Global Output Regulation with Minimum Information of Uncertain Nonlinear Systems Subject to Exogenous Signals

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Abstract

This paper deals with global output regulation of uncertain nonlinear systems affected by disturbances which are generated from a known exosystem. The information needed for the control design is that the uncertain system satisfies the geometric conditions for the output feedback form with stable zero-dynamics and the parameterization is linear for unknown constant parameters and the unmeasurable disturbances. This information is minimum compared with the conditions specified in the literature for global output regulation.

1 Introduction

Output regulation of nonlinear systems has attracted the attention of control engineers in recent years. When the exogenous signals (including both reference signals and disturbances) are generated by known autonomous exosystems, local results have been shown in [1, 2] using the full information including the measurements of exogenous signals as well as states. The necessary and sufficient conditions for existing a local full information solution are specified as that the linearized system is stabilizable and there exists a certain invariant manifold [1]. A semiglobal extension to these results for a class of feedback linearizable systems is reported in [3]. Output regulation of error feedback is solved recently [4, 5] with the application of system immersion technique. The uncertainty parameterized by unknown constant parameters are treated as special cases of exogenous signals and the solution, extended from the error feedback regulation, is referred to as structurally stable regulation. A semiglobal adaptive output feedback control is presented in [6] for nonlinear systems represented by input-output models, using a high-gain observer. Global solutions for output regulation using state or partial state feedback are shown for strict feedback systems in [7] and for extended strict feedback systems in [8].

In this paper, we consider the globally asymptotic output tracking of any smooth trajectories using output feedback for a class of uncertain systems with disturbances generated from known exosystems. We assume that the uncertain nonlinear system satisfies the conditions specified in [9] so that it can be transformed to the output feedback form used in [9, 10]. Another assumption is that the zero-dynamics is stable. No other assumptions, such as the sign of the high frequency gain and the bound of the unknown parameters, are needed in this paper for the control design. For systems in the output feedback form whose unknown parameters are constants with known sign of the high frequency gain, adaptive control algorithms have been proposed to achieve asymptotic output tracking in [9, 10] using adaptive backstepping. With the unknown parameters being interpreted as disturbances, almost disturbance decoupling is shown in [11, 12] at the cost of asymptotic output tracking. A combination of adaptive output tracking and almost disturbance decoupling is achieved in [13]. For the method proposed in this paper, asymptotic tracking via output feedback is achieved provided that the unknown parameters (or disturbances) are constants or they are generated from a known exosystem.

The proposed controller is a dynamic one that only needs the measurement of the system output. The controller dynamics includes the estimation of both state variables and the unknown parameters or disturbances. We propose a new state estimator, different from those estimators used for adaptive output tracking in [9, 10], to take into the consideration of time-varying nature of unknown disturbances, and at the same time, to achieve the dynamic swapping enjoyed by the K-filters used for adaptive output tracking in [10]. Since the sign of the high frequency gain is also unknown, we propose a Nussbaum gain in the control design. Based on the proposed state estimator and the Nussbaum gain, the control input for the output regulation is designed using adaptive backstepping with tuning functions to avoid over-parameterization.

2 Problem Formulation

We consider a single-input-single-output nonlinear system which can be transformed into the output feedback form

$$\begin{aligned} \dot{x} &= A_c x + b\sigma(y)u + \phi_0(y) + \sum_{i=1}^{p} \phi_i(y)a_i \\ &:= A_c x + b\sigma(y)u + \phi_0(y) + \Phi(y)a \\ y &= e_1^T x \end{aligned} \tag{1}$$

with

$$A_c = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}, \quad a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix},$$

$$b = [0 \ \dots \ 0 \ b_\rho \ \dots \ b_n]^T := \left[0_{1\times(\rho-1)} \ \ \bar{b}^T\right]^T,$$

where $x \in R^n$ is the state vector, $u \in R$ is the control, e_i denotes the i column of the identity matrix I, ϕ_i, $0 \le i \le p$, are known smooth vector fields in R^n, $\sigma : R \to R$ is a smooth function and $\sigma(y) \neq 0$, $\forall y \in R$, b is an unknown constant vector with $b_\rho \neq 0$, a denotes uncertain system parameters as well as disturbances, and it is generated from an exosystem

