Open–Loop Feedback Control for Hybrid Manufacturing Systems

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Abstract

In this paper we discuss a novel formulation for the optimal control of discrete–event dynamic processes representing manufacturing systems characterized by unreliable machines, finite buffers and time–varying predictable demands. We approximatively represent the dynamics of the system with a hybrid model and derive an optimum control strategy for parts routing and machines scheduling embedded in a two–levels hierarchical control framework. At the higher level the discrete flows of parts are described by first–order fluid approximations and an optimum receding horizon control policy for the machines production rates is obtained by solving a sequence of linear programming problems. At the lower level a discrete–event real time dispatcher will be used to track the solution of the upper level controller as closely as possible.

1 Introduction

This paper focuses on the problem of optimum control of manufacturing systems consisting of unreliable machines and finite buffers holding costs, arranged in a feed–forward configuration. The system is subject to a time–varying exogenous demand, thus incurring additional shortfall/inventory costs. The process allows the production of several part–types with general service time distributions and routing policies. Each part has to perform its own orderly sequence of operations, and the same operation can be performed on alternative machines. The same machine can perform operations on different part classes, eventually with different service times. The objective is to determine an optimum control strategy for the machine production rates so as to minimize over a finite time horizon the sum of buffer holding costs and system shortfall/inventory costs, subject to machine/buffer capacity constraints and a predictable demand.

Machine breakdowns, planned and unplanned maintenance, operator unavailability, setup times, etc., make the manufacturing environment stochastic. However, in this paper we present a preliminary result considering deterministic fail and repair events, such as in the case of planned maintenance programs, and assuming that the uncertainty of the machine service times is small enough so that it does not affect the sequence of a limited number of events (machine starvation, blockage, breakdown and repair, material release times and due dates), that we call *macro–events*. Basically, we derive an optimum control strategy for the machine production rates, that is critically based on a hybrid model which approximatively represents the discrete behavior of the system.

As in all hybrid models, we distinguish between time–driven and event–driven dynamics. The continuous time–driven dynamics is described by first–order fluid processes where the average machine production rates are piecewise constant control variables. It has been shown [1, 13] that continuous–flow models are generally good approximations of asynchronous discrete models. The event–driven dynamics is defined by the transitions of the system through a sequence of operational states, that we call *macro–states*, upon the occurrence of the macro–events.

By this approximation approach, originally presented in [1] and briefly outlined in the next section, the system dynamics can be easily described with a single formalism by a discrete–time, time–varying, state variable model, where the average flows of parts through the machines enter the model as non–linear control variables and are kept constant within each interval of time between the occurrence of consecutive macro–events (*macro–period*).

The control task can be principally formulated as a deterministic mathematical optimization problem. A satisfactory, although suboptimal, solution can be achieved by embedding the optimization problem into a feedback structure, i.e., within a receding horizon scheme where the problem is repeatedly solved with updated demand predictions and updated initial conditions. This approach is usually called in the literature *open–loop feedback control* (OLFC) [9].

Particularly, we show that the control problem of minimizing over a finite time horizon the sum of buffer holding costs, and system shortfall/inventory costs, has a necessary condition such that the evaluation of the optimum control policy can be achieved by solving a sequence of linear programming problems, one for each macro–period. This result is particularly interesting because it formally confirms the intuition resulting from the myopic approach proposed in [1] where each macro–period was treated independently from the others.

1.1 Previous Work

There has been significant work on optimal control of failure–prone manufacturing systems. Gershwin [6] developed a general model suggesting a hierarchical approach for scheduling and planning. For single–machine single–class systems, the optimal solution (called *hedging–point* policy) can be explicitly determined. However, it is difficult to solve an optimum stochastic control problem for complex systems and with general constraints.

Sethi and Zhang [12] established a multi–level hierarchical control framework for production systems with stochastic demand and stochastic production capacity. They obtained an asymptotically optimal open–loop control policy for the machine production rates in order to maximize the expected total profit over a finite horizon. In [11] Presman *et al.* considered the problem of choosing the production rates of an N–machines flow–shop by formulating a stochastic dynamic programming problem. Perkins and Kumar [10] studied a pull model of a manufacturing system showing that optimum control problems can be reduced to a set of quadratic programming problems. Yao *et al.* [5] considered the problem of scheduling manufacturing systems based on a deterministic fluid network model. Hsu and Shamma [7] proposed a framework for solving optimal scheduling problems for re–entrant and transfer lines based on approximated cost–to–go functions and a receding horizon control.

In recent works [1, 3], Balduzzi *et al.*, developed a discrete–time, time–varying stochastic state variable model for the fluid approximation of flexible manufacturing systems. Then, by using perturbation analysis techniques they obtained average values and variances of both performance measures and their gradients with respect to the system parameters, in order to perform optimal design of the system configuration. In particular [2], they developed an optimum control scheme for maximizing productivity over a finite time horizon while guaranteeing at the final time a desired production mix. The developments presented in this paper follow the basic model presented in [1] and extend the results obtained in [2].

2 Description of the System

The manufacturing system considered in this work consists of a set of *elementary services* (ES) arranged in a feed–forward L–stages configuration. Each stage $l = 1, \ldots, L$ is composed of $P_l \geq 1$ alternative ES, denoted $S_{l,i}$, for $i = 1, \ldots, P_l$. An elementary service is composed of a single–server station $M_{l,i}$ coupled with a finite buffer $B_{l,i}$ for storing arriving parts. We assume that the input buffers within the first stage represent infinite reservoirs of raw parts, and the last stage is framed into output buffers collecting finished products. The total amount over all output buffers at time t represents the cumulative production at that time.

For simplicity of notation, we consider in this paper a manufacturing system producing a single class of products, and we assume for the first and the last stage that $P_1 = P_L = 1$, thus simply denoting the input and output buffers B_1 and B_L, respectively. Parts move from service $S_{l,i}$ to $S_{l+1,j}$ according to their production cycle and, along their routes, are queued in the buffers located at the entrance of the ES's.

