Robust Stability and H_∞ Control of Discrete-Time Jump Linear Systems with Time-Delay: An LMI Approach

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Abstract: This paper considers the class of discrete-time jump linear system with time-delay and polytopic uncertain parameters. The problems of delay-independent robust stability, stabilization and H_∞ control are cast into the framework of linear matrix inequality (LMI) and thus solved by LMI Toolbox of Matlab. By using the supplementary technique, the system with time-delay is converted into a higher dimension Markov system without time-delay, and thus can be handled as a standard jump linear system with uncertain parameters.

Keywords: Jump linear system, polytopic uncertainty, time delay, LMI.

1 Introduction

Jump linear system (JLS) is very popular to characterize a class of abrupt changes in the structure of the system, which occur frequently in manufacturing system, economics system, communication system and power system etc. For jump linear systems with norm-bounded uncertainties, the stability, stabilization, H_2, H_∞ and their robustness have been addressed by Boukas and Yang [3], Costa and Boukas [4], Shi and Boukas [7], Souza and Fragoso [8] and the references therein.

Time-delay is a great source of instability and poor performance. During the past decades, the stability of linear system with time-delay has received considerable attention. For the progress on the research of deterministic system with time-delay, the reader is referred to Jeung et al. [5] and references therein. For JLS with time-delay, Benjelloun and Boukas [1] established a sufficient condition for the stochastic stability in the mean square sense. The robust stabilization problem was addressed by Benjelloun, Boukas and Yang [2]. This paper is to study the robust stability and stabilization problems of discrete-time jump linear time delay system with polytopic uncertainty The rest of this paper is organized as follows. In section 2, the model is specified. Section 3 establishes the robust stability of the system. Section 4 addresses the robust H_∞ control problem. By using supplementary variables, in Section 5 the time-delay system is converted into a standard jump linear system with higher dimension.

2 Model Description

Let $\{r_k, k \geq 0\}$ be a Markov chain with state space $\mathcal{S} = \{1, \cdots, N\}$ and state transition matrix $P = [p_{ij}]_{i,j\in\mathcal{S}}$, i.e. the transition probabilities of $\{r_k, k \geq 0\}$ are as follows:

$$P[r_{k+1} = j | r_k = i] = p_{ij}, \forall i, j \in \mathcal{S}.$$

Consider a discrete-time hybrid system with N modes. Suppose the system mode switching is governed by $\{r_k, k \geq 0\}$ and the system parameters contain polytopic uncertainties. The system dynamics is

$$\begin{cases} x_{k+1} = A(r_k)x_k + A_d(r_k)x_{k-\tau} + B_1(r_k)w_k \\ \qquad\quad + B_2(r_k)u_k, \\ x_t = \alpha_t, t = -\tau, \cdots, -1 \\ z_k = C(r_k)x_k + D(r_k)w_k \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state of the system, for each $i \in \mathcal{S}$, $A(i), A_d(i), B_1(i), B_2(i), C(i)$ and $D(i)$ are matrices with appropriate dimensions but unknown, τ is a constant, denoting the time-delay of the system.

In this paper, the matrices $A(i), A_d(i), B_1(i), B_2(i), C(i)$ and $D(i)$ are assumed to be in a convex set $\mathcal{M}(i)$ defined as follows:

$$\mathcal{M}(i) = \Big\{(A(i), A_d(i), B_1(i), B_2(i), C(i), D(i)) = \sum_{j=1}^{\mu} t_j(A(i,j), A_d(i,j), B_1(i,j), B_2(i,j), C(i,j), D(i,j)), t_j \geq 0, \sum_{j=1}^{\mu} t_j = 1\Big\} \quad (2)$$

with $A(i,j), A_d(i,j), B_1(i,j), B_2(i,j), C(i,j)$ and $D(i,j)$ being known matrices with appropriate dimensions.

