Tuning of a Combustion Controller by Extremum Seeking: A Simulation Study[1]

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Abstract

Attenuation of thermoacoustic instabilities by active control has lately received considerable attention. Many experimentally successful endeavors have used phase-shifting controllers feeding back a phase-shifted/time-delayed version of the dominant harmonic of acoustic oscillations through appropriate actuation (loudspeakers or fuel flow modulation). The phase-shifters are tuned by trial and error to produce a minimum oscillation amplitude. Hence, for widespread practical application of these controllers, there exists a need for adaptive tuning of the phase-shift to produce a minimum oscillation amplitude in a dynamic environment. The present simulation study demonstrates the efficacy of the extremum seeking algorithm and a modified version in optimally tuning the phase-shift in a model of a controlled combustion process identified from data. Step and ramp changes in the optimal phase-shift are successfully tracked, and oscillation amplitude kept at its minimum value.

1 Introduction

It is recognized that thermoacoustic instabilities in combustion processes cause many problems such as high-cycle fatigue, increased wall heat-transfer and possibilities of flame reversals [4, 10, 11]. The need for active control in attenuating thermoacoustic oscillations has been recognized [5, 11, 12] and its utility demonstrated [3, 10, 12, 14, 17]. The application of well-known control techniques requires sufficiently accurate reduced-order models of the complex processes involved, capturing the essential dynamics. Though there exist several efforts in this direction [5, 6, 7, 13, 17], and even in the specific case of lean premixed combustion [15], there is no model yet, which inspires a confidence sufficient for its general acceptance. What is, however known from experimental efforts in several conditions is that the oscillation is dominated by a single frequency,

$$p = p_0 + A\sin(\Omega t) + \text{h.o.t.}, \tag{1}$$

where p denotes the pressure, p_0 the static pressure, and $A\sin(\Omega t)$ denotes the dominant mode of oscillation, the higher order terms (h.o.t.) in the oscillation being small. Many successful experimental efforts in the area have used a simple phase-shifting controller to attenuate the oscillations [2, 3, 11, 12, 14, 18]. The control utilizes sensing of fluctuating pressure and actuates a loudspeaker or fuel-injector using a time-delayed/phase shifted version of the input signal. In the model that we use,

$$\theta = \Omega T, \tag{2}$$
$$\dot{m}_f(t) = -Gp(t - \theta/\Omega), \tag{3}$$

where θ is the phase-shift, Ω is the frequency of thermoacoustic oscillations, T is the controller delay, G the controller gain, and $\dot{m}_f(t)$ the fuel mass-flow rate.

A question that naturally arises is whether the phase-shift θ can be optimized in a dynamic environment. In this work, we start with a model identified from a controlled combustion process in a lean premixed gas combustor [1]. We simulate the standard extremum seeking algorithm and a modified version on the model, and demonstrate how the phase-shift parameter from [2, 3, 11, 12, 14, 18] can be optimized online.

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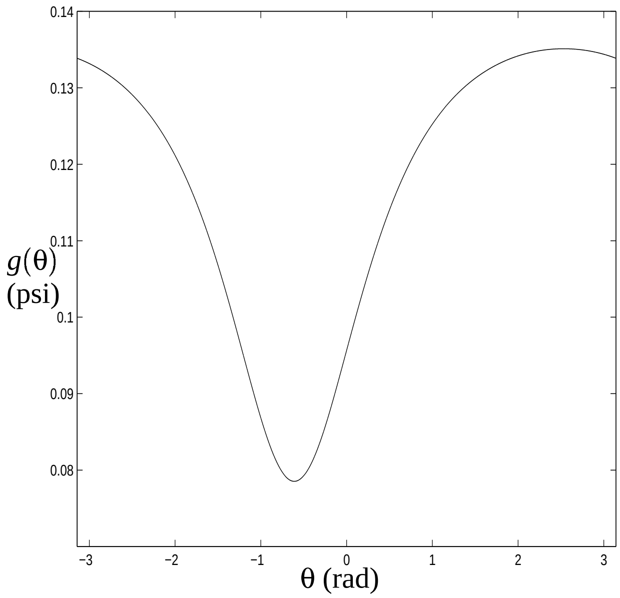