We consider a fluid model of the production system, such that each machine $M_{l,i}$ can process parts at an average rate up to $V_{l,i}$ parts per unit time (maximum machine production rate). All buffers $B_{l,i}$, for $l = 2, \ldots, L$, have finite capacity $C_{l,i}$ and all machines are unreliable. We consider operation–dependent failures and we define for each machine $M_{l,i}$ the *production volumes before a machine fails* and the *repair times*, denoted by the sequences $f_{l,i} = \{f(1)_{l,i}, f(2)_{l,i}, \ldots\}$ and $r_{l,i} = \{r(1)_{l,i}, r(2)_{l,i}, \ldots\}$, respectively, which are both assumed deterministic values as in the case of planned maintenance programs.

The system is subject to a predictable time–varying demand. If the cumulative production $x_L(t)$, given by the initial amount in B_L and the cumulative input to B_L up to time t, exceeds the cumulative depletion $d(t)$ due to the exogenous demand, then the system at that time t undergoes an *inventory* of finished products awaiting for shipping. Otherwise, the system goes through a *shortfall*. We assume that a unit of material in buffer $B_{l,i}$, $l = 2, \ldots, L-1$, incurs a holding cost of w_l units per unit time (no holding costs for B_1), and each unit of inventory (shortfall) incurs a cost of w_L^+ (w_L^-) units per unit time. Here, we assume that holding costs are non–decreasing along the part routes, i.e., $0 = w_1 \leq w_2 \leq \ldots \leq w_{L-1} \leq w_L^+$, and $w_L^- \geq 0$.

2.1 A Hybrid Model

The evolution in time of the production process is discussed within a framework that distinguishes two levels of aggregation. The lower layer represents the microscopic behavior of arrivals and departures of parts to/from each machine (micro–events). It will be modeled in an aggregated view by using first–order fluid approximations [1, 8]. At the higher layer a discrete event model, such as a finite automaton, will represent the transitions of the process through a sequence of macro–states, at the occurrence of the macro–events.

Let $\rho_k = [t_k, t_{k+1})$, for $k = 0, 1, 2, \ldots$, be the interval of time between the occurrence of consecutive macro–events at time t_k and t_{k+1} (called *macro–period*), and let $0 \leq v_{l_i,(l+1)_j}(k) \leq V_{l,i}$ be the average flow rates of parts moving from service $S_{l,i}$ to $S_{l+1,j}$ ($v_{i,j}(k)$ for short), which are constant values within ρ_k. The microscopic behavior of a production system during a macro–period can be approximated by suitable fluid processes defined for each service $S_{l,i}$ as described below.

• The level of buffer $B_{l,i}$:

$$x_{l,i}(t) = x_{l,i}(t_k) + [v_{in,i}(k) - v_{out,i}(k)](t - t_k). \quad (1)$$

• The production volume processed by machine $M_{l,i}$ since the last repair:

$$\chi_{l,i}(t) = \chi_{l,i}(t_k) + v_{out,i}(k)(t - t_k). \quad (2)$$

• The time spent by machine $M_{l,i}$ under repair since the last failure:

$$s_{l,i}(t) = s_{l,i}(t_k) + (t - t_k). \quad (3)$$

These processes are defined for all $t \in \rho_k$, and $v_{in,i}(k) = \sum_h v_{h,i}(k)$, $v_{out,i}(k) = \sum_j v_{i,j}(k)$ represent the inflow and outflow rates of parts of each service. Note that Equations (2) and (3) will be reset to 0 after each repair and failure event, respectively.

At the macroscopic level the evolution in time of the system through a sequence of macro–states can be described by a finite automaton with states given by a finite set of admissible configurations of machines status (*operational* or *down*) and buffer status (*full*, *not full–not empty*, *empty*), and with transitions represented by the macro–events (*failure*, *repair*, *buffer full* and *buffer empty*). The transitions of this finite automaton define the interlacing of Equations (1)–(3) through the sequence of macro–periods, and drive the evolution of a hybrid system whose dynamics can be described with a single formalism by the following discrete–time, time–varying state variable model [1]:

$$\begin{cases} \mathbf{x}(k+1) = \mathbf{D}(k)\big[\mathbf{x}(k) + \mathbf{b}\big(\mathbf{u}(k)\big)\Delta\big(\mathbf{u}(k), \mathbf{x}(k)\big)\big] \\ \mathbf{x}(0) = \mathbf{x}_0 \end{cases} \quad (4)$$

with samples corresponding to the occurrence of the macro–events at times t_k. The state vector is

$$\mathbf{x}(k) = [\ldots, x_{l,i}(t_k), \chi_{l,i}(t_k), s_{l,i}(t_k), \ldots]^T \quad (5)$$

with entries given by the values of the fluid processes (1)–(3) at the occurrence of the macro–events. The control vector is $\mathbf{u}(k) = [\ldots, v_{i,j}(k), \ldots]^T$, with entries given by the constant average machine outflow rates. Matrix $\mathbf{D}(k)$ is diagonal with entries 0 and 1, and $\mathbf{b}\big(\mathbf{u}(k)\big) = \mathbf{R}\,\mathbf{u}(k) + \mathbf{e}_\nu$, where $\mathbf{e}_\nu$ is a vector with entries $0, 1$ accounting for the state variables $s_{l,i}(t)$ which do not explicitly depend on the control vector $\mathbf{u}(k)$, and $\mathbf{R}$ is the constant weight matrix for the machine outflow rates with entries 0, 1 and -1. The scalar function $\Delta\big(\mathbf{u}(k), \mathbf{x}(k)\big)$ denotes the length of the k–th macro–period.