$$\dot{a} = S_a(y)a \tag{2}$$

Remark 1: The coordinate-free geometric conditions for the existence of state transform for transforming a nonlinear system into (1) are specified in [9]. $b_\rho \neq 0$ indicates the nonlinear system before the transformation has a constant relative degree of ρ. In the adaptive output tracking in [9], a and b are assumed to be unknown constant vectors, while in the almost disturbance decoupling in [11], a is assumed to be bounded disturbances, with b known. In [13], a contains constant unknown system parameters and unknown bounded disturbances, and b is an unknown constant vector.

Assumption 1: The system is of minimum phase, i.e., the polynomial $\mathcal{B}(s) = \sum_{i=\rho}^{n} b_i s^{n-i}$ is Hurwitz.

Assumption 2: There exists a *constant* positive definite matrix P_a such that

$$P_a S_a + S_a^T P_a = 0 \tag{3}$$

Remark 2: We do not assume that the sign of the high frequency gain b_ρ, unlike most of the results in adaptive control literature.

Remark 3: Assumption 2 is similar to the neutral stable assumption of the exosystem adopted in [1], to ensure the boundedness of exo-signals. From Assumption 2, we can state, without loss of generality, that S_a is skew-symmetric, i.e.,

$$S_a + S_a^T = 0 \tag{4}$$

which can always be achieved by re-defining a and $\Phi(y)$. If the original unknown parameter vector is $\bar{a}$ and its affined nonlinear matrix is $\bar{\Phi}(y)$, with $\dot{\bar{a}} = \bar{S}_a \bar{a}$ and $P_a \bar{S}_a + \bar{S}_a^T P_a = 0$, then we can re-define

$$a = (P_a)^{1/2} \bar{a} \tag{5}$$

$$\Phi(y) = \bar{\Phi}(y)(P_a)^{-1/2} \tag{6}$$

In the remaining part of the paper, we assume the above transforms have been done and (4) is referred to whenever Assumption 2 is needed.

Remark 4: Comparing the exosystem with the one in [1, 4, 5], Assumption 2 allows the exosystem dynamics depend on the system output, as long as P_a is a constant positive definite matrix, satisfying (3). Similar to the assumption on the exosystem made in [4, 5], Assumption 2 also accommodates sinusoids of known frequencies, but with unknown amplitudes and phases. For example, if a sinusoid $\eta(t)$ is of the frequency ω_d, then it must satisfy

$$\ddot{\eta} + \omega_d^2 \eta = 0 \tag{7}$$

if we let $a = [\eta, \eta/\omega_d]^T$, it is easy to see

$$\dot{a} = \begin{bmatrix} 0 & \omega_d \\ -\omega_d & 0 \end{bmatrix} a \tag{8}$$

The output regulation problem we are going to solve is based on the definition below.

Definition 1: The output regulation is said to be *globally solvable* for uncertain nonlinear system (1) if there is a finite dimensional system

$$\begin{aligned} \dot{\mu} &= \nu(\mu, y(t), y_r, \dots, y_r^{\rho}), \quad \mu \in R^r, \\ u &= u(\mu, y(t), y_r, \dots, y_r^{\rho}) \end{aligned} \tag{9}$$

such that for every $x(0) \in R^n$, $\mu(0) \in R^r$ and for every bounded smooth output reference y_r with bounded derivatives up to order ρ, $x(t)$, $\mu(t)$ and $u(t)$ are bounded $\forall t \ge 0$, and the output tracking error, $y - y_r$, asymptotically approaches zero.

3 Design of Filters for State Estimation

For dynamic output feedback control, state variables need to be estimated for the control design directly or indirectly. Because the separation principle and certainty equivalence control are not applicable to nonlinear systems in general, state estimation is expected to be different from observers for linear systems. For the output feedback system with constant uncertain parameters, MT-filters and K-filters have been proposed for state estimation or dynamic swapping in [9, 10], but they cannot be used for the system (1) that we are considering, as a, the unknown vector for parameters and disturbances, is time-varying.