Figure 1: Map of closed loop equilibria

2 Model of the Controlled Combustion Process

The model that is simulated here has been identified from experiments on a controlled combustion process [1]. The reduced-order model identified from the closed loop system is:

$$\dot{A} = -\alpha(\theta)\,(A - g(\theta))\,, \tag{4}$$

where A is the amplitude of the acoustic oscillations in the combustion chamber. $g(\theta)$ is the map of closed loop equilibria parametrized by θ, obtained in [1] as a curve fit to experimental data:

$$g(\theta) = \Gamma \left\{ \frac{1 + L\sin(\theta - \theta^* - \frac{\pi}{2})}{1 + M\sin(\theta - \theta^* - \frac{\pi}{2})} \right\}. \tag{5}$$

The parameters herein are $\Gamma = 0.1246$, $L = 0.7659$, $M = 0.6286$. It can be shown, that the map has a minimum at $\theta = \theta^*$. The map $g(\theta)$ is shown in Fig. 1. $\alpha(\theta)$ denotes the decay rate of the exponential relaxation process of the system at a given phase-shift θ. (One has to keep in mind that α must be cyclic, therefore a sinusoid is fitted to the data.) The curve fit for $\alpha(\theta)$ used in the simulations is:

$$\alpha(\theta) = -a_1 \sin(\theta - \theta^* - a_2) - a_3, \tag{6}$$

where $a_1 = -26.15$, $a_2 = -0.6094$, $a_3 = -44.8750$. Here it can be noted that the model satisfies all conditions for the application of the extremum seeking algorithm [8, 9]. The following section provides a brief overview of this method and the structure of the simulations.

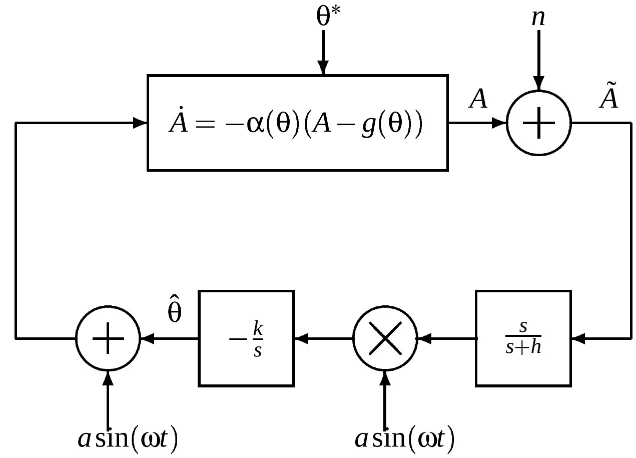

Figure 2: Implementation of the extremum seeking algorithm.

3 Optimizing Control with the Extremum Seeking Algorithm

Extremum seeking is well suited for tuning controller parameters in uncertain and noisy environments. The implementation of the extremum seeking algorithm is shown in Fig. 2. Sensor noise n is included. For an introductory presentation of extremum seeking, the reader may consult [8]. For the configuration in Fig. 2, the frequency ω of the probing signal should be small relative to the reciprocal of the largest time constant of the plant, and large relative to k and a. These conditions severely restrict the speed of convergence which is controlled by k. While larger k would speed up the convergence, it would also make the system more sensitive to noise and possible unstable.

The restriction on k was removed in [9] by introduction of the dynamic compensator $C(s)$ shown in Fig. 3. This compensator, if properly designed, can lift the requirement on the plant to be fast, thus ensuring stability for much larger values of k, for which convergence is faster. An additional extension offered in [9], which we employ here, is compensation of non-step changes in the optimal operating point. For example, if this point changes as a ramp function of time, i.e., $\theta^*(s) = \lambda_\theta/s^2$, where λ_θ is a constant, the adaptation law will incorporate a double integrator $\Gamma_\theta(s) = 1/s^2$. Of course, the presence of the double integrator in the loop will require a more careful choice of $C(s)$ to guarantee reliability. As we shall see in our problem, a ramp in θ^* will require a second order polynomial for $C(s)$ (for a step in θ^*, a constant $C(s)$ would suffice).