Let us consider a finite horizon $T = \cup_{k=0}^{N-1}[t_k, t_{k+1}]$ (N denotes the final event), and assume that an "admissible" control policy $u(0,T) = \big(\mathbf{u}(0), \ldots, \mathbf{u}(N-1)\big)$ is applied to the system, thus providing the values of the machine production rates upon the occurrence of the macro–events. Therefore, we can fully characterize the sequence of the macro–events and the corresponding state trajectory $x(0,T) = \big(\mathbf{x}(0), \ldots, \mathbf{x}(N-1)\big)$. Furthermore, with this model any macro–event occurs when a suitable state variable reaches a specified value. Precisely, when machine $M_{l,i}$ fails (gets repaired) then it must result $\chi_{l,i}(t_{k+1}) = f(\gamma)_{l,i}$ ($s_{l,i}(t_{k+1}) = r(\beta)_{l,i}$) for some γ (β). When buffer $B_{l,i}$ gets full (gets empty) then the condition $x_{l,i}(t_{k+1}) = C_{l,i}$ ($x_{l,i}(t_{k+1}) = 0$) must be satisfied.

Let us denote by $e(k) \in \{0, C_{l,i}, f(\gamma)_{l,i}, r(\beta)_{l,i}\}$, where $f(\gamma)_{l,i} \in f_{l,i}$, $r(\beta)_{l,i} \in f_{l,i}$, $\gamma, \beta \in \mathbb{N}^+$, the deterministic sequence of values reached by the state variables upon the occurrence of the macro–events. Let us further define $\mathbf{h}(k) = \mathbf{e}_j^T(k)\mathbf{D}(k)$, $q\big(\mathbf{u}(k)\big) = \mathbf{h}(k)\mathbf{b}\big(\mathbf{u}(k)\big)$ and $K\big(\mathbf{u}(k)\big) = \frac{1}{q(\mathbf{u}(k))}$, where $\mathbf{e}_j(k)$ is a vector with entries 0 and 1 which selects that state variable leading to the current macro–state transition. Then, at the occurrence of the macro–events the following equality holds:

$$\begin{aligned} e(k+1) &= \mathbf{e}_j^T(k)\mathbf{x}(k+1) \\ &= \mathbf{h}(k)\mathbf{x}(k) + q\big(\mathbf{u}(k)\big)\Delta\big(\mathbf{u}(k), \mathbf{x}(k)\big), \end{aligned} \quad (6)$$

and the length of the k–th macro–period is defined by:

$$\Delta\big(\mathbf{u}(k), \mathbf{x}(k)\big) = K\big(\mathbf{u}(k)\big)\big[e(k+1) - \mathbf{h}(k)\mathbf{x}(k)\big]. \quad (7)$$

We observe that each macro–state defines a feasible region, denoted $\mathcal{U}(k)$, for the average machine production rates $v_{i,j}(k)$ that enter model (4) as non–linear control variables.

Definition 2.1 *Let $I_o(k)$ and $I_d(k)$ be the sets of indices of operational and down machines, $I_f(k)$ and $I_e(k)$ the sets of indices of full and empty buffers during the k–th macro–period, respectively. A control $\mathbf{u}(k) \in \mathcal{U}(k)$ is* admissible *if it is a feasible solution of the following set of linear inequalities:*

$$\begin{cases} (a) \quad 0 \le \sum_j v_{i,j}(k) \le V_{l,i}, & \forall i \in I_o(k) \\ (b) \quad \sum_j v_{i,j}(k) = 0, & \forall i \in I_d(k) \\ (c) \quad \sum_h v_{h,i}(k) \le \sum_j v_{i,j}(k), & \forall i \in I_f(k) \\ (d) \quad \sum_j v_{i,j}(k) \le \sum_h v_{h,i}(k), & \forall i \in I_e(k) \\ v_{i,j}(k) \ge 0 & \end{cases} \quad (8)$$

The consistency constraint set *(CCS) (8) will be denoted* $\mathbf{g}\big(k, \mathbf{u}(k)\big) \le \mathbf{0}$. ■

Constraints of the form (8.a) bound the machine production rates at their maximum value and apply for all operational machines. Constraints of the form (8.b) apply for all machines under repairing. Constraints of the form (8.c) have to be satisfied for all services with full buffers, and constraints (8.d) for all services whose buffer's level is 0.

The region $\mathcal{U}(k)$ defined by $\mathbf{g}(k, \mathbf{u}(k)) \le 0$ is a convex polyhedron whose vertices are basic solutions of any linear programming problem with objective function of the form $J(k) = \mathbf{a}^T(k)\mathbf{u}(k)$ and subject to the CCS. Any admissible control policy $\mathbf{u}(k)$ corresponds to a point within the feasible region $\mathcal{U}(k)$ and the boundary represents all those control policies aimed at optimizing a given linear objective function. Thus the optimum solution $\mathbf{u}^o(k)$ will always lay on the boundary of the feasible region.

3 The Dynamic Control Problem

The dynamic control policy developed in this work provides the sequence $u(0,T)$ of the machine production rates $\mathbf{u}(k) \in \mathcal{U}(k)$ as the solution of an optimum control problem aimed at minimizing over a finite time horizon T the sum of buffer holding costs, and system shortfall/inventory costs, subject to a time–varying predictable demand. Let N be the final event, $T = \cup_{k=0}^{N-1}[t_k, t_{k+1}]$ a finite time horizon, and let

$$J\big(\mathbf{x}_0, u(0,T)\big) = \sum_{k=0}^{N}\sum_{l=1}^{L-1}\sum_{i=1}^{P_l} \alpha_{l,i}(k) x_{l,i}(k) + \alpha_L(k)\,|x_L(k) - d(k)| \tag{9}$$

be the total expected cost during the interval $[0,T]$, where $x_L(k)$ and $d(k)$ denote the cumulative production and the cumulative exogenous demand up to time t_k, respectively. The weighting coefficients in Eq. (9) are defined as follows:

$$\alpha_{l,i}(k) = \begin{cases} \frac{w_l}{2}\Delta\big(\mathbf{u}(0),\mathbf{x}(0)\big), \quad k=0 \\ \frac{w_l}{2}\Big[\Delta\big(\mathbf{u}(k-1),\mathbf{x}(k-1)\big) + \Delta\big(\mathbf{u}(k),\mathbf{x}(k)\big)\Big], \\ \quad k = 1,\ldots,N-1 \\ \frac{w_l}{2}\Delta\big(\mathbf{u}(N-1),\mathbf{x}(N-1)\big), \quad k=N \end{cases} \tag{10}$$