We re-arrange the system as

$$\dot{x} = A_c x + \phi_0(y) + F^T(y,u)\theta \tag{10}$$

where the $q = n - \rho + 1 + p$ vector θ is defined by

$$\theta = \begin{bmatrix} \bar{b} \\ a \end{bmatrix}$$

and

$$F(y,u)^T = \left[\begin{bmatrix} 0_{(\rho-1)\times(n-\rho+1)} \\ I_{n-\rho+1} \end{bmatrix} \sigma(y)u, \; \Phi(y) \right]$$

Notice that θ, containing unknown system parameters and unknown disturbances, is not constant, and in fact we can write

$$\dot{\theta} = S(y)\theta \tag{11}$$

where $S(y) = \text{diag}\{0_{(n-\rho+1)\times(n-\rho+1)}, S_a(y)\}$.

Following the idea of K-filter design in [10], we aim at an exponentially decaying estimation error $\epsilon = x - \hat{x}$ with

$$\hat{x} = \xi + \Omega^T\theta. \tag{12}$$

where ξ and Ω are two signals generated from filters to be designed. The filter for ξ follows the design as in K-filters as

$$\dot{\xi} = A_0\xi + ky + \phi_0(y), \tag{13}$$

where

$$k = [k_1, \ldots, k_n]^T, \quad A_0 = A_c - ke_1^T \tag{14}$$

with k being chosen so that A_0 is Hurwitz.

For Ω, we have

$$\dot{\Omega}^T = A_0\Omega^T + F(y,u)^T - \Omega^T S(y) \tag{15}$$

where the last term is added in to accommodate the time-varying nature of the unknown vector θ. Similar to the K-filters in [10], the order of the filters can be reduced by

$$\begin{aligned} \Omega^T &= [v_\rho, \ldots, v_n, \Xi], & (16)\\ \dot{\Xi} &= A_0\Xi + \Phi(y) - \Xi S_a(y), & (17)\\ \dot{\lambda} &= A_0\lambda + e_n\sigma(y)u, & (18)\\ v_j &= A_0^{n-j}\lambda, \text{ for } j = \rho, \ldots, n. & (19) \end{aligned}$$

The following lemma summaries the properties of the filters for later use in the paper.

Lemma 3.1: The state estimation based on the filters (13) and (15) has the following properties:

(i) Let the estimate of the state be given by

$$\hat{x} = \xi + \Omega^T\theta \tag{20}$$

and define the estimation error by

$$\epsilon = x - \hat{x} \tag{21}$$

Then estimation error satisfies

$$\dot{\epsilon} = A_0\epsilon \tag{22}$$

(ii) The filters (13) and (17) are bounded-input-bounded-output (BIBO), i.e., if y is bounded, then ξ, Ξ are also bounded.

Proof: The property (i) follows from a straightforward evaluation of $\dot{\epsilon} = \dot{x} - \dot{\hat{x}}$ using (12), (13) and (15). The boundedness of ξ follows from the fact that A_0 is Hurwitz. The remaining part of the proof is to establish that Ξ-system (17) is BIBO.

Let us introduce a number of notations for convenience. If M is an $p \times q$ matrix, where $M = [m_1, \ldots, m_q]$, with m_i as the ith column vector of M, then vec(M) denotes the vector obtained by rolling out the column vectors of M, i.e.,

$$\text{vec}(M) = \begin{bmatrix} m_1 \\ \vdots \\ m_q \end{bmatrix} \tag{23}$$

The direct product of two matrices M and N, which is also known as the Kronecker product, is denoted by $M \otimes N$. The following useful properties concerning the vector of a matrix and direct product of matrices are extracted from [14]:

$$\begin{aligned} \text{vec}(ABC^T) &= (A\otimes C)\text{vec}(B) & (24)\\ (A\otimes B)^T &= A^T \otimes B^T & (25)\\ (A\otimes B)(C\otimes D) &= (AC)\otimes(BD) & (26) \end{aligned}$$

where A, B, C and D are with proper dimensions so that their products can be evaluated.