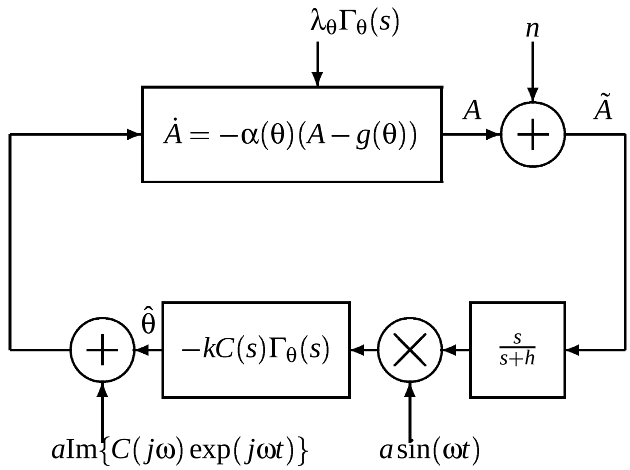

Figure 3: Implementation of the extremum seeking algorithm with compensator and extension for non-step changes in θ^*.

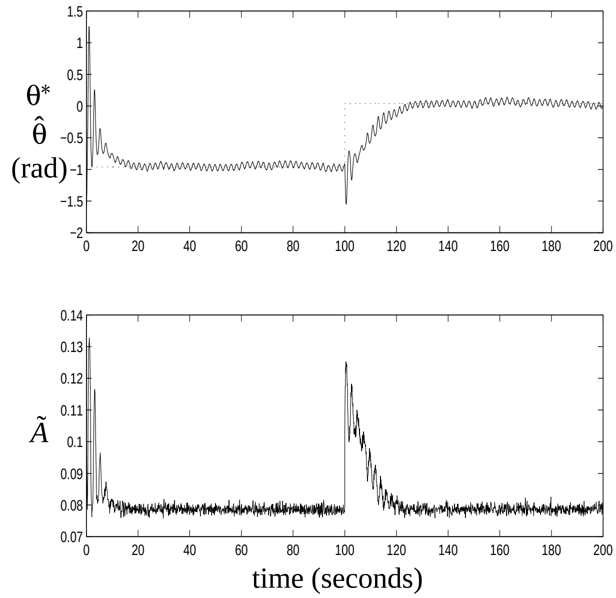

Figure 4: Results for a unit step in θ^* (dotted).

4 Simulations

The following simulation study deals with the case where step or ramp changes occur in the minimizer θ^* of the equilibrium map $g(\theta)$[1]. The step and ramp changes in θ^* can be viewed, for example, as changes in the operating conditions of the power plant such as abrupt changes in external load, or the ramping up to full power during starting. The capability of the extremum seeking algorithm to track the minimum and keep the system output at its optimal state will be demonstrated.

4.1 Step Changes in the Operating Conditions

The minimization of the plant output has been carried out by the extremum seeking algorithm mentioned in Section 3. The tracking of a step change of 1 rad in θ^* is shown in Fig. 4. The initial condition of A is 0.13, which is almost the maximum output of the system. The parameter values are chosen as $h = 0.3$, $\omega = 3$, $a = 0.05$, $k = 1000$. In the figure the values of θ^* and its estimate $\hat{\theta}$ are simultaneously plotted for comparison. $\tilde{A}$ is the output with the noise n. The algorithm achieves excellent tracking in the presence of noise. The amount of noise added to the system is measured in decibels (dB) according to the definition of Signal to Noise Ratio (SNR):

$$SNR = 10\log_{10}\left(\frac{<\tilde{A}^2>}{<n^2>}\right), \tag{7}$$

where $<\tilde{A}^2>$ denotes signal variance and $<n^2>$ denotes noise variance. In this case $SNR = 19.03$ dB. Roughly, the same level of noise has been maintained in all the following simulations.