for all $i = 1,\ldots,P_l$, and $\alpha_L(k)$ is also defined by Eq. (10) given $w_L = w_L^+$ if $x_L(k) - d(k) \geq 0$ and $w_L = w_L^-$ if $x_L(k) - d(k) < 0$. The optimum control problem is to determine the control sequence $u^o(0,T) = \big(\mathbf{u}^o(0),\ldots,\mathbf{u}^o(N-1)\big)$, for $\mathbf{u}^o(k) \in \mathcal{U}(k)$, which minimizes the performance functional

$$\sum_{k=0}^{N}\Big[\boldsymbol{\alpha}^T(k)\mathbf{C}\mathbf{x}(k) + \alpha_L(k)\,\big|\mathbf{e}_L^T\mathbf{x}(k) - d(k)\big|\Big] \tag{11}$$

over the finite time horizon T. Here $\boldsymbol{\alpha}(k) = [\ldots,\alpha_{1,i}(k),\ldots,\alpha_{L-1,i}(k),\ldots]^T$, matrix $\mathbf{C}$ and vector $\mathbf{e}_L^T$ of appropriate dimensions have entries 0 and 1 selecting all buffer levels $x_{l,i}(k)$, for $l = 1,\ldots,L-1$, and the output buffer level $x_L(k)$ within the state vector $\mathbf{x}(k)$, respectively. Note that $\boldsymbol{\alpha} = \boldsymbol{\alpha}\big(\mathbf{u}(k),\mathbf{x}(k)\big)$.

The proposed optimum control problem can be formulated as follows:

$$\begin{aligned} &\min_{u(0,T)} \sum_{k=0}^{N}\Big[\boldsymbol{\alpha}^T(k)\mathbf{C}\mathbf{x}(k) + \alpha_L(k)\,\big|\mathbf{e}_L^T\mathbf{x}(k) - d(k)\big|\Big] \text{ s.t.} \\ &(a) \begin{cases} \mathbf{x}(k+1) = \mathbf{D}(k)\big[\mathbf{x}(k) + \mathbf{b}\big(\mathbf{u}(k)\big)\Delta\big(\mathbf{u}(k),\mathbf{x}(k)\big)\big] \\ \mathbf{x}(0) = \mathbf{x}_0 \end{cases} \\ &(b) \;\; \mathbf{u}(k) \in \mathcal{U}(k), \quad \forall k = 0,\ldots,N-1 \end{aligned} \tag{12}$$

Equation (12.a) represents the dynamics of system (4) and Equation (12.b) is the control vector feasibility condition. To solve this problem, we introduce the *Hamiltonian* sequence

$$\begin{aligned} H(k) &= \boldsymbol{\alpha}^T(k)\mathbf{C}\mathbf{x}(k) + \alpha_L(k)\,\big|\mathbf{e}_L^T\mathbf{x}(k) - d(k)\big| \\ &\quad + \boldsymbol{\lambda}^T(k+1)\big\{\mathbf{D}(k)\big[\mathbf{x}(k) + \mathbf{b}\big(\mathbf{u}(k)\big)\Delta\big(\mathbf{u}(k),\mathbf{x}(k)\big)\big]\big\} \end{aligned}$$

where $\boldsymbol{\lambda}(k)$ is the costate vector. Let us assume the existence of an optimal trajectory $x^o(0,T)$ and corresponding control sequence $u^o(0,T)$, that satisfy a number of technical assumptions. Then, for the *minimum principle* [4], in order that $u^o(0,T)$ be optimal, there exists a costate vector $\boldsymbol{\lambda}(k)$, such that $\boldsymbol{\lambda}(k)$ and $\mathbf{x}^o(k)$ are a solution of the system

$$\begin{cases} \mathbf{x}^o(k+1) = \mathbf{D}(k)\Big[\mathbf{x}^o(k) + \mathbf{b}\big(\mathbf{u}^o(k)\big)\Delta\big(\mathbf{u}^o(k),\mathbf{x}^o(k)\big)\Big] \\ \boldsymbol{\lambda}^T(k) = \boldsymbol{\alpha}^T(k)\mathbf{C} + \nabla_{\mathbf{x}^o(k)}\boldsymbol{\alpha}(\mathbf{u}^o(k),\mathbf{x}^o(k))\mathbf{C}\mathbf{x}^o(k) \\ \quad + \operatorname{sgn}\big(\mathbf{e}_L^T\mathbf{x}^o(k) - d(k)\big)\alpha_L(k)\mathbf{e}_L^T \\ \quad + \nabla_{\mathbf{x}^o(k)}\alpha_L(\mathbf{u}^o(k),\mathbf{x}^o(k))\,\big|\mathbf{e}_L^T\mathbf{x}^o(k) - d(k)\big| \\ \quad + \boldsymbol{\lambda}^T(k+1)\mathbf{D}(k)\big[\mathbf{I} + \mathbf{b}\big(\mathbf{u}^o(k)\big)\nabla_{\mathbf{x}^o(k)}\Delta(\mathbf{u}^o(k),\mathbf{x}^o(k))\big] \end{cases}$$

with boundary conditions $\boldsymbol{\lambda}^T(N) = \mathbf{0}$ and $\mathbf{x}^o(0) = \mathbf{x}_0$, and such that $H(k)$ is minimized, i.e.,

$$\begin{aligned} H^o(k) &= \min_{\mathbf{u}(k)}\Big\{\boldsymbol{\alpha}^T(k)\mathbf{C}\mathbf{x}^o(k) + \alpha_L(k)\,\big|\mathbf{e}_L^T\mathbf{x}^o(k) - d(k)\big| \\ &\quad + \boldsymbol{\lambda}^T(k)\mathbf{D}(k)\Big[\mathbf{x}^o(k) + \mathbf{b}\big(\mathbf{u}(k)\big)\Delta\big(\mathbf{u}(k),\mathbf{x}^o(k)\big)\Big]\Big\} \\ &\text{s.t.} \quad \mathbf{g}\big(\mathbf{u}(k)\big) \leq \mathbf{0} \end{aligned}$$

for $k = 0,\ldots,N-1$. Note that, the problem of minimizing $H^o(k)$ is a non–linear programming problem with linear constraints, which is rather difficult to solve analytically. To reduce the complexity, in the next section we will consider a linearized version of this problem.