Using the notations introduced above together with (24), we re-write the Ξ-system (17) as

$$\frac{d}{dt}\text{vec}(\Xi) = [A_0\otimes I - I\otimes S_a^T(y)]\text{vec}(\Xi) + \text{vec}(\Phi(y)) \tag{27}$$

Since A_0 is Hurwitz, there exists a positive definite matrix P satisfying

$$PA_0 + A_0^T P = -I \tag{28}$$

Define

$$V_\Xi = \frac{1}{2}\text{vec}(\Xi)^T(P\otimes I)\text{vec}(\Xi) \tag{29}$$

where $(P \otimes I)$ is positive definite. Evaluating its derivative along (27), and using (25), (26) and Assumption 2, we have

$$\begin{aligned}
\dot{V}_\Phi &= \frac{1}{2}\mathrm{vec}(\Xi)^T[(P\otimes I)(A_0\otimes I) \\
&\quad + (A_0\otimes I)^T(P\otimes I)]\mathrm{vec}(\Xi) \\
&\quad -\frac{1}{2}\mathrm{vec}(\Xi)^T[(P\otimes I)(I\otimes S_a^T(y)) \\
&\quad + (I\otimes S_a^T(y))^T(P\otimes I)]\mathrm{vec}(\Xi) \\
&\quad +\mathrm{vec}(\Xi)^T(P\otimes I)\mathrm{vec}(\Phi(y)) \\
&= \frac{1}{2}\mathrm{vec}(\Xi)^T[(PA_0+A_0P)\otimes I \\
&\quad - P\otimes(S^T+S)]\mathrm{vec}(\Xi) \\
&\quad +\mathrm{vec}(\Xi)^T(P\otimes I)\mathrm{vec}(\Phi(y)) \\
&= -\|\mathrm{vec}(\Xi)\|^2 + \mathrm{vec}(\Xi)^T(P\otimes I)\mathrm{vec}(\Phi(y)) \\
&\leq -\frac{1}{2}\|\mathrm{vec}(\Xi)\|^2 + \frac{1}{2}\|(P\otimes I)\mathrm{vec}(\Phi(y))\|^2 \\
&\leq -\frac{1}{2\lambda_{max}(P)}V_\Xi + \frac{1}{2}\|(P\otimes I)\mathrm{vec}(\Phi(y))\|^2
\end{aligned} \tag{30}$$

where $\lambda_{max}(P)$ denotes the maximum eigenvalue of P. Applying Grown-Bellman lemma, we have

$$\begin{aligned}
V_\Xi(t) &\leq V_\Xi(0)e^{-\frac{t}{2\lambda_{max}(P)}} \\
&\quad + \frac{1}{2}\int_0^t e^{-\frac{t-\tau}{2\lambda_{max}(P)}}\|(P\otimes I)\mathrm{vec}(\Phi(y))\|^2 d\tau
\end{aligned} \tag{31}$$

Therefore, the boundedness of y ensures the boundedness of V_Ξ and Ξ-system is BIBO. △

Remark 5: The observer based on (13), (15) and (20) is not implementable, because it depends on θ which is unknown. However, it does provide a nice static relationship between the state variable x and θ:

$$x = \xi + \Omega^T\theta + \epsilon \tag{32}$$

which will be used in the backstepping design to replace the unmeasurable state variables in the same way as in [10]. The terms in $\Omega^T\theta$ will be grouped together with other terms associated with θ in the design, and adaptive control technique is then used to deal with the uncertainty of θ.

An estimator for the unknown vector θ is designed as

$$\dot{\hat{\theta}} = S(y)\hat{\theta} + \tau_\rho \tag{33}$$

where $\hat{\theta}$ is an estimate of θ, and τ_ρ is an interlace function to be decided later in the control design. Let $\tilde{\theta} = \theta - \hat{\theta}$, then (33) can be written as

$$\dot{\tilde{\theta}} = S(y)\tilde{\theta} - \tau_\rho \tag{34}$$

4 Control Design

Since the time-varying effects of the exosystem has been absorbed in the filter design of (15) and in the observer (34) in the last section, control design can be carried out using adaptive backstepping together with a Nussbaum gain. Define