The goal of this paper is to establish the robust stability and stabilization in the means square sense and robust H_∞ stability of system (1) by using LMI.

In the sequel, the standard notations are used. For a matrix X, $X > 0(\geq)$ means that X is symmetric and positive (semi-positive) definite, and $\lambda_{max}(X)$, $\lambda_{min}(X)$ denote respectively the maximal and minimal eigenvalues.

3 Robust Stability and Stabilization

In this section, we will consider the stability problem of system (1). First, let us give some definitions of stability.

Definition 3.1 *System (1) is said to be mean stochastically stable (MSS) if there exist a constant C such that*

$$\sum_{k=0}^{\infty} E\|x_k\|^2 \leq C\Big(\mathbb{E}(\|x_0\|^2) + \sum_{l=1}^{\tau} \|\alpha_l\|^2 + \sum_{k=0}^{\infty} \|w_k\|^2\Big) \quad (3)$$

for any $w(\cdot) \in l^2$, where $l^2 = \{\{a_k, k \geq 0\} | \sum_{k=0}^{\infty} a_k^2 < \infty\}$.

Definition 3.2 *System (1) is called mean square exponentially stable (MSES) if there exist constant $C > 0, \delta \in (0,1)$ such that*

$$\mathbb{E}[\|x_k\|^2] \leq C\delta^k. \quad (4)$$

Obviously, mean square geometrical stability implies mean stochastic stability.

In the sequel of this section we will restrict our study to the case of $B_1(r_k) = 0$, i.e.

$$\begin{cases} x_{k+1} = A(r_k)x_k + A_d(r_k)x_{k-\tau} + B_2(r_k)u_k, \\ x_t = \alpha_t, t = -\tau, \cdots, -1 \end{cases} \quad (5)$$

Definition 3.3 *System (5) with $u_k \equiv 0, \forall k \geq 1$ is said to be robustly mean square quadratically stable (MSQS) if there exist matrices $P(i) > 0, Q > 0$ such that for all $i \in \mathcal{S}$*

$$\Theta_0(i) = \begin{pmatrix} A^\top(i)G(i)A(i) - P(i) + Q & A^\top(i)G(i)A_d(i) \\ A_d^\top(i)G(i)A(i) & A_d^\top(i)G(i)A_d(i) - Q \end{pmatrix} < 0 \quad (6)$$

for any $(A(i), A_d(i)) \in \mathcal{M}(i)$, where $G(i) = \sum_{j=1}^{N} p_{ij}P(j)$.

To study the relation between MSQS and MSES, we assume the following assumption.

Assumption 3.1 *There exist a symmetric matrix $M \geq 0$, such that the state trajectory of system (5) satisfies for any $-\tau \leq \theta \leq 0$,*

$$\|x_{k+\theta}\|^2 \leq \begin{pmatrix} x_k^\top & x_{k-\tau}^\top \end{pmatrix} M \begin{pmatrix} x_k \\ x_{k-\tau} \end{pmatrix}. \quad (7)$$

Remark 3.1 *Assumption 3.1 means that the system state between k and $k - \tau$ is bounded by a function of x_k and $x_{k-\tau}$. Setting $M = \begin{pmatrix} h^2 & 0 \\ 0 & 0 \end{pmatrix}$ with $h > 0$ leads to the assumption 4 in [6]*

$$\|x_{t+\theta}\| \leq h\|x_t\| \quad (8)$$

which was pointed out not to be restrictive since h can be chosen arbitrarily. Assumption 3.1 is more logic than (8). In the case of the system being stable, (8), in fact, restricts the convergence rate of the system.

Under this assumption, we have the following theorem.

Theorem 3.1 *If system (5) is robustly MSQS, then it is robustly MSES.*

Theorem 3.1 states that MSQS is a sufficient condition for system (5) with $u_k \equiv 0, \forall k$ to be MSES. However, noting that in $\Theta_0(i)$, the parameters $A(i), A_d(i)$ are unknown and thus the results concerning the robust MSQS is not easy to use directly. Now we proceed to establish an equivalent condition for MSQS, which is provided by LMI and can be easily checked by LMI toolbox of Matlab. For this purpose, let us introduce some notations.