This class of optimal control formulations has been applied to a number of manufacturing systems, e.g., [12, 10, 7]. The major difficulty is that there is no available techniques for deriving closed–form solutions. However, as it will be clear in the following sections, a general finding is that the optimal control sequence $u^o(0,T)$ can be seen as a switching policy over the control variables. Control takes place upon the occurrence of the macro–events, and each solution $\mathbf{u}^o(k)$ corresponds to an extreme point of the CCS, i.e., the desirable operating point. Hence, the controller will always attempt to drive the system there and keep it.

4 The Linear Optimization Problem

Since problem (12) is analytically untractable, we consider a linearized version. Let us assume the existence of an optimal solution, i.e., $u^o(0,T)$ is the optimal control sequence and $x^o(0,T)$ is the corresponding optimal state trajectory.

Let us consider small perturbations $\delta\mathbf{x}(k)$ of the optimal trajectory $x^o(0,T)$ produced by admissible infinitesimal perturbations $\delta\mathbf{u}(k)$ of the optimal control sequence $u^o(0,T)$. Furthermore, let us assume that the perturbations $\delta\mathbf{u}(k)$ are small enough so that the sequence of the macro–events will not change. Formally, the dynamics of the perturbations $\delta\mathbf{x}(k)$ can be obtained by linearizing (4) around its optimal trajectory as follows:

$$\begin{aligned}\delta\mathbf{x}(k+1) &= \mathbf{D}(k)\delta\mathbf{x}(k) + \mathbf{D}(k)\mathbf{R}\Delta\big(\mathbf{u}^o(k),\mathbf{x}^o(k)\big)\delta\mathbf{u}(k)\\ &+ \mathbf{D}(k)\mathbf{b}\big(\mathbf{u}^o(k)\big)\Big[\nabla_{\mathbf{u}^o(k)}\Delta\big(\mathbf{u}^o(k),\mathbf{x}^o(k)\big)\delta\mathbf{u}(k)\\ &+ \nabla_{\mathbf{x}^o(k)}\Delta\big(\mathbf{u}^o(k),\mathbf{x}^o(k)\big)\,\delta\mathbf{x}(k)\Big]\end{aligned}$$

where $\delta\mathbf{x}(0)=\delta\mathbf{x}_0$ (e.g., $\delta\mathbf{x}_0=0$). Now, if we let

$$\begin{aligned}\mathbf{A}_x(k) &= \mathbf{D}(k)\Big[\mathbf{I} + \mathbf{b}\big(\mathbf{u}^o(k)\big)\nabla_{\mathbf{x}^o(k)}\Delta\big(\mathbf{u}^o(k),\mathbf{x}^o(k)\big)\Big]\\ \mathbf{B}_x(k) &= \mathbf{D}(k)\Big[\mathbf{R}\Delta\big(\mathbf{u}^o(k),\mathbf{x}^o(k)\big)\\ &\quad + \mathbf{b}\big(\mathbf{u}^o(k)\big)\nabla_{\mathbf{u}^o(k)}\Delta\big(\mathbf{u}^o(k),\mathbf{x}^o(k)\big)\Big]\end{aligned}$$

then we simply obtain

$$\delta\mathbf{x}(k+1) = \mathbf{A}_x(k)\delta\mathbf{x}(k) + \mathbf{B}_x(k)\,\delta\mathbf{u}(k). \tag{13}$$

Let us define $\tilde{\mathbf{x}}(k) = \mathbf{x}^o(k) + \delta\mathbf{x}(k)$ and $\tilde{\mathbf{u}}(k) = \mathbf{u}^o(k) + \delta\mathbf{u}(k)$. The linear perturbed system dynamics is given by:

$$\begin{cases}\tilde{\mathbf{x}}(k+1) = \mathbf{A}_x(k)\tilde{\mathbf{x}}(k) + \mathbf{B}_x(k)\tilde{\mathbf{u}}(k) + \mathbf{G}_x(k)\\ \tilde{\mathbf{x}}(0) = \tilde{\mathbf{x}}_0\end{cases} \tag{14}$$

where

$$\begin{aligned}\mathbf{G}_x(k) &= \big[\mathbf{D}(k)\mathbf{R}\Delta\big(\mathbf{u}^o(k),x^o(k)\big) - \mathbf{B}_x(k)\big]\mathbf{u}^o(k)\\ &\quad - \mathbf{D}(k)\mathbf{b}\big(\mathbf{u}^o\big)\nabla_{\mathbf{x}^o(k)}\Delta\big(\mathbf{u}^o(k),\mathbf{x}^o(k)\big)\mathbf{x}^o(k).\end{aligned}$$

Now, the control problem is to find the optimal sequence $\tilde{u}^o(0,T) = \big(\tilde{\mathbf{u}}^o(0),\dots,\tilde{\mathbf{u}}^o(N-1)\big)$, for $\tilde{\mathbf{u}}^o(k)\in\mathcal{U}(k)$, which minimizes the performance functional $J\big(\mathbf{x}_0,\tilde{u}(0,T)\big)$ over the finite time horizon T. This optimization problem can be formulated as follows:

$$\begin{aligned}&\min_{\tilde{u}(0,T)} \sum_{k=0}^{N}\Big[\boldsymbol{\alpha}^T(k)\mathbf{C}\tilde{\mathbf{x}}(k) + \alpha_L(k)\left|\mathbf{e}_L^T\tilde{\mathbf{x}}(k) - d(k)\right|\Big] \text{ s.t.}\\ &(a)\ \begin{cases}\tilde{\mathbf{x}}(k+1) = \mathbf{A}_x(k)\tilde{\mathbf{x}}(k) + \mathbf{B}_x(k)\tilde{\mathbf{u}}(k) + \mathbf{G}_x(k)\\ \tilde{\mathbf{x}}(0) = \tilde{\mathbf{x}}_0\end{cases}\\ &(b)\ \tilde{\mathbf{u}}(k)\in\mathcal{U}(k),\quad \forall k = 0,\dots,N-1\end{aligned} \tag{15}$$