4.2 Ramp Changes in the Operating Conditions

We first simulate classical extremum seeking in the presence of the ramp change in θ^*. Fig. 5 shows a simulation where θ^* begins ramping at $t = 100$ seconds. The slope is 0.04 rad/sec. The tracking of the ramp is not perfect but it is acceptable. The error is due to violation of the standard internal model principle property of LTI systems which requires a double integrator (rather than a single integrator in the basic extremum seeking scheme) for perfect tracking of a ramp. An increase in the slope of the ramp to 1 rad/sec leads not only to a larger steady state error, but to an actual loss of tracking ability, as shown in Fig. 6. The output of the system now cycles through all possible values of the equilibrium map. To demonstrate that tracking cannot be achieved by increasing k, we show Fig. 7 where it is clear that some form of stability loss (with boundedness of A preserved) occurs for the overall nonlinear feedback system. Note that Figures 6 and 7 were obtained with $n = 0$ to emphasize the tracking difficulty even in the absence of

[1] Note from Eqns. (5) and (6) that $g(\theta^*)$ and $\alpha(\theta^*)$ do not change when θ^* changes.

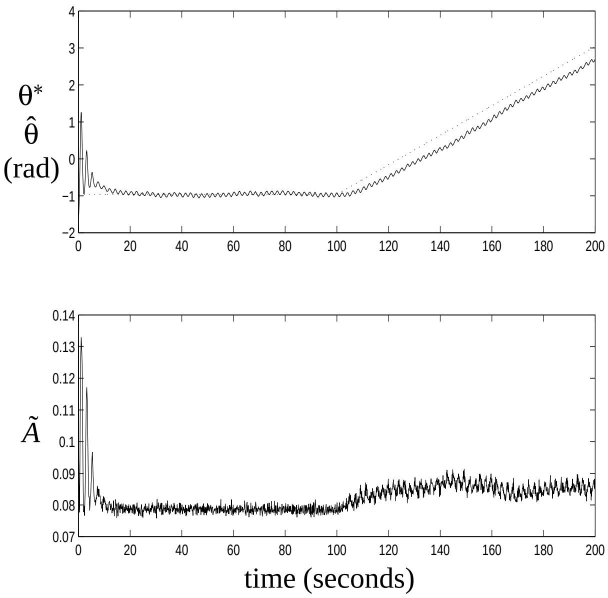

Figure 5: Results for a ramp in θ^* (dotted). Ramp slope= 0.04 rad/sec.

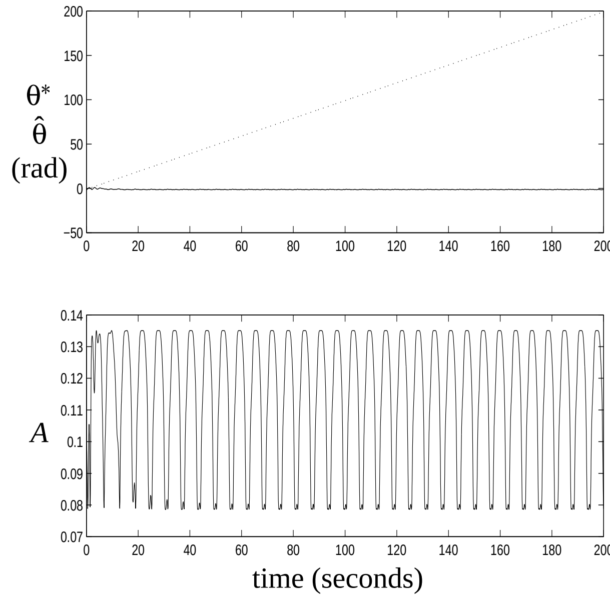

Figure 6: Results for a ramp in θ^* (dotted). Ramp slope= 1 rad/sec. $k = 1000$

noise. Many other choices of k and a were tried (not shown here), without success. A way to cope with this problem is the introduction of a double-integrator as $\Gamma_\theta(s) = 1/s^2$ and a compensator as indicated in Fig. 3. The introduction of a double integrator makes it harder to achieve closed-loop stability. The choice $C(s) = 1$ no longer works, and neither does a PD compensator $C(s) = ds + 1$. We succeed in our task with a compensator $C(s) = d_2 s^2 + d_1 s + 1$, for $d_2 = 0.001$ (very small, but important) and $d_1 = 0.1$. Note that, even though $C(s)$ is improper, $C(s)\Gamma_\theta(s) = d_2 + d_1/s + 1/s^2$ is proper. The loop is well-posed because the plant is relative degree one.