For any matrices $X_i > 0, i \in \mathcal{S}$, define $J_1(X), y_{ij}(X), Z_i(X)$ as follows

$$J_i(X) = \begin{pmatrix} -X_i & 0 & X_i \\ 0 & -Q & 0 \\ X_i & 0 & -Q^{-1} \end{pmatrix},$$

$$y_{ij}(X) = \begin{pmatrix} X_i A^\top(i,j) \\ A_d^\top(i,j) \\ \mathbf{0} \end{pmatrix} \begin{pmatrix} \sqrt{p_{i1}} & \cdots & \sqrt{p_{iN}} \end{pmatrix}$$

$$Z(X) = \begin{pmatrix} X_1 & & \\ & \ddots & \\ & & X_N \end{pmatrix}.$$

Theorem 3.2 *System (5) with $u_k \equiv 0, \forall k$, is robustly MSQS if and only if there exists matrix $Q > 0$ such that*

$$\begin{pmatrix} J_i(X) & y_{ij}(X) \\ y_{ij}^\top(X) & -Z(X) \end{pmatrix} < 0, j = 1, \cdots, \nu \qquad (9)$$

are feasible for some $X_i > 0, i \in \mathcal{S}$.

Proof: *Necessity*: Since system (5) is robustly MSQS, by the definition of robustly MSQS there exist $P(i) > 0, i \in \mathcal{S}, Q > 0$ such that (6) is valid for any $(A(i), A_d(i)) \in \mathcal{M}(i)$. In particular, for all the vertices of $\mathcal{M}(i)$,

$$\Theta_0(i,j) = \begin{pmatrix} A^\top(i,j)G(i)A(i,j) - P(i) + Q & A^\top(i,j)G(i)A_d(i,j) \\ A_d^\top(i,j)G(i)A(i,j) & A_d^\top(i,j)G(i)A_d(i,j) - Q \end{pmatrix} < 0 \qquad (10)$$

holds. Letting $X_i = P^{-1}(i)$, pre- and post- multiplying (10) by $\begin{pmatrix} X_i & 0 \\ 0 & I \end{pmatrix}$ gives

$$\begin{pmatrix} X_i A^\top(i,j)G(i)A(i,j)X_i - X_i + X_i Q X_i & A_d^\top(i,j)G(i)A_d(i,j) - Q \\ A_d^\top(i,j)G(i)A(i,j)X_i & X_i A^\top(i,j)G(i)A_d(i,j) \end{pmatrix} < 0 \qquad (11)$$

which combined with Schur complement yields

$$\begin{pmatrix} X_i A^\top(i,j)G(i)A(i,j)X_i - X_i & X_i A^\top(i,j)G(i)A_d(i,j) & X_i \\ A_d^\top(i,j)G(i)A(i,j)X_i & A_d^\top(i,j)G(i)A_d(i,j) - Q & \mathbf{0} \\ X_i & \mathbf{0} & -Q^{-1} \end{pmatrix} < 0. \qquad (12)$$

Note that

$$G(i) = \sum_{j=1}^{N} p_{ij} P(j) = \sum_{j=1}^{N} p_{ij} X_j^{-1}$$

$$= \begin{pmatrix} \sqrt{p_{i1}} \cdots \sqrt{p_{iN}} \end{pmatrix} Z^{-1}(X) \begin{pmatrix} \sqrt{p_{i1}} \\ \vdots \\ \sqrt{p_{iN}} \end{pmatrix}$$

and thus the left hand side of (12) is

$$J_i(X) - y_{ij}(X) Z^{-1}(X) y_{ij}^\top(X).$$

By using Schur complement again, we get (9).