$$z_1 = y - y_r \tag{35}$$

$$z_i = v_{\rho,i} - \alpha_{i-1}, \quad i = 2,\ldots,\rho \tag{36}$$

$$z_{\rho+1} = 0 \tag{37}$$

where α_i, $i = 1,\ldots,\rho$, are stabilizing functions to be decided in the control design. Consider the dynamics of z_1

$$\dot{z}_1 = x_2 + \phi_{0,1} + \Phi_{(1)}\theta - \dot{y}_r \tag{38}$$

where the subscript (i) is used to denote the ith row of a matrix. From the design of the state estimator, in particular, from (32), we have $x_2 = \xi_2 + \Omega_{(2)}^T\theta + \epsilon_2$, and we use this to replace the unmeasurable x_2 in (38), resulting at

$$\begin{aligned}
\dot{z}_1 &= \xi_2 + \Omega_{(2)}^T\theta + \epsilon_2 + \phi_{0,1} + \Phi_{(1)}\theta - \dot{y}_r \\
&= b_\rho v_{\rho,2} + \omega_0 + \bar{\omega}^T\theta + \epsilon_2 - \dot{y}_r
\end{aligned} \tag{39}$$

where $\omega_0 = \xi_2 + \phi_{0,1}$ and $\bar{\omega} = [0, v_{\rho+1,2},\ldots v_{n,2}, \Phi_{(1)} + \Xi_{(2)}]^T$. Since the sign of the high frequency gain, b_ρ, is unknown, we employ a Nussbaum gain, N, in the control design. Let

$$\alpha_1 = N(\kappa)\bar{\alpha}_1 \tag{40}$$

where κ is a parameter generated by

$$\dot{\kappa} = \gamma_\kappa z_1\bar{\alpha}_1 \tag{41}$$

with γ_κ being a positive real design parameter. The Nussbaum gain N is a function (e.g. $N(\kappa) = \kappa^2\cos\kappa$) which satisfies the two-sided Nussbaum properties [15, 16]

$$\lim_{\kappa\to\pm\infty}\sup\frac{1}{\kappa}\int_0^\kappa N(s)ds = +\infty \tag{42}$$

$$\lim_{\kappa\to\pm\infty}\inf\frac{1}{\kappa}\int_0^\kappa N(s)ds = -\infty \tag{43}$$

where $\kappa \to \pm\infty$ denotes $\kappa \to +\infty$ and $\kappa \to -\infty$ respectively.

We design $\bar{\alpha}_1$ as

$$\bar{\alpha}_1 = -c_1z_1 - d_1z_1 - \omega_0 - \bar{\omega}^T\hat{\theta} - \dot{y}_r \tag{44}$$

where c_1 and d_1 are two of the positive real design parameters c_i and d_i, $i = 1,\ldots,\rho$. Notice that

$$b_\rho v_{\rho,2} = b_\rho(z_2 + \alpha_1) = \hat{b}_\rho z_2 + b_\rho\alpha_1 + \tilde{b}_\rho z_2 \tag{45}$$

where $\tilde{b}_\rho = b_\rho - \hat{b}_\rho$. Then from (44) and (39), we have

$$\begin{aligned}
\dot{z}_1 &= \hat{b}_\rho z_2 + b_\rho(N(\kappa)-1)\bar{\alpha}_1 + (\bar{\omega} + z_2e_1)^T\tilde{\theta} \\
&\quad -c_1z_1 - d_1z_1 + \epsilon_2
\end{aligned} \tag{46}$$

Based on (46), we design the first tuning function

$$\tau_1 = (\bar{\omega} + z_2 e_1) z_1 \tag{47}$$

After the design of α_1 the remaining stabilizing functions can be designed in a similar way to the standard adaptive backstepping shown in [10]. Omitting the deriving procedures, we briefly describe the final results

