Global inverse modeling for nonlinear non-affine system control by wavelet network

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Abstract

This paper presents a control scheme which learns the inverse mapping of a dynamic system by an orthonormal wavelet network. To compensate the modeling error caused by the model paramterization, feedback is added. The inverse mapping of dynamic system proposed here is defined as a mapping between the output trajectory and input trajectory. Training samples are chosen such that they can cover input trajectory space uniformly both in the amplitude domain and frequency domain . Here amplitude domain depends on the actuator while the frequency domain depends on sampling period of control system. For trajectory training, there are a lot of sample data(not sample trajectory) which enhance the complexity of modeling problem. Hence data compression is used by wavelet threshold which is a method frequently used in signal processing. The performance of proposed algorithm is illustrated by compute simulation experiment.

1 Introduction

Many practical control systems have complex, nonlinear dynamic behavior. Usually, the available a prior information is not sufficient enough to design a perfect control especially for highly nonlinear non-affine system. Even though the nominal model of dynamic system is known, for those non-affine system, it is hard to find a suitable control input because of nonlinearity.

Inverse modeling which finds the relation between output and control input is a good way to find suitable control input for nonlinear dynamic systems if the desired output trajectory is given. This kind of modeling belongs to nonparametric modeling in which black-box method is often used because the functional form of inverse model is unknown. Many methods have been done on this area including neural network, fuzzy system, radial basis network and wavelet network[11-13]. These methods all based on learning from training input/output data which covers only parts of input/output space. However, the function learning is based on the point learning which means samples are input/output data. However, control process is a continuous and dynamic, the main concern of control problem is not to find input point but input trajectory. Each trajectory can be viewed as a point in input/output space. Compared with ordinary methods in inverse modeling problem, the information contained in trajectory learning is much richer. Thus the model can represent the dynamic system much better.

However, more information means more complicated modeling process. Thus a method with compacted structure is prefered.

Wavelets theory are fairly new but simple mathematical tool with a great variety of possible applications. Though they are widely used in signal analysis, numerical analysis, time-series analysis,and operator research, not much work has been done on control applications especially for dynamic system. Using wavelet theory for nonlinear system identification and nonparametric estimation have been discussed by Zhang and Benveniste[1] , Zhang[2] and Zhang[3]. They take the advantage of both wavelet theory and neural network in wavelet network. They also proved that wavelet network is an universal approximator (Zhang [3]) and have more compact structure and less training than normal BP network. Wavelet was also used in control problem by Sureshbabu and Farrell[4]. They took advantage of orthonormal prosperity and multiresolution property of wavelet in system identification of dynamics plant. For control problem presented in this paper, inverse modeling is done by wavelet network by sample trajectories. Orthonormal wavelet (Daubechies wavelet[5]) is used in wavelet network, the structure of wavelet network is based on modeling error bound ϵ.

However the performance of modeling problem is highly depends on the distribution of training trajectories. If the training trajectories are uniformly distributed in function domain, the estimated function can be directly used. If these condition can not satisfied, using the trained model will cause trouble. For trajectories, uniformly distribution means that the trajectories are uniformly distributed both in frequency and amplitude.The domain of amplitude and frequency depend on the actuator's character and sampling interval of system. At the same time, in modeling, uniformly distribution in output space is more important than that in input space because more attention is paid to output in modeling. In inverse modeling here, the model output is control input which can be chosen by designer.

For trajectory learning problem, the modeling complexity problem is caused because each trajectory includes a lot of sample input/output data. In this paper data compressing method based on wavelets analysis is utilized to solve this problem which is often used in signal processing. By data compression, the trajectory can be represented by least sample data.

It is obviously that after parameterization in black-box modeling method, error can be produced. Also, there exists modeling error bound. Hence error feedback is used to enhance the control performance.

The paper is organized as follows. In Section 2, the problem is formulated. After a brief review of theory of wavelets in Section 3,there are training algorithm and inverse model control in Section 4. In Section 5 the data compression is discussed. Experiment results are discussed in Section 6, followed by summary in Section 7.

2 Problem Formulation

The basic idea of this paper can be explained clearly by a scalar nonlinear system:

$$\begin{aligned} \dot{x} &= f(x, u) \\ y &= x \end{aligned} \qquad (1)$$

where the system output trajectory $y(t)$ is the state $x(t)$ of dynamic system, $x(t) \in \mathbf{R}$,$u(t)$ is system input trajectory. Each trajectory represent a point in trajectory space. Hence $x(t)$ and $u(t)$are written as x and u respectively for similarity.

