Finite-time Tracking for Robot Manipulators with Singularity-free Continuous Control: A Passivity-based Approach

V. Parra-Vega and G. Hirzinger
Inst. of Robotics and Mechatronics–DLR, P.O Box 1116, 82230 Wessling, Germany
E-mail: Vicente.Parra-Vega@dlr.de, Gerd.Hirzinger@dlr.de

Abstract

Terminal attractors are introduced in dynamic sliding mode error coordinates in order to obtain two control schemes for robot manipulators that guarantee globally finite-time convergence of tracking errors. The simple passivity-based design yields a chattering-free controller with singularity-free closed-loop dynamics. The controllers render better stability properties in comparision to an ill-posed class of static terminal sliding mode control, with simpler control structures in comparision to a class of dynamic sliding mode controllers. Simulation data show the performance of the controllers.

1 Introduction

In real-time implementation there are always model uncertainties as well as round-off errors and sensor noise, and in some vital applications such as medical, micro biotechnology, and space robotics, infinite time convergence (*i.e. asymptotic stability*) may not be enough. Infinite stability, such as the induced by terminal attractors, may guarantee the succesful operation of delicated tasks. As technology advances and precision requirements increase, the problem of designing a controller that renders *finite-time convergence* (FTC) *of tracking errors* (FTT=Finite Time Tracking) has become increasingly important.

The philosophy of design of terminal sliding mode control is basically a conventional static sliding mode controller with a nonlipchitz sliding surface, where the dynamics of this surface exhibits an attractor with FTC (called terminal attractor [(1)]) and thus tracking errors converge in finite time. There is an attractive singularity located precisely in zero position error and this may render unbounded control in bounded domain with internal inestability of the differential equation. This puzzling behavior seems then disastrous, and not surprinsigly, the consequence of this unusual and unconventional formulation is the few control algorithms available for physical systems. Although terminal attractors has been subject of intensive research in the numerical and neural networks research community, where the application is bound to computer computations [(2)], that is not the case for applications on physical systems such as robot manipulators.

In particular, terminal sliding mode controllers available in the literature need a discontinuous control input to achieve FTC, besides the reasonable assumption of the knowledge of the parametric bounds, and faces singularity problems in position tracking errors, at least during reaching phase, which renders unbounded control input [(3)-(7)]. Furthermore, since a saturation function is proposed to realize the controller, the no sliding mode arises and the singularity problem arises all the time, resulting in ill-posed controllers. However to induce a sliding mode with a continuous control input, an involved dynamic sliding mode control has been proposed in

In this paper, we propose a simple and rather different approach in comparision to [(8)] in order to design a passivity-based terminal dynamic sliding mode control that guarantees global FTC for robot manipulators. Using a novel continuous *dynamic sliding mode error coordinate system* similar to [(9)], we design a saturated conventional sliding mode control structure [(10)]. Then we derive two controllers, adaptive and sliding mode-type; both controller are continuous and do not require acceleration measurement to induce a dynamic sliding mode for all time, in contrast to [(8)]. Then, a terminal attractor is introduced to achieve FTT. The structure of our controller keeps similar structure to [(11), (12)], and therefore this paper aims on extending this class of conventional passivity-based controllers, into dynamic error coordinates in order to attain better stability properties, with better real-time performance. The algorithm exposes the evident advantages of terminal sliding mode control by parametrizing the whole error equation in terms of *terminal error coordinates.* A comparative simulation study on a two DOF direct drive arm confirms the predicted stability properties of the proposed controller, and yields better performance, in same conditions, in comparision to the static terminal sliding mode controller for robot manipulators proposed by [(3)]. This paper is oganized as follows. Section 2 shows the robot dynamics in the error space. Section 3 presents the non-adaptive controller, and in Section 4 the adaptive version is presented. A simulation study is discussed in Section 5. Finally, some conclusions are offered in Section 6.