$$\begin{aligned}
\alpha_2 &= -\hat{b}_\rho z_1 - c_2 z_2 - d_2 \left(\frac{\partial \alpha_1}{\partial y}\right)^2 z_2 + k_2 v_{\rho,1} \\
&\quad + \frac{\partial \alpha_1}{\partial X_1}\dot{X}_1 + \frac{\partial \alpha_1}{\partial y}(\omega_0 + \omega^T \hat{\theta}) + \frac{\partial \alpha_1}{\partial \hat{\theta}}\tau_2 \quad (48) \\
\alpha_i &= -z_{i-1} - c_i z_i - d_i \left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^2 z_i + k_i v_{\rho,1} \\
&\quad + \frac{\partial \alpha_{i-1}}{\partial X_{i-1}}\dot{X}_{i-1} + \frac{\partial \alpha_{i-1}}{\partial y}(\omega_0 + \omega^T \hat{\theta}) \\
&\quad + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\tau_i - \sum_{j=2}^{i-1} \sigma_{j,i} z_j, \quad i = 3, \ldots, \rho \quad (49)
\end{aligned}$$

where

$$\begin{aligned}
\omega &= [v_{\rho,2}, \ldots, v_{n,2}, \Phi_{(1)} + \Xi_{(2)}]^T && (50) \\
\bar{\lambda}_i &= [\lambda_1, \ldots, \lambda_i]^T, && (51) \\
\bar{y}_i &= [y_r, \dot{y}_r, \ldots, y_r^{(i)}]^T, && (52) \\
X_i &= [\xi^T, \mathrm{vec}(\Xi)^T, \kappa, \bar{\lambda}_i^T, \bar{y}_i^T]^T, && (53) \\
\sigma_{j,i} &= \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}}\Gamma \frac{\partial \alpha_{i-1}}{\partial y}\omega. && (54)
\end{aligned}$$

In the above, the interlace functions τ_i are defined by

$$\tau_i = \tau_{i-1} - \Gamma \frac{\partial \alpha_{i-1}}{\partial y}\omega z_i \qquad i = 2, \ldots, \rho \tag{55}$$

where Γ is a constant positive definite matrix satisfying $\Gamma^{-1}S(y) + S(y)^T\Gamma^{-1} = 0$, and in fact we can take $\Gamma = \gamma_\theta I$, $\gamma_\theta > 0$, following Assumption 2 and (4). The control input is given by

$$u = \frac{1}{\sigma(y)}(\alpha_\rho - v_{\rho,\rho+1}). \tag{56}$$

Theorem 4.1: The output regulation problem is globally solvable for system (1) under Assumptions 1 and 2.

Proof: Based on the stabilizing functions shown in (48) and (49), the dynamics of z_i, for $i = 2, \ldots, \rho$, can be written as

$$\begin{aligned}
\dot{z}_2 &= -\hat{b}_\rho z_1 - c_2 z_2 - d_2 \left(\frac{\partial \alpha_1}{\partial y}\right)^2 z_2 + z_3 \\
&\quad - \frac{\partial \alpha_1}{\partial y}\omega^T \tilde{\theta} - \frac{\partial \alpha_1}{\partial y}\epsilon_2 + \sum_{j=3}^{\rho} \sigma_{2,j} z_j, \quad (57) \\
\dot{z}_i &= -z_{i-1} - c_i z_i - d_i \left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^2 z_i + z_{i+1} \\
&\quad - \frac{\partial \alpha_{i-1}}{\partial y}\omega^T \tilde{\theta} - \frac{\partial \alpha_{i-1}}{\partial y}\epsilon_2 + \sum_{j=i+1}^{\rho} \sigma_{i,j} z_j \\
&\quad - \sum_{j=2}^{i-1} \sigma_{j,i} z_j \qquad i = 3, \ldots, \rho \quad (58)
\end{aligned}$$

Let

$$V = \frac{1}{2}\sum_{i=1}^{\rho} z_i^2 + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1}\tilde{\theta} + \sum_{i=1}^{\rho} \frac{1}{4d_i}\epsilon^T P \epsilon. \tag{59}$$