The control objective is to find a control $u_d(t)$ such that

$$\dot{x}_d(t) = f(x_d(t), u_d(t)), \qquad (2)$$

where $x_d(t)$ is desired trajectory.$u_d(t)$ is called desired control input trajectory. In real application, time t is bounded $t <= T$.

The following assumption regarding the control system and control are made:

Assumption 1. $f : \mathbf{R^2} \to \mathbf{R}$ is a known function, which is continuous and differentiable. And f is also highly non-affine function which means u can not solved directly even if f is known.

Assumption 2. The magnitude of control u is bounded by $|u| \leq U$ which U is determined by the character of actuator.And the frequency of control u is also bounded by $f_u \leq F$ which F is determined by sampling period of the system. These means $u \in \Omega$ and $\Omega = \{|u| \leq U, f_u \leq F\}$.

Assumption 3.There exists $\varepsilon > 0$ such that $|\partial f/\partial u| > \varepsilon$ for all u satisfies the assumption 2. If there is any u that satisfies $|\partial f/\partial u| = 0$, this u will be eliminated form sample trajectory.

Assumption 4. For a given x_d, there exists unique control input trajectory $u_d(t)$ satisfies Eq.2 and $u_d \in \Omega$.

Assumption 5. The magnitude of system output x and $\dot{x}$ are bounded by Xm and $\dot{X}m$ respectively.

The assumption 3 is a key controllability condition ,without which the system would be locally uncontrollable(see in Goh [6]). The dynamic system output also depends on the initial value of system state. For simplicity, assume that the initial value of system state of different input trajectory is fixed and $x_0 = x(0)$.

Under above assumption, by using implicit function theorem (see in Mali [7]),for any point$(x, \dot{x}, u)$ satisfied Eq.1 there exists the unique function $u = g(x, \dot{x})$ in a neighborhood of this point $(x, \dot{x}, u)$ in trajectory space such that $\dot{x} = f(x, g(x, \dot{x}))$. It should be noted that the function g is the relation between output trajectory and input trajectory.

Although implicit function theorem is only a local result, there exists a global implication in Ω because of assumption 3 and 4. Thus a controller can be designed directly if g and desired trajectory are known. From above section, wavelet network is has more compacted structure and less learning, thus it is suitable for trajectory-based learning.

3 Wavelet and Wavelet Network Review

Wavelets have been found to be useful in many applications such as physics, signal processing and statistics. Wavelets are a type of building block for constructing function by the conception of multiresolution approximation. The building block is formed by shifting and dilating base function called "mother wavelet"ψ and a "father wavelet"ϕ.

3.1 Multiresolution Analysis

Multiresolution analysis was proposed by Mallet [10] which provided a mathematically tool to describe the "increment in information" need to go from a coarse approximation to higher resolution approximation.

This multiresolution analysis consists of a sequence of successive approximation space $V_j \in L^2(R)$ which satisfies:

$$\ldots V_{-2} \subset V_{-1} \subset V_0 \subset V_1 \subset V_2 \ldots \qquad (3)$$

$$\bigcap_{m \in Z} V_m = \{\mathbf{0}\} \qquad (4)$$

$$\bigcup_{m \in Z} V_m = L^2(R) \quad (5)$$

$$f(x) \in V_j \Leftrightarrow f(2x) \in V_{j+1} \quad (6)$$

$$f(x) \in V_j \Rightarrow f(x - 2^{-j}k) \in V_j \quad k \in Z \quad (7)$$

$$V_m = span\{\phi_{mn}, n \in Z\} \quad (8)$$

where $\phi_{mn}(x) = 2^{m/2}\phi(2^m x - n)$ and $\phi_{mn} \in V_m$. Here ϕ_{mn} is an orthonormal basis of V_m and is called scaling function or "father wavelet".

For every $j \in Z$, define W_j to be the orthogonal complement of V_j in V_{j+1}, then

$$V_{j+1} = V_j \bigoplus W_j \quad (9)$$

$$W_j \perp W_i, \quad if\ i \neq j \quad (10)$$

If $f(x) \in L^2(R)$, $f_j(x) = \sum_{k \in Z} < \phi_{j,k}(x), f(x) > \phi_{j,k}$, $f_j(x)$ is projection of $f(x)$ in the space of V_j which is called approximation at resolution 2^j. The additional information in an approximation at resolution 2^{j+1} compared with the resolution 2^j is contained the space Wj.This additional information is called "details". As $j \to \infty$, $V_j \to L^2(R)$.