Let us assume, without loss of generality, that the sequences $\boldsymbol{\alpha}(k)$, $\alpha_L(k)$ and $d(k)$ do not change after linearization. In this case the objective function and the constraints are linear in the state and control variables. Therefore we are dealing with a linear programming problem and the optimum solution, if it exists, will always require the control variables to be laying on the boundary of the feasible region. This approach can be seen as an extension of the *bang–bang principle* [4] to the multi–dimensional case. In particular, we will show that an optimum solution of problem (15) can be obtained by solving a sequence of linear programming problems, one for each macro–period.

By the same developments as in Section 3 we introduce the *Hamiltonian* sequence

$$\begin{aligned}H(k) &= \boldsymbol{\alpha}^T(k)\mathbf{C}\tilde{\mathbf{x}}(k) + \alpha_L(k)\left|\mathbf{e}_L^T\tilde{\mathbf{x}}(k) - d(k)\right|\\ &\quad + \boldsymbol{\lambda}^T(k+1)\big[\mathbf{A}_x(k)\tilde{\mathbf{x}}(k) + \mathbf{B}_x(k)\tilde{\mathbf{u}}(k) + \mathbf{G}_x(k)\big]\end{aligned}$$

and we define the costate dynamics

$$\begin{cases}\boldsymbol{\lambda}^T(k) = \boldsymbol{\alpha}^T(k)\mathbf{C} + \operatorname{sgn}\big(\mathbf{e}_L^T\tilde{\mathbf{x}}(k) - d(k)\big)\alpha_L(k)\mathbf{e}_L^T\\ \qquad + \boldsymbol{\lambda}^T(k+1)\mathbf{A}_x(k)\\ \boldsymbol{\lambda}^T(N) = \mathbf{0}\end{cases} \tag{16}$$

Therefore, the optimal solution of problem (15) can be obtained by computing the optimal sequence $H^o(k)$, for $k = 0,\dots,N-1$, as follows:

$$\begin{aligned}H^o(k) &= \min_{\tilde{\mathbf{u}}(k)}\Big\{\boldsymbol{\alpha}^T(k)\mathbf{C}\tilde{\mathbf{x}}(k) + \alpha_L(k)\left|\mathbf{e}_L^T\tilde{\mathbf{x}}(k) - d(k)\right|\\ &\quad + \boldsymbol{\lambda}^T(k)\big[\mathbf{A}_x(k)\tilde{\mathbf{x}}(k) + \mathbf{B}_x(k)\tilde{\mathbf{u}}(k) + \mathbf{G}_x(k)\big]\Big\}\\ &\text{s.t.}\quad \mathbf{g}\big(\tilde{\mathbf{u}}(k)\big)\le\mathbf{0}\end{aligned} \tag{17}$$

where $\boldsymbol{\lambda}^T(k)$ is the solution of Eq. (16). We observe that, since the optimal trajectory $x^o(0,T)$ is known, matrices $\mathbf{A}_x(k)$ are also known. This fact allows us to compute $\boldsymbol{\lambda}(k)$ and thus solving problem (17) iteratively. Finally, we note that the Hamiltonian sequence is linear in $\tilde{\mathbf{u}}(k)$, thus problem (15) reduces to the solution of a sequence of linear programming problems of the following form:

$$\min_{\tilde{\mathbf{u}}(k)}\left\{\mathbf{c}^T(k)\,\tilde{\mathbf{u}}(k)\right\}\qquad \text{s.t.}\qquad \mathbf{g}\big(\tilde{\mathbf{u}}(k)\big)\le\mathbf{0} \tag{18}$$

for $k = 0,\dots,N-1$, where the cost coefficient vectors are defined as $\mathbf{c}^T(k) = \boldsymbol{\lambda}^T(k)\mathbf{B}_x(k)$. The following theorem proves that the optimum solution of the linear problem (15) is also a solution for the non–linear problem (12).

Theorem 4.1 *Consider the following constrained non–linear programming problem*

$$\min_{\mathbf{u}}\ J(\mathbf{u})\qquad s.t.\qquad \mathbf{g}(\mathbf{u})\le\mathbf{0} \tag{19}$$

and the following constrained linear programming problem

$$\min_{\mathbf{u}}\ \mathbf{c}^T\mathbf{u}\qquad s.t.\qquad \mathbf{g}(\mathbf{u})\le\mathbf{0} \tag{20}$$

Let $\mathcal{U}$ be the feasible region for both problems, $\mathbf{u}^o$ an optimal solution for problem (20) and suppose $\mathbf{u}^o$ is a regular point of the constraint set $\mathbf{g}(\mathbf{u}) \leq \mathbf{0}$. If

$$\nabla_{\mathbf{u}} J(\mathbf{u}^o) \neq \mathbf{0}, \quad \forall\, \mathbf{u} \in \mathcal{U} \tag{21a}$$

$$\nabla_{\mathbf{u}} J(\mathbf{u}^o) = \mathbf{c}^T \tag{21b}$$

then $\mathbf{u}^o$ satisfies the necessary conditions for the solution of problem (19).