Sufficiency: Noting that (9) is affine in $A(i,j), A_d(i,j)$ and reversing the above procedure, one can prove the sufficiency part of Theorem 3.2. $\nabla\nabla\nabla$

Based on Theorem 3.2, we can find a set of gain matrices $K(i), i \in \mathcal{S}$ such that the controller $u_k = K(r_k)x_k$ robustly stabilizes system (5) in the MSQS sense.

Theorem 3.3 *For given matrices X_i, Y_i, let*

$$\Xi_{ij}(X,Y) = \begin{pmatrix} X_i A^\top(i,j) + Y_i^\top B_1^\top(i,j) \\ A_d^\top(i,j) \\ 0 \end{pmatrix} \cdot \begin{pmatrix} \sqrt{p_{i1}} & \cdots & \sqrt{p_{iN}} \end{pmatrix}$$

If there exists $Q > 0$ such that

$$\begin{pmatrix} J_i(X) & \Xi_{ij}(X,Y) \\ \Xi_{ij}^\top(X,Y) & -Z(X) \end{pmatrix} < 0, j = 1, \cdots, \nu \qquad (13)$$

are feasible for $X_i > 0, Y_i > 0, i \in \mathcal{S}$, then controller $u_k = K(r_k)x_k$ with $K_i = Y_i X_i^{-1}$ robustly stabilizes (5) in the MSQS sense.

Proof: Replacing $A(i,j)$ in Theorem 3.2 by $A(i,j) + B(i,j)K(i)$ and letting $Y_i = K(i)X_i$ leads to Theorem 3.3. $\nabla\nabla\nabla$

4 H_∞ Disturbance Attenuation

Section 3 has solved the robust stability and stabilization problem, this section will be devoted to studying the H_∞ control problem of system (1). To this end, let us introduce some definitions.

Definition 4.1 *System (1) with $u_k \equiv 0$ is said to robustly MSQS with noise attenuation level γ, if there exists a constant C such that*

$$\|z\|_2 \leq \gamma \left[\|w\|_2^2 + C \left(\mathbb{E}\|x_0\|^2 + \sum_{t=1}^{\tau} \|\alpha_t\|^2 \right) \right]^{1/2} \quad (14)$$

for any noise disturbance $w(\cdot) \in l^2$,where $\|z\|_2 = \left[\sum_{k=0}^{\infty} \mathbb{E}[z_k^\top z_k]\right]^{1/2}$

Definition 4.2 *System (1) with $u_k \equiv 0$ is said to be internally MSQS if (6) holds for all $(A(i), A_d(i), B_1(i)) \in \mathcal{M}(i)$.*

By definition, internally robustly MSQS is in fact robustly MSQS in the case of zero noise disturbance. The following Theorem 4.1 shows that if system (5) is robustly MSQS and an l^2 disturbance noise is added to its input, then the system is robustly MSS. Before presenting this theorem let us give two lemmas.

Lemma 4.1 *If system (5) with $u_k \equiv 0$ is robustly MSQS, then there exists $\varepsilon > 0$ such that $\forall (A(i), A_d(i)) \in \mathcal{M}(i), i \in \mathcal{S}$*

$$\Theta_1(i) = \Theta_0(i) + \varepsilon \begin{pmatrix} A^\top(i)A(i) & \mathbf{0} \\ \mathbf{0} & A_d^\top(i)A_d(i) \end{pmatrix} < 0 \quad (15)$$

are feasible for some $P(i) > 0, Q > 0$.

Theorem 4.1 *Suppose system (1) with $u_k \equiv 0$ is internally MSQS and the noise disturbance $w(\cdot) \in l^2$, then system (1) is MSS, i.e. there exists a positive constant C, such that*

$$\sum_{k=0}^{\infty} \mathbb{E}[\|x_k\|^2] \leq C \Big(\mathbb{E}[\|x_0\|^2] + \sum_{l=1}^{\tau} \|\alpha_l\|^2 + \sum_{k=1}^{\infty} \|w_k\|^2 \Big) \quad (16)$$

which implies $\lim_{k\to\infty} E[\|x_k\|^2] = 0$.