In order to distinguish between θ^* and $\hat{\theta}$, θ^* is plotted as a broad black straight band behind the thin white curve representing $\hat{\theta}$. With the compensators $\Gamma_\theta(s)$ and $C(s)$, the ability to track a ramp is recovered and the system's output stays at the minimum (Fig. 8). Tracking is almost perfect and $\tilde{A}$ resides under an exponentially decaying envelope.

5 Conclusions

The simulations show that the extremum seeking algorithm is able to drive the oscillation amplitude to its minimum in the presence of changing plant conditions. Step and ramp changes, which occur during the operation of a power plant, were successfully handled. The extremum seeking control can cope even with very steep slopes. The magnitude of sensor noise tolerated is quite remarkable, although we note that the modified extremum seeking algorithm is more sensitive to noise than the standard algorithm.

References

[1] K. B. Ariyur, A. Banaszuk, M. Krstic,“Identification of averaged dynamics of a controlled combustion instability”, *UCSD Technical Report*, Oct. 1999.

[2] A. Banaszuk, C. A. Jacobson, A. I. Khibnik, P. G. Mehta,“Linear and nonlinear analysis of controlled combustion processes. Part I: Linear analysis,” *Proc. of the IEEE Conf. on Control Applications*, Kohala-Coast, Hawai’i, pp. 199–205, Aug. 1999.

[3] J. M. Cohen, N. M. Rey, C. A. Jacobson, T. J. Anderson, “Active control of combustion instability in a liquid-fueled low-NO_x combustor,” *ASME/IGTI Gas turbine Expo and Congress*, Stockholm, Sweden, June 1998.

[4] F. E. C. Culick, “Technical evaluation report, Combustion Instabilities in Liquid-fueled Propulsion Systems,” 72nd *Propulsion and Energetics Panel Specialists’ Meeting*, Bath, UK, 1988.

[5] Y. T. Fung and V. Yang, “Active control of nonlinear pressure oscillations in combustion chambers,”