Proof: *If $\mathbf{u}^o$ satisfies the necessary condition for the solution of problem (19), then there exists a vector $\tilde{\boldsymbol{\mu}} \geq \mathbf{0}$ such that (*Kuhn-Tucker conditions*):*

$$\begin{aligned} &\nabla_{\mathbf{u}} J + \tilde{\boldsymbol{\mu}}^T \nabla_{\mathbf{u}}\, \mathbf{g}(\mathbf{u}^o) = \mathbf{0} \\ &\tilde{\boldsymbol{\mu}}^T \mathbf{g}(\mathbf{u}^o) = \mathbf{0}. \end{aligned} \tag{22}$$

However by hypothesis $\mathbf{u}^o$ is a solution for problem (20), then it must satisfy for a certain $\boldsymbol{\mu} \geq \mathbf{0}$ the following conditions:

$$\begin{aligned} &\mathbf{c}^T + \boldsymbol{\mu}^T \nabla_{\mathbf{u}}\, \mathbf{g}(\mathbf{u}^o) = \mathbf{0} \\ &\boldsymbol{\mu}^T \mathbf{g}(\mathbf{u}^o) = \mathbf{0} \end{aligned} \tag{23}$$

From condition (21b) it is immediate to verify that (22) is equal to (23) with $\tilde{\boldsymbol{\mu}} = \boldsymbol{\mu}$. ■

Theorem 4.1 provides a necessary condition for the optimum solution of problem (12), i.e., if $\mathbf{u}^o$ is an optimum solution of problem (15) then it is also a solution of problem (12). In fact, if the linear problem admits a unique optimum solution $\mathbf{u}^o$, and condition (21a) of the previous theorem allows us to avoid degenerate solutions, then $\mathbf{u}^o$ must be a stationary point for problem (12).

4.1 Open–Loop Feedback Control Strategy

An efficient method to transform open–loop optimal control decisions into a closed–loop feedback structure can be obtained by implementing a receding horizon control technique (also known as *open–loop feedback control*). Precisely, the optimization problem is solved at each time instant k with updated initial conditions $\mathbf{x}(k)$, but only the first value $\mathbf{u}(k)$ of the resulting control policy $u(k, T)$ is actually applied to the process.

We describe the proposed strategy in the form of an iterative procedure which can be further refined by sampling within each macro–period at a given number of time instants $t_{k_j} \in \Delta(k)$, for $j = 0, 1, 2, \ldots$ The number of sampling instants and their values are usually problem dependent. Let us denote by $\Pi_T\big(\mathbf{x}(0), t_0\big)$ the optimal control problem with initial conditions $\mathbf{x}(0)$, t_0 and horizon T, as described in Section 4, and let

$$J_N^o\big(\mathbf{x}(0), t_0\big) = \min_{u(0,T)} J\big(\mathbf{x}(0), u(0,T)\big)$$

be the optimal finite horizon cost. The receding horizon optimal control strategy can be either defined as

$$\phi\big(\mathbf{x}(k), N\big) = \arg \min_{\mathbf{g}(\mathbf{u}(k)) \leq \mathbf{0}} \Big\{ \Big[\boldsymbol{\alpha}^T(k) \mathbf{C}\mathbf{x}(k) + \alpha_L(k) \cdot \big|\mathbf{e}_L^T \mathbf{x}(k) - d(k)\big|\Big] + J_{N-1}^0(\mathbf{x}(k)) \Big\}$$

or by the following iterative procedure.

1. Let $k = 0$, $\mathbf{x}(t_k) = \mathbf{x}_0$ and $T_k = T$.
2. Solve problem $\Pi_{T_k}\big(\mathbf{x}(t_k), t_k\big)$ over the finite horizon $[t_k, t_k + T_k]$. This yields the optimal trajectory $x^o(t_k, t_k + T_k)$ and corresponding optimal control policy $u^o(t_k, t_k + T_k) = \big(\mathbf{u}^o(t_k), \mathbf{u}^o(t_{k+1}), \ldots\big)$.
3. Provide the control input $\mathbf{u}^o(t_k)$ to the real–time dispatcher that will track it as closely as possible over $[t_k, t_{k+1})$.
4. Let $k = k + 1$, $\mathbf{x}(t_k) = \mathbf{x}^o(t_k)$ and choose a new T_k (e.g., $T_k = T$). Repeat the procedure starting from 2.

The optimization of problem $\Pi_{T_k}\big(\mathbf{x}(t_k), t_k\big)$ in step 2 yields an open–loop control that depends on the initial state $\mathbf{x}(t_k)$. Thus, when computing the solution for the new horizon $[t_k, t_k + T_k]$, we are in fact closing the loop. Note that, the size of $\Delta_k = t_{k+1} - t_k$, as already mentioned, as well as the duration of the time horizon T_k, usually depend on the system dynamics and the magnitude of random disturbances.

4.2 A Case Study

Let us consider a simple transfer line producing a single class of products, composed of 4 services in tandem, with an output buffer B_5, and the input buffer B_1 acting as an infinite reservoir of raw parts. Assume that the intermediate buffers B_i, $i = 2, 3, 4$, have infinite capacity and that all machines M_i, $i = 1, 2, 3, 4$, will not fail during the time horizon $T = 18$. The system is subject to an exogenous demand of constant rate $\delta = 1$ and the maximum machine production rates vector is $\mathbf{V} = [1.5, 3, 2, 3]^T$.

Let $\mathbf{x}(k) = [x_1(k), x_2(k), x_3(k), x_4(k), x_5(k)]^T$ denote the state vector, where $x_i(k)$ is the level of buffer B_i at time t_k ($x_5(k)$ represents the cumulative production), and let $\mathbf{u}(k) = [v_1(k), v_2(k), v_3(k), v_4(k)]^T$ denote the control vector. Assume that buffer holding costs are $w_2 = 1$, $w_3 = 2$, $w_4 = 3$, inventory/shortfall costs are $w_5^+ = 3$ and $w_5^- = 4$, the initial buffer levels are $\mathbf{x}(t_0) = [\cdot, 6, 12, 24, 0]^T$, and the cumulative demand is $d(t_0) = 24$, i.e., initial shortfall.