With these preparations, we can now establish the robust H_∞ stability of system (1). For given $X_i > 0$, define matrices $J_{2ij}(X), \Xi_\infty(i,j)$ as follows

$$J_{2ij}(X) = \begin{pmatrix} -X_i & \mathbf{0} & C^\top(i,j) & X_iC^\top(i,j) & X_i \\ \mathbf{0} & -Q & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ C(i,j) & \mathbf{0} & -\gamma^2 I & D(i,j) & \mathbf{0} \\ C(i,j)X_i & \mathbf{0} & D^\top(i,j) & -I & \mathbf{0} \\ X_i & \mathbf{0} & \mathbf{0} & \mathbf{0} & -Q^{-1} \end{pmatrix}$$

$$\Xi_\infty(i,j) = \begin{pmatrix} X_iA^\top(i,j) \\ A_d^\top(i,j) \\ B_1^\top(i,j) \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \begin{pmatrix} \sqrt{p_{i1}} & \cdots & \sqrt{p_{iN}} \end{pmatrix}$$

Theorem 4.2 *If there exists matrix $Q > 0$, such that for all $i \in \mathcal{S}, j = 1, \cdots, \nu$*

$$\Theta_\infty(i,j) = \begin{pmatrix} J_{2ij}(X) & \Xi_\infty(i,j) \\ \Xi_\infty^\top(i,j) & -Z_i(X) \end{pmatrix} < 0 \quad (17)$$

are feasible for some $X_i > 0, i \in \mathcal{S}$, then

$$\|z\|_2 \leq \gamma \Big[\|w\|_2^2 + C_0 \Big(\mathbb{E}[\|x_0\|^2] + \sum_{l=1}^{\tau} \|\alpha_l\|^2 \Big) \Big]^{1/2} \quad (18)$$

where $C_0 = \max\{\lambda_{max}(P(i)), \lambda_{max}(Q)\}$.

Theorem 4.2 establishes the robust H_∞ of system (1). In fact, based on Theorem 4.2, we can develop an algorithm to design controller $u_k = K(r_k)x_k$ that robustly stabilizes system (1) and minimizes the noise attenuation level. To this end, let us define

$$J_{2ij}^v(X) = \begin{pmatrix} -X_i & \mathbf{0} & C^\top(i,j) & X_iC^\top(i,j) & X_i \\ \mathbf{0} & -Q & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ C(i,j) & \mathbf{0} & -vI & D(i,j) & \mathbf{0} \\ C(i,j)X_i & \mathbf{0} & D^\top(i,j) & -I & \mathbf{0} \\ X_i & \mathbf{0} & \mathbf{0} & \mathbf{0} & -Q^{-1} \end{pmatrix}$$

$$\Xi_{ij}^{\infty}(X,Y) = \begin{pmatrix} X_i A^\top(i,j) + Y_i^\top B^\top(i,j) \\ A_d^\top(i,j) \\ B_1^\top(i,j) \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \cdot \begin{pmatrix} \sqrt{p_{i1}} & \cdots & \sqrt{p_{iN}} \end{pmatrix}$$

By using Theorem 4.2, we have the following theorem.

Theorem 4.3 *Consider the following optimization problem*

$$\mathcal{P}_1 : \qquad \min_{X_i, Y_i} \ v$$
$$s.t. \begin{cases} X_i > 0, Y_i > 0 \\ \begin{pmatrix} J_{2ij}^v(X) & \Xi_{ij}^{\infty}(X,Y) \\ \Xi_{ij}^{\infty\top}(X,Y) & -Z(X) \end{pmatrix} < 0 \end{cases}$$

Suppose γ_0, X_i, Y_i are a set of feasible solution of $\mathcal{P}_1$, then state feedback control $u_k = K(r_k)x_k$ with gain matrices $K_i = Y_i X_i^{-1}$ robustly stabilizes the system (1) in the MSQS sense with noise attenuation level $\sqrt{\gamma_0}$.