3.2 Orthonormal bases of wavelet

By virtue of Eq.5 and Eq.9, this implies

$$L^2(R) = \bigoplus_{j \in Z} W_j \quad (11)$$

Assume that for fixed j,$\{\psi_{j,k}; k \in Z\}$ constitutes an orthonormal basis for W_j, by Eq.11, $\{\psi_{j,k}; j, k \in Z\}$ constitutes orthonormal basis of $L^2(R)$ where $\psi_{j,k}(x) = 2^{j/2}\psi(2^j x - k)$ and is called "mother wavelet". Thus any function in $L^2(R)$ can be uniquely written in the following form

$$f(x) = \sum_{j,k \in Z} < \psi_{j,k}, f > \psi_{j,k}(x) \quad (12)$$

$$\begin{aligned} f(x) &= \sum_{k \in Z} < \psi_{j,k}, f > \phi_{J,k}(x) \\ &\quad + \sum_{j \geq J, k \in Z} < \psi_{j,k}, f > \psi_{j,k}(x) \quad (13) \\ &= \sum_{k \in Z} w_{J,k}\phi_{J,k}(x) + \sum_{j \geq J, k \in Z} w_{j.k}\psi_{j,k}(x) \end{aligned}$$

where $< . >$ denotes inner product in L^2 and J is arbitrary integer, representing the lowest resolution.

It is easy to extended to multi-dimensional, i.e.to $L^2(\mathbf{R^d})$,where $d > 1$ is an integer by tensor product of one-dimension "father wavelet" ϕ and "mother wavelet" ψ

$$\phi(\mathbf{x}) = \phi(x_1, x_2, \ldots, x_d) = \prod_{j=1}^{d} \phi(x_j) \quad (14)$$

$$\psi(\mathbf{x}) = \psi(x_1, x_2, \ldots, x_d) = \prod_{j=1}^{d} \psi(x_j) \quad (15)$$

We extends the wavelet theory to trajectory space. For above theory, the independent variable x in Eq. 12 and Eq. 13 can be replaced by a function of time domain $x(t)$, a trajectory in trajectory space. The

4 Wavelet network structure for trajectory-based learning

In Section 3,the number of orthonormal wavelets basis are infinite, in real application, it is impossible to use all wavelet basis. Thus infinite wavelet basis must be truncated into finite set i.e. unknown function only be estimated up to a certain "resolution" (or "scale") of details. Thus the certain " scale" can be determined the modeling error.

Because of the orthonormality of wavelets at different scales, adding new scale do not influence the coefficients of existing wavelets basis. Thus a coarse scale, for example $Jp = 2$ which Jp means the finest resolution by truncation is first used. For a predefined modeling error bound ϵ, adding the scale one by one until the desired predefined modeling error bound is satisfied.

There is another truncation in translation because the orthonormal basis in space V_j is also infinite. In modeling, a grid form partition in input space in used as in fuzzy input partition. Clearly, the thinner the grid, the better the modeling performance, the more complicated of modeling process. There must be a trade off between the precision and complexity. However, the truncation in translation will be compensated somehow by the increase of the scales by modeling error bound, thus it will cause not much trouble.

The structure of wavelet network is shown in Fig.1.

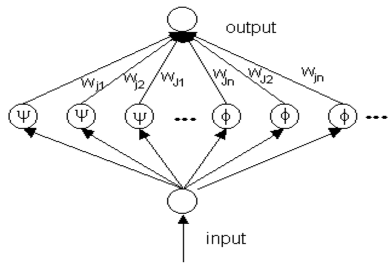

Figure 1: The structure of the wavelet network

4.1 Wavelet-network training

For the inverse modeling problem in control system, given samples $\{x, \dot{x}, u\}$ Which $u \in \Omega$, find a function $\hat{g}$ such that for the L^2 cost function

$$J(\theta) = \int_{x \in R \dot{x} \in R} ||g(x, \dot{x}) - \hat{g}(x, \dot{x}, \theta)||^2 dx d\dot{x} \quad (16)$$

there exist a $\theta^\star \in \mathbf{R^N}$ such that $\theta^\star = argmin_\theta J(\theta)$, where N is dimension of parameters which is determined by the level of performance discussed in above section. Since $g(x,\dot{x})$ is unknown, it is impossible to get $\theta^\star$ directly. Thus summation form of cost function is used.

$$J_1(\theta) = \sum_{i=1}^{K} ||g(x_i, \dot{x}_i) - \hat{g}(x_i, \dot{x}_i, \theta)||^2, \qquad (17)$$

where K is the number of samples. Here in control problem, the sample means a trajectory instead of point in the input/output space.