The control problem of minimizing the sum of buffer holding costs and system shortfall/inventory costs can be formulated as follows:

$$\min_{u(0,T)} \quad \sum_{k=0}^{N} \Big[\boldsymbol{\alpha}^T(k)\mathbf{x}(k) + \alpha_5(k)\, \big|\mathbf{e}_5^T \mathbf{x}(k) - d(k)\big|\Big] \text{ s.t.}$$

$$(a) \begin{cases} \mathbf{x}(k+1) = \mathbf{D}\big[\mathbf{x}(k) + \mathbf{R}\mathbf{u}(k)\Delta(k)\big] \\ \mathbf{x}(0) = \mathbf{x}(t_0) \end{cases}$$

$$(b)\ \ \mathbf{u}(k) \in \mathcal{U}(k), \quad \forall k = 0, \ldots, N-1$$

where $\mathbf{D} = \text{diag}(1, 1, 1, 1, 1)$, $\mathbf{e}_5 = [0, 0, 0, 0, 1]^T$, $\mathbf{R} = \text{diag}_2\big[(-1, -1, -1, -1), (1, 1, 1, 1)\big] \in \mathbb{N}^{5\times 4}$ is 2–

diagonal, and $\boldsymbol{\alpha}(k)$, $\alpha_5(k)$ are given by Eq. (10). Reference [10] considered this manufacturing system showing that for each machine M_i the optimal production rates are $v_i(k) = 0$ if $t_k \leq \bar{t}^i$ or $v_i(k) = V_i$ if $t_k > \bar{t}_i$, where $\bar{t}_i$ are "deferral times" that can be computed through a decomposition of the system according to the bottlenecks machines.

We solve this control problem by considering $N = 5$ sampling times t_k, that correspond to the occurrence times of some macro–events (e.g., buffer empty) and the deferral times $\bar{t}^4 = 0$, $\bar{t}^2 = \bar{t}^3 = 6.23$, $\bar{t}^1 = 18$ as given in [10]. The solution of this problem provides the optimal policy $u^o(t_0, T)$ defined throughout each macro–period as follows:

- $\rho_0 = [0, 6.23]$ starts at time $t_0 = \bar{t}^4$. The CCS is $\mathcal{U}(0) = \{\mathbf{u}(0) \geq \mathbf{0} \,|\, \mathbf{u}(0) \leq \mathbf{V}\}$ and the optimal control is $\mathbf{u}^o(0) = [0, 0, 0, V_4]^T$ yielding $J_1^o(\mathbf{x}(t_0), t_0) = 995.72$;
- $\rho_1 = [6.23, 8.23]$ starts at time $t_1 = \bar{t}^2 = \bar{t}^3$. The CCS is $\mathcal{U}(1) = \{\mathbf{u}(1) \geq \mathbf{0} \,|\, \mathbf{u}(1) \leq \mathbf{V}\}$ and the optimal control is $\mathbf{u}^o(1) = [0, V_2, V_3, V_4]^T$ yielding $J_2^o(\mathbf{x}(t_0), t_0) = 1176.5$;
- $\rho_2 = [8.23, 11.54]$ starts at time $t_2 = 8.23$ when buffer B_2 gets empty. The CCS is $\mathcal{U}(2) = \{\mathbf{u}(2) \geq \mathbf{0} \,|\, \mathbf{u}(2) \leq \mathbf{V}\,;\, v_2(2) \leq v_1(2)\}$ and the optimal control is $\mathbf{u}^o(2) = [0, 0, V_3, V_4]^T$ yielding $J_3^o(\mathbf{x}(t_0), t_0) = 1241$;
- $\rho_3 = [11.54, 15.23]$ starts at time $t_3 = 11.54$ when buffer B_4 gets empty. The CCS is $\mathcal{U}(3) = \{\mathbf{u}(3) \geq \mathbf{0} \,|\, \mathbf{u}(3) \leq \mathbf{V}\,;\, v_2(3) \leq v_1(3)\,;\, v_4(3) \leq v_3(3)\}$ and the optimal control is $\mathbf{u}^o(3) = [0, 0, V_3, V_3]^T$ yielding $J_4^o(\mathbf{x}(t_0), t_0) = 1267.9$;
- $\rho_4 = [15.23, 18]$ starts at time $t_4 = 15.23$ when buffer B_3 gets empty. The CCS is $\mathcal{U}(4) = \{\mathbf{u}(4) \geq \mathbf{0} \,|\, \mathbf{u}(4) \leq \mathbf{V}\,;\, v_2(4) \leq v_1(4)\,;\, v_4(4) \leq v_3(4)\,;\, v_3(4) \leq v_2(4)\}$ and the optimal control is $\mathbf{u}^o(4) = [0, 0, 0, 0]^T$ yielding $J_5^o(\mathbf{x}(t_0), t_0) = 1267.9$.

The minimum value of the performance functional at time $T = 18$ is $J_5^o(\mathbf{x}(t_0), t_0) = 1267.9$. Note that, for the next macro–period starting at time $t_5 = \bar{t}^1$ ($x_5(t_5) = d(t_5)$), the CCS will be $\mathcal{U}(5) = \{\mathbf{u}(5) \geq \mathbf{0} \,|\, \mathbf{u}(5) \leq \mathbf{V}\,;\, v_2(5) \leq v_1(5)\,;\, v_4(5) \leq v_3(5)\,;\, v_3(5) \leq v_2(5)\}$ and the optimal control will be $\mathbf{u}^o(5) = [V_1, V_1, V_1, V_1]^T$.

We observe that, the optimal control policy $u^o(t_0, T)$ is the same as that computed in [10], where this optimization problem was solved over an infinite time horizon. However, with this approach we can address finite horizon problems, apply open–loop feedback control strategies, and finally, we can solve multi–class production systems with finite buffers and unreliable machines.

5 Conclusions

In this paper we have approached the control problem of determining optimal machines production rates so as to minimize the sum of buffer holding costs, and system shortfall/inventory costs, subject to machine/buffer capacity constraints and a predictable demand. We have shown that this problem has a necessary condition which lends itself to the evaluation of the optimum machine production rates for each sample period through a sequence of linear programming problems. A discrete–time optimum control problem has been formulated within a two–levels hierarchical hybrid control framework, and the use of a receding horizon control scheme has been proposed in order to tackle planning and scheduling issues encountered in production systems. The results obtained so far, allow us to consider this approach as a promising avenue. Our main future goal will be to extend the approach to the control of stochastic manufacturing systems.

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