Theorem 4.3 provides a robust H_∞ control design technique, this design technique is a convex optimization and thus can be easily solved by using the LMI toolbox of Matlab. In Theorem 4.3 matrix $Q > 0$ is in fact unknown but not design parameters. In practice, we can choose $Q = \alpha I$ and the optimal α can be obtained by using one-dimensional search technique. By this way, a feasible suboptimal solution can be obtained.

5 Delay-Dependent Robust stability and H_∞ Control

Since x_k is dependent on $x_{k-\tau}$, $\{(x_k, r_k), k \geq 0\}$ is not a Markov chain. However, an augmented Markov system with higher dimension can be constructed from (5) by using the supplementary variables technique. Indeed, define

$$y_k = \begin{pmatrix} x_k \\ x_{k-1} \\ \vdots \\ x_{k-\tau} \end{pmatrix}, A_0(i) = \begin{pmatrix} A(i) & \mathbf{0} & A_d(i) \\ I & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & I_\tau & \mathbf{0} \end{pmatrix}$$

where I, I_τ are identity matrices with dimensions n and $(\tau - 1) \times n$.

Then system (5) becomes into

$$\begin{cases} y_{k+1} = A_0(r_k) y_k, \\ y_0 = \begin{pmatrix} x_0 \\ \alpha_1 \\ \vdots \\ \alpha_\tau \end{pmatrix} \\ z_k = \tilde{C}(r_k) y_k + D(r_k) w_k, \end{cases} \qquad (19)$$

where $\tilde{C}(i) = \begin{pmatrix} C(i) & \mathbf{0} \end{pmatrix}$. This system is a $n(\tau+1)$-dimensional system without time-delay.

Evidently, $\|x_k\| \leq \|y_k\|$, thus if $\{y_k, k \geq 0\}$ is shown to be MSES, then the mean square exponential stability of $\{x_k, k \geq 0\}$ follows.

Theorem 5.1 *Let*

$$U_{ij}(X) = X_i A_0^\top(i,j) \begin{pmatrix} \sqrt{p_{i1}} & \cdots & \sqrt{p_{iN}} \end{pmatrix}.$$

If for all $j = 1, \cdots, \nu$, LMIs

$$\begin{pmatrix} -X_i & U_{ij}(X) \\ U_{ij}^\top(X) & -Z(X) \end{pmatrix} < 0, i \in \mathcal{S} \qquad (20)$$

are feasible for some $X_i > 0$, then $\{x_k, k \geq 0\}$ is robustly MSES.

Remark 5.1 *By using supplementary variables, the system (5) with time-delay is converted into (19), which is a standard jump linear system. Thus the results of jump linear system can be applicable. However, the system dimension increases greatly. In fact, if the system (5) is of dimension n, the feasible problem (9) to judge the robust stability contains $[Nn(n+1)]/2$ unknown variables. Nevertheless, the feasible problem corresponding to the robust stability of (19) contains $(N/2)(\tau+1)n[(\tau+1)n+1]$ unknowns. Thus the delay-dependent technique is only suitable for system with small time-delay.*