Learning algorithm is based on back-propagation algorithm. But there are some modification.

Assume the finest scale $Jp = 2$, for given samples $\{x_i, \dot{x}_i, u_i\}$, define $xm = max(x_i)$, $xm \in Z$,$xn = min(x_i)$, $xn \in Z$, $\dot{x}m = max(\dot{x}_i)$ and $\dot{x}m \in Z$, $\dot{x}n = min(\dot{x}_i)$ and $\dot{x}n \in Z$, then partition $[xn, xm]$ and $[\dot{x}n, \dot{x}m]$ into N grid which implies that the truncation of shifing wavelet is chosen to N. By tensor product one dimension to dimension, there are N^2 orthnormal basis at each resolution.

The weight updating law is :

$$w_{j,k} = \begin{cases} w_{j,k} - \lambda \frac{\partial J1}{\partial w_{j,k}} & if \quad w_{i,j} > w0 \\ w0 & else \end{cases}$$

where λ is learning rate which is determined by a fuzzy system(Xu, and Tan [8]), and $w0$ is predefined parameters in order to avoid meaningless wavelet basis.

The error in the training comes from two source, one is from coarse weight, the another is form the coarse structure (coarse scale). After the fixed number of training, if there is no sign of error decreasing in weights update process , the new resolution is added to improve the approximation performance.

After adding one resolution, new N^2 weights is added to the system. For orthonormality character of wavelet basis, updating the new weights is enough.

$$g(x,\dot{x}) \approx \sum_{j=0}^{Jp}\sum_{k=0}^{N^2} w_{j,k}\psi_{j,k} + \sum_{k=0}^{N^2} w_{Jp+1,k}\psi_{Jp+1,k}$$

$$= V_{Jp}(x,\dot{x}) + \sum_{k=0}^{N^2} w_{Jp+1,k}\psi_{Jp+1,k} \qquad (18)$$

$$J2 = \sum_{i=1} K||(g(x_i,\dot{x}_i) - V_{Jp}(x_i,\dot{x}_i)||^2 \qquad (19)$$

$$w_{Jp+1,k} = \begin{cases} w_{Jp+1,k} - \lambda \frac{\partial J2}{\partial w_{Jp+1,k}} & if w_{Jp+1,j} > w0 \\ w0 & else \end{cases} \qquad (20)$$

This process continued until the predefined modeling error bound ϵ is satisfied.

4.2 Inverse modeling control

Inverse model of control system just provide a feedforward control of system. Because of parameterization of unknown function by wavelet basis, there must exist error because of parameterization. Thus error feedback is also considered in the control system. For inverse model $u = g(x,\dot{x})$, after parameterization the model becomes $\hat{u} = \hat{g}(x,\dot{x},\theta^\star$. For a given trajectory x_d,there exists $u_d = g(x_d, \dot{x}_d)$, while for trained function $\hat{u} = \hat{g}(x_d, \dot{x}_d, \theta^\star)$ can be obtained. Define $\triangle u = u_d - \hat{u}$, $e = \hat{x} - x_d$,where $\hat{x}$ is output of plant when $\hat{u}$ is the plant input.

$$\begin{aligned}\dot{\hat{x}} &= f(\hat{x},\hat{u}) \\ &= f(x_d + e, u_d - \triangle u) \\ &\approx f(x_d, u_d) + \frac{\partial f}{\partial x}e - \frac{\partial f}{\partial u}\triangle u \\ &\approx \dot{x}_d + \frac{\partial f}{\partial x}e - \frac{\partial f}{\partial u}\triangle u \qquad (21)\end{aligned}$$

$$\triangle u = \frac{\partial f}{\partial u}^{-1}((f(x_d,u_d) - f(x,u)) + \frac{\partial f}{\partial x}e)\ if \frac{\partial f}{\partial u} \neq 0 \qquad (22)$$

By using feedback the performance of control problem is enhanced.