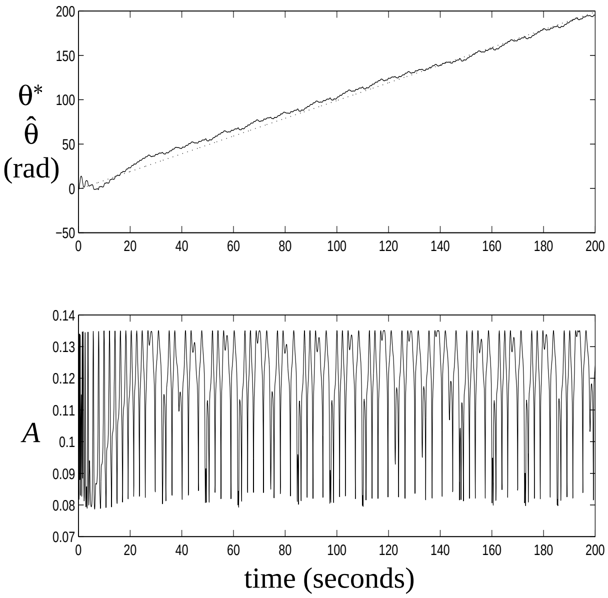

Figure 7: Results for a ramp in θ^* (dotted). Ramp slope= 1 rad/sec. $k = 5000$

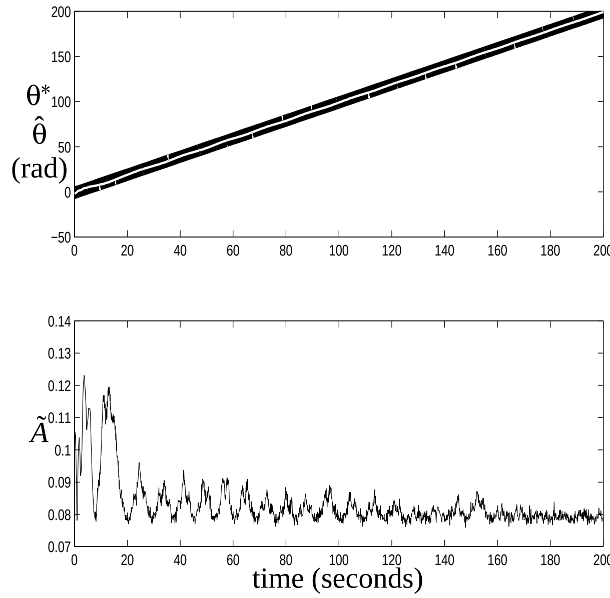

Figure 8: Results for a ramp increase of θ^* (black band, $\hat{\theta}$ white line) by 1 rad/sec.

Journal of Propulsion and Power, vol. 8, pp. 1282–1289, 1992.

[6] W. M. Haddad, A. Leonessa, J. R. Corrado, V. Kapila,"State space modeling and robust reduced-order control of combustion instabilities", *Proc. of the American Control Conference*, Albuquerque, NM, pp. 3125–3129, June 1997.

[7] J. P. Hathout, A. M. Annaswamy, M. Fleifil, A. F. Ghoniem,"A model-based active control design for thermoacoustic instability", *Combustion Sci. and Tech.*, vol. 132, pp. 99–110, 1998.

[8] M. Krstic and H. Deng, *Stabilization of Nonlinear Uncertain Systems*, Springer, London, 1998.

[9] M. Krstic, "Performance improvement and limitations in extremum seeking control," *System and Control Letters*, to appear.

[10] W. Lang, T. Poinsot, S. Candel,"Active control of combustion instability", *Combustion and Flame*, vol. 70, pp. 281–289, 1987.

[11] P. J. Langhorne, A. P. Dowling, N, Hooper, "Practical active control system for combustion oscillations," *Journal of Propulsion*, vol. 6, pp. 324–333, 1990.

[12] K. R. McManus, J. C. Magill, M. F. Miller,"Control of unstable combustion oscillations in liquid-fueled gas turbines", *Proc. of the IEEE Conf. on Control Applications*, Trieste, Italy, pp. 1170–1174, Sept. 1998.

[13] Y. Neumeier, B. T. Zinn,"Active control of combustion instabilities using real-time identification of combustor modes", *Proc. of the IEEE Conf. on Control Applications*, Albany, NY, pp. 691–698, Sept. 1995.

[14] C. O. Paschereit, E. Gutmark, W. Weisenstein,"Acoustic control of combustion instabilities and emissions in a gas-turbine combustor", *Proc. of the IEEE Conf. on Control Applications*, Trieste, Italy, pp. 1175–1179, Sept. 1998.

[15] A. A. Peracchio and W. M. Proscia, "Nonlinear heat-release/acoustic model for thermoacoustic instability in lean premixed combustors," *ASME/IGTI Turbo Expo '98*, Stockholm, Sweden, June 1998.

[16] G. A. Richards and M. C. Janus, "Characterization of oscillations during premix gas turbine combustion," *Trans. of the ASME, Journal of Eng. for Gas Turbines and Power*, vol. 120, pp. 294–310, 1998.

[17] J. W. Rumsey, M. Fleifil, A. M. Annaswamy, A. F. Ghoniem,"Low-order nonlinear models of thermoacoustic instabilities and model-based control ", *Proc. of the IEEE Conf. on Control Applications*, Trieste, Italy, pp. 1419–1423, Sept. 1998.

[18] K. Yu, K. J. Wilson, K. C. Schadow, "Liquid fueled active instability suppression," *27^{th} International Symposium on Combustion*, The Combustion Institute, Pittsburgh, PA, pp. 2039–2046, 1998.