Remark 5.2 *Since $A_0(i,j)$ depends on the delay τ, the upper bound of time-delay τ can be obtained by solving*

$$\max_{X_i > 0} \ \tau$$
$$s.t. \ (20)$$

Theorem 5.2 *For $X_i \in \mathbb{R}^{n(\tau+1)\times n(\tau+1)}$, $Y_i \in \mathbb{R}^{n\times n}$, $i \in \mathcal{S}$, define matrices $\tilde{J}_1(i,j)$, $\tilde{\Xi}(i,j)$ and $\tilde{y}_\infty(i,j)$ as follows*

$$\tilde{J}_1(i,j) = \begin{pmatrix} -X_i & \mathbf{0} & X_i \tilde{C}^\top(i,j) \\ \mathbf{0} & -\gamma^2 I & D(i,j) \\ \tilde{C}(i,j) X_i & D^\top(i,j) & -I \end{pmatrix},$$

$$\tilde{\Xi}(i,j) = \begin{pmatrix} X_i A_0^\top(i,j) \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \begin{pmatrix} \sqrt{p_{i1}} & \cdots & \sqrt{p_{iN}} \end{pmatrix}$$

$$\tilde{y}_\infty(i,j) = Y_i^\top B^\top(i,j) \begin{pmatrix} \sqrt{p_{i1}} & \cdots & \sqrt{p_{iN}} \end{pmatrix}.$$

Suppose $X_i, Y_i, i \in \mathcal{S}$ is a set of feasible solution of the following LMIs

$$\begin{pmatrix} \tilde{J}_1(i,j) & \tilde{\Xi}(i,j) \\ \Xi^\top(i,j) & Z(X) \end{pmatrix} + \begin{pmatrix} \mathbf{0} & \tilde{y}_\infty(i,j) \\ \tilde{y}_\infty^\top(i,j) & \mathbf{0} \end{pmatrix} < 0. \quad (21)$$

Let $K_i = Y_i X_{11}^{-1}(i)$, where

$$X_i = \begin{pmatrix} X_{11}(i) & X_{12}(i) \\ X_{12}^\top(i) & X_{22}(i) \end{pmatrix}.$$

Then the control $u(t) = K(r_t)x_t$ robust stabilizes the system (5) in the MSQS sense and the closed-loop system verifies the noise attenuation level γ.

By the same technique as in Theorem 4.2, letting $v = \gamma^2$ in (21) we can obtain the optimal noise attenuation level by solving the following optimization problem

$$\mathcal{P}_2: \quad \min_{X_i, Y_i} \; v \quad \text{s.t. } (21).$$

Remark 5.3 *When the control also contains time-delay, i.e. the system dynamics is as follows*

$$\begin{cases} x_{k+1} = A(r_k)x_k + A_d(r_k)x_{k-\tau_1} + B_1(r_k)w_k \\ \qquad + B_2(r_k)u_k + B_d(r_k)u_{k-\tau_2}, \\ x_t = \alpha_t, t = -\tau_1, \cdots, -1 \\ z_k = C(r_k)x_k + D(r_k)w_k \end{cases} \quad (22)$$

where $\tau_1, \tau_2 (\leq \tau_1)$ are the time-delays in the state and control respectively, the results of previous sections are not valid. In fact, suppose $\tau_1 = \tau_2$, if we consider the control $u(t) = K(r_t)x_t$, then the closed-loop system has dynamics

$$\begin{cases} x_{k+1} = [A(r_k) + B_2(r_k)K(r_k)]x_k + [A_d(r_k) \\ \qquad + B_d(r_k)K(r_{k-\tau})]x_{k-\tau} + B_1(r_k)w_k, \\ x_t = \alpha_t, t = -\tau, \cdots, -1 \end{cases}$$

In view of Theorem 3.2, to handle the stabilization problem one has to deal with $K(r_k)$ and $K(r_{k-\tau})$ at the same time, which seems to be impossible, since $\{r_k\}$ evolves randomly.

6 Conclusion

In this paper, Some LMI-based sufficient conditions for the delay-independent robust stability, stabilization and H_∞ control problems of time-delay jump linear system with polytopic uncertain parameters are developed. By using the supplementary technique, the system with small time-delay is converted into a higher dimension system without time-delay, and thus can be handled as a standard jump linear system with uncertain parameters. Simulation results show that by using the supplementary technique one can obtain a control law such that the closed-loop system verifies smaller noise attenuation, however the system dimension increases greatly.

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