5 Data Compression

5.1 Data compression

In inverse modeling problem in control, in order to make the trained model has character of generalization in region Ω. Attention here, the state of nonlinear depends on both the control system and system input. Training samples cover all possible control input that can be provided. If the input of desired trajectory is not in the region Ω, the desired trajectory can not be tracked under the system condition of actuator and sampling period. Thus it is assumed that $u_d \in \Omega$ (Assumption 4).

Training samples are chosen to cover all control input domain Ω both in amplitude and frequency because the output of dynamic control system is depends on amplitude and frequency of control input. Thus there are large number of samples which each samples means a trajectory in input/output space. For training problem, the larger number of samples, the more complexity of training. Here data compression by wavelet which is often used in signal processing is utilized here to reduce the complexity of wavelet network training.

In wavelet decomposition, signal is passed through two filters: low pass filter and high pass filter(in wavelet theory, low pass filter means "father wavelet" while high pass filter means "mother wavelet"). Downsampling process means sampling period increase as $T = 2T$.The signal is decomposed into two part: approximation which is passed through low-pass filter and detail which is passed through high-pss filter. If there are 1000 samples in original signal, after filtering and downsampling , there lefts 500 samples if high frequency part of original signal is small enough (less than some predefined threshold) to be discarded. Thus information is kept in 500 samples. The process is shown in Fig.2. This process can be iterated, with successive approximations being decomposed in turn and information is compressed in less samples. For details, see Schumaker and Webb [9].

The general compression algorithm is following: *1. Decompose the signal(training trajectory) into approximation and detail;*
2. If the coefficients of detail are less than predefined threshold H, ,sampling period is in creased by $T = 2T$, data is compressed. Else there is no compression. 3. Repeat the procedure of 1 and 2 until the coefficients are big than threshold.

Thus the data compression is highly depends on the threshold H. The larger the H is, the more data is compressed. How to choose the threshold H depends on the modeling error and if there exists noise, it also depends on ratio between noise and signal.

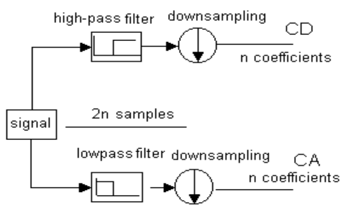

Figure 2: Signal compression

6 Application

A nonlinear non-affine model is assumed to be the following one-dimensional nonlinear differential equation

$$\dot{x} = u^2 sin(x) + cos(u)e^x \tag{23}$$

The range of frequency range is in $[0, 100hz]$ because the sampling period is $T = 0.01s$. The amplitude range of control input is in $[0, 20]$. The control input are selected to cover $\Omega = \{0 \leq u \leq 20,\ 0 \leq f_u \leq 100\}$ by uniform grid. Thus $u_{i,j} = (i+1)\triangle i sin((j+1)\triangle j)$ where $\triangle i = 20/Ni$, $\triangle j = 100/Nj$,Ni and Nj is uniform spacing between two adjacent lattice point. In this example, both Ni and Nj equal 20. Hence there are all together 400 training trajectories.

For every sample trajectory $x_i(t)$, it is impossible to learn the trajectory in all time domain. Thus time t is also restricted in a region $[0, T_0]$. Here T_0 equals 1. Under this condition, there are 100 points in every trajectory. Thus a lots of information needs to be dealt with.

6.1 data compression

First, data compression method is used to reduce the information. By choosing different threshold, the compression result is different.

Table 1. Compression results

th	scales	wavelet used	n1
0.4	5	db3	3120
0.3	5	db3	7823
0.2	5	db3	11564
0.1	5	db3	29924
0.0			40400

where th is the value of threshold and $n1$ is the number of the training samples after data compression. In order to compare the performance of different threshold,the training samples after different threshold compression are all trained respectively by wavelet network proposed in Sec.3. The modeling error bound $\epsilon = 0.05$.

6.2 Wavelet network structure

In identifying wavelet network structure, input partition should be considered first. In this example $N = 10$, hence for each resolution, there are $10^2 = 100$ weights needed to be tuned. The initial $Jp = 2$. In this wavelet network,Daubechies's wavelet $db3$ is used. There are many other orthonormal wavelet basis such as Haar wavelet and shannon wavelet. The choose of orthonormal wavelet basis is not main concern of this paper, thus it is beyond our discussion.

