and Stability Analysis

The only parameter that needs to be tuned on-line is the gain r of the discontinuity surface, which is to be adjusted to the value of ω^2. The frequency ω of the disturbance can be detected by monitoring the zero-crossings of the absorber acceleration signal. This adaptive algorithm requires only two feedback signals, $\ddot{x}_a$ and $\ddot{x}$, which are within the structure of the absorber. The stability of the adaptive control system is guaranteed, provided that: (i) the adaptive discontinuity surface is attractive and (ii) the roots of $D(s)$ always have negative real parts during adaptation. Condition (i) can be met by the choice of the control law as shown in (10), while condition (ii) can be satisfied by judiciously shaping the root locus of $D(s)$ when the gain r varies over the desired suppression frequency band. This process can be done off-line. Note that as $r = \omega^2$, this gain would be very large if the suppression band is in the range of hundreds of rad/s. This may cause some technical difficulties for practical implementation. However, in practice, due to a reduced order of the system during sliding mode, one of the gains of the sliding surface (4) can be chosen freely, i.e., independent of the constraint imposed by the pole-placement requirement. This means that the gain r, together with p and q, in (4) can be scaled down by a factor $\mu > 0$.

VI. Robustness & Transient Response Analysis

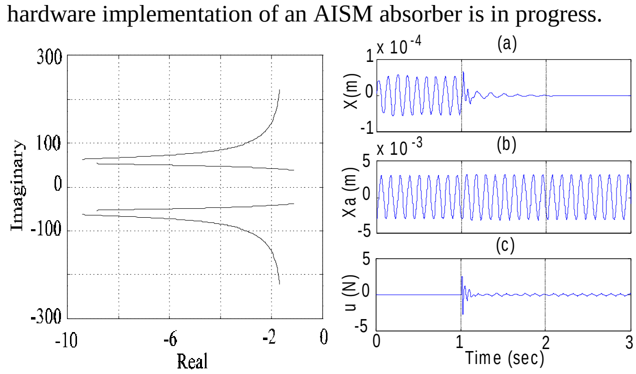
As seen from (14a,b), the performance of the controller during sliding mode is completely insensitive to changes in the stiffness k_a and damping c_a of the absorber. In addition, one of the most important factors in designing an active absorber is its settling time, i.e., the time required for the absorber's damping action to be effective. For a given primary system [m c k], this settling time is determined by the choice of the three gains p, q, and r in (14a,b), the absorber mass, and the required suppression frequency band. In this proposed control scheme, the absorber's time constant, τ, is dictated by the dominant poles, i.e., the dominant zeros of the characteristic polynomial $D(s)$, during the adaptive process. The settling time is then estimated to be 4.6τ.

VII. Computer Simulation and Discussion

The performance of the proposed tuning scheme is tested via simulation using Matlab Simulink. Parameters of the two trios are: [m,c,k]=[5kg, 100Ns/m, 16000N/m], and [m_a,c_a,k_a]=[1kg, 1Ns/m, 3200N/m]. Each of these two trios has a resonant frequency at ω=56.57rad/s. Suppose that the active absorber is expected to operate over the frequency range [40rad/s, 200rad/s]. The characteristic polynomial $D(s)$ is designed to have two pairs of complex conjugate roots with their real parts equal to -1 and -9 when $r = 40^2$, i.e., at the lower bound of the required frequency band. This gives p = 1, q = 10. The root loci of $D(s)$ as r varies over the suppression band is plotted in Figure 1(i). It is evident that the global system is guaranteed to be stable during adaptation, and the settling time can also be estimated at each operating frequency. In order to avoid control chattering due to finite switching speed, a boundary layer switching scheme is employed [6]. The switching gain is set to K = 200, and the boundary layer has a thickness of 0.0001. The coupled system is excited by a cyclic load at the resonant frequency with an amplitude of 10N, and operates in passive mode steady state for 1 second. At time t = 1sec, the AISM controller is turned on. Figure 1(ii) shows that the vibration is completely nullified after 1 second, and that the displacement of the absorber as well as the magnitude of the control force are considered as acceptable.

VIII. Conclusions

A new robust adaptive tuning scheme based on integral sliding mode control for active vibration absorber design is presented. The proposed tuning algorithm, which guarantees robust performance to modelling uncertainty, has a fast transient response that can be shaped by the choice of the discontinuity surface gain. Current studies are focused on examining the effects of measurement noise, time delay, and optimal pole-placement algorithm for maximising the tuning range. A hardware implementation of an AISM absorber is in progress.



(i) (ii)

Figure 1. (i): Closed-loop root loci of $D(s)$. (ii): Performance of the AISM absorber - (a) displacement of the primary mass, (b) displacement of the absorber mass, and (c) control u.

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