From (46), (57), (58), (34), (47) and (55), it can be shown that

$$\begin{aligned}
\dot{V} &= b_\rho (N(\kappa) - 1) z_1 \bar{\alpha}_1 - \sum_{i=1}^{\rho}\left[c_i + d_i \left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^2\right] z_i^2 \\
&\quad - \sum_{i=1}^{\rho} z_i \frac{\partial \alpha_{i-1}}{\partial y}\epsilon_2 + \sum_{i=1}^{\rho} \frac{1}{4d_i}\|\epsilon\|^2 \quad (60)
\end{aligned}$$

where we set $\frac{\partial \alpha_0}{\partial y} = -1$. Noting (41) and

$$|z_i \frac{\partial \alpha_{i-1}}{\partial y}\epsilon_2| \leq \frac{1}{4d_i}\|\epsilon_2\|^2 + d_i \left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^2 z_i^2 \tag{61}$$

we have

$$\dot{V} \leq b_\rho (N(\kappa) - 1)\dot{\kappa}/\gamma_\kappa - \sum_{i=1}^{\rho} c_i z_i^2 \tag{62}$$

The boundedness of V can be establishes based on the Nussbaum gain properties (42) and (43) in a similar way as in [16] via an argument of contradiction. In fact, integrating (62) gives

$$\begin{aligned}
V(t) + \int_0^t \sum_{i=1}^{\rho} c_i z_i^2 dt &\leq b_\rho/\gamma_\kappa \int_0^{\kappa(t)} N(s) ds \\
&\quad - \kappa/\gamma_{\kappa(t)} + V(0) \quad (63)
\end{aligned}$$

If $\kappa(t)$, $\forall t \in R^+$, is not bounded from above or below, then from (42) and (43) it can be shown that the right hand side of (63) will be negative at some instances of time, which is a contradiction, since the left hand side of (63) is non-negative. Therefore κ is bounded, which implies the boundedness of V.

The boundedness of V implies that z_i, $i = 1, \ldots, \rho$, $\tilde{\theta}$ and ϵ are bounded. The boundedness of $\tilde{\theta}$ further implies that $\hat{\theta}$ is bounded, based on Assumption 2. Since $y = z_1 + y_r$, the boundedness of ξ and Ξ follows from Lemma 3.1 (ii). The boundedness of λ can be established from Assumption 1. Therefore, u is bounded and we can conclude that all the variables of the feedback control system are bounded.

By applying the invariant set theorem to (62), we have $\lim_{t\to\infty} z_i(t) = 0$, $i = 1, \ldots, \rho$, which ensures the asymptotic output tracking, $\lim_{t\to\infty}(y(t) - y_r(t)) = 0$. $\triangle$

5 Conclusions

Output regulation using measurement feedback has been solved for uncertain nonlinear systems with the minimum information: the systems can be transformed to the output feedback form with stable zero dynamics and linear parameterization of unknown parameters and disturbances. The uncertainties parameterized by unknown constant parameters and by the unmeasurable disturbances generated from known exosystems are treated in a unified way, by considering the unknown constant parameters as the disturbances generated from the exosystems with all their derivatives equal to zero. The state filters are redesigned to absorb the dynamics of the unknown parameters, and the traditional parameter adaptive law has been replaced with an parameter estimator which is similar to the traditional state observer, containing the dynamics of the exosystems. A Nussbaum gain has been used to tackle the unknown sign of the high frequency gain in the control design. The proposed algorithm extends the application of backstepping technique to the situation where the unknown parameters or disturbances are possibly time-varying. Compared with the existing output regulation algorithms including structural stable regulation, the proposed algorithm offers a better alternative with the global output tracking for the class of the uncertain systems considered. Assumption 2 made on the disturbances is not very restrictive, compared with the standard neutral stable assumption made in local output regulation. The exosystems satisfying Assumption 2 can be viewed as linearized version of the neutral stable systems around the operating point, and this is needed for global regulation. Our assumption does allow some interaction between the disturbances and the output, which is not permitted by the common autonomous neutral stable assumption made for local regulation. Based on Assumption 2, the disturbances are allowed to be sinusoids with known frequencies but with unknown amplitudes unknown phases, which can be rejected completely by the proposed algorithm using the output measurement only. In practice, the dominant frequency components in disturbances can be identified by spectral analysis, and once the frequencies are identified, the corresponding components can be completely suppressed by the proposed algorithm. In case that there are errors left over after the extraction of dominant frequency components, they can be tackled using the robust algorithm or almost disturbance decoupling technique, as long as they are bounded.

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