The result from the training of wavelet network is shown if Table 2.

Table 2. Training results

th	$n1$	$e1$	Jp	$n2$	$e2$
0.4	3120	0.049	5	1243	0.678
0.3	7823	0.049	6	3057	0.342
0.2	11564	0.049	7	5182	0.174
0.1	29924	0.049	8	6924	0.096
0.0	40400	0.049	10	8849	0.049

where $n2$ is the training steps, Jp is the finest scale, $e1 = \frac{1}{K'}\sum_{i=1}^{K'} ||u_i - \hat{g}(x_i, \dot{x}_i)||^2$ and $e2 = \frac{1}{K}\sum_{i=1}^{K} ||u_i - \hat{g}(x_i, \dot{x}_i)||^2$. K' and K are samples compressed and without compressed respectively.

From the above result, the larger threshold, the more compression, the less training, the less precision. Thus there must be a trade off between the modeling precision and training complexity. However, there are error compensation in control feedback, the larger threshold can not cause much trouble. Figure 3 shows the tracking error when threshold is set to 0.4. Figure 4 shows the tracking error when there is no data compression. The simulation results show our conclusion. In this example, the desired trajectory is $x(t) = tcost + 2sintln(t + 0.1) + 3t^2$ and initial value is fixed $x_0 = 0$.

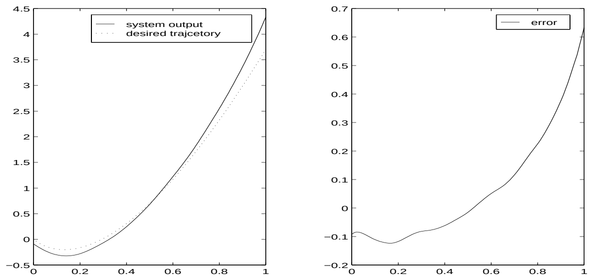

Figure 3: The system output with error feedback when threshold equals to 0.4

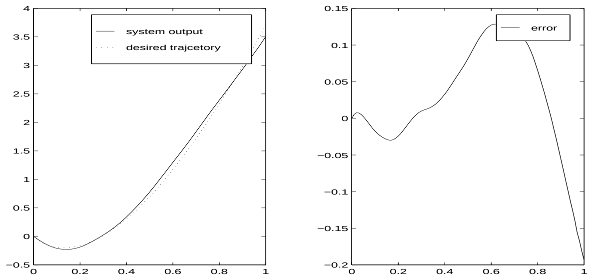

Figure 4: The system output without data compression

A typical PD control is also used to compare the performance of proposed method. $u = -kc(xd - x) - kd(\dot{x}d - \dot{x})$. Here $kc = 4$ and $kd = 6$ because of maximum amplitude of actuator. The result is shown in Fig.5.

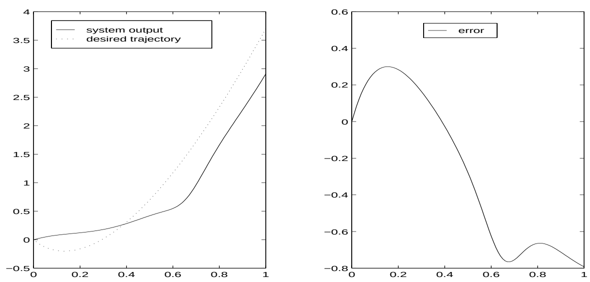

Figure 5: The system output by PD control

From above Figures, it is clearly shown that the model with no compression has a better performance compared with model with compression and also better than common PD controller. However after adding feedback, the performances of different model are all enhanced and are all better that PD control. The difference between different model is not as clearly as control system without feedback. Thus higher threshold can be chosen to simplify the training process while good performance is also kept by adding feedback. This method is shown to be effective in control problem.

7 Summary

In this paper, inverse modeling of control system using wavelet network is proposed. This kind of network take advantage of both neural network learning and orthonormal wavelet basis. It also avoiding the problem of meaningless wavelet basis which is frequently used in wavelet basis network. For practical point of control and modeling, training samples are designed to cover both the amplitude and frequency domain Ω which makes it possible the better generalization of modeling in this domain. At the same time, data compression method which is a popular method is signal processing is utilized to reduce the number of training samples caused by coverage of Ω.Moreover, error feedback is proposed to compensate the error cause by parameterized modeling. The application results shows that our method is effective in control problem.

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