2 Robot Error Dynamics

2.1 Robot dynamics

The dynamic model of a rigid serial n-link robot manipulator with all revolute joints described in generalized joint coordinates $(q,\dot{q})^T \in R^{2n}$ can be written as follows

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + B_0\dot{q} + G(q) = U, \quad (1)$$

where $H(q)$ denotes a $n \times n$ symmetric positive definite inertial matrix, B_0 stands for a diagonal $n \times n$ positive definite matrix of friction damping coefficients, $C(q,\dot{q})$ stands for a $n \times n$ matrix of Coriolis and centrifugal forces, $G(q)$ models the gravity forces, and U is the torque input. Since equation (1) is linearly parametrizable [(12)], then it can be written in terms of a nominal reference $(\dot{q}_r, \bar{\ddot{q}}_r)^T \in R^{2n}$ as follows

$$H(q)\bar{\ddot{q}}_r + \{C(q,\dot{q}) + B_0\}\dot{q}_r + G(q) = Y_r\Theta, \quad (2)$$

where the regressor $Y_r = Y_r(q,\dot{q},\dot{q}_r,\bar{\ddot{q}}_r) \in R^{n\times p}$ is composed of known nonlinear functions, and $\Theta \in R^p$ is assumed to represent unknown but constant parameters, with $\dot{q}_r, \bar{\ddot{q}}_r$ to be defined yet. Applying (2) to (1) yields the following error dynamics

$$H(q)\bar{\dot{S}}_r + \{C(q,\dot{q}) + B_0\}S_r = U - Y_r\Theta, \quad (3)$$

where

$$S_r = \dot{q} - \dot{q}_r, \quad \text{and} \quad \bar{\dot{S}}_r = \ddot{q} - \bar{\ddot{q}}_r \quad (4)$$

There exists several studies on redesign $\dot{q}_r$ and $\bar{\ddot{q}}_r$ and thus the baseline controller [(12), (11)], however, those do not explore novel parametrizability of the whole algorithm, which is proved to be of crucial importance to obtain FTC.

2.2 Terminal Error Coordinates

Consider the following *continuous* second order change of coordinates

$$\begin{aligned} \dot{q}_r &= \dot{q}_d - \alpha\Delta q^p + S_d - K_i\sigma & (5)\\ \dot{\sigma} &= sign(S_q) & (6)\\ & & (7) \end{aligned}$$

and

$$\bar{\ddot{q}}_r = \ddot{q}_d - p\alpha\Delta q^{p-1}\Delta\dot{q} + \dot{S}_d - K_i tanh(\lambda S_q), \quad (8)$$

where

$$\begin{aligned} S_q &= S - S_d \\ S &= \Delta\dot{q} + \alpha\Delta q^p \end{aligned} \quad (9)$$

and $\Delta q = q - q_d$, and $\Delta\dot{q} = \dot{q} - \dot{q}_d$ stand for joint position and joint velocity tracking errors, respectively. Feedback gains α, K_i, and λ are $n \times n$ diagonal symmetric positive definite matrices, the $sign(x)$ and the $tanh(x)$ are the discontinuous $signum(x)$ and the continuous hyperbolic tanget functions of vector $x \in R^n$, respectively; function S_d can be designed as long as it fulfills the following four conditions: $S_d \in C^1$, $S_d(t_0) = S(t_0)[\equiv \Delta\dot{q}(t_0) + \alpha\Delta q^p(t_0)]$, $S_d(t)$ be monotonously decreacent function, and it exhibits FTC; parameter p is defined below

$$p = \frac{p_n}{p_d}, p_n, p_d \in Z_+, p_n < p_d, \frac{1}{2} < p < 1, \quad (10)$$

with p_n, p_d odd. Equations (5)-(8) yield $\ddot{q}_r, \bar{\ddot{q}}_r \in C^1$ and therefore the parametrization (2) is continuous. Equations (5)-(8) into (4) give rise to

$$\begin{aligned} S_r &= S_q + K_i\sigma, & (11)\\ \bar{\dot{S}}_r &= \dot{S}_q + K_i tanh(\lambda S_q), & (12) \end{aligned}$$

and (11)-(12) into (3) render the following open-loop error dynamics

$$\begin{aligned} H(q)\dot{S}_r &= -\{C(q,\dot{q}) + B_0\}S_r + U - Y_r\Theta \\ &\quad + H(q)K_i Z, & (13) \end{aligned}$$

where

$$Z = tanh(\lambda S_q) - sign(S_q) \quad (14)$$

has the following properties $Z \geq -1$, $Z \leq 1$, $Z_{S_q\to 0_-} = -1$, $Z_{S_q \to 0_+} = +1$ and $Z_{S_q\to\infty} = 0$.

Statement of the problem. *Design a continuous singularity-free control law U for (13) such that*

$$q(t) - q_d(t) = 0, \quad t \geq t_s > 0$$

with measurable state $(q,\dot{q})$ for any known $q_d(t) \in C^2$, assuming that the regressor Y_r available and Θ is unknown, but the upper bound of Θ is known.

3 Terminal Dynamic Sliding Mode Control

Consider the following continuous control law

$$U = -K_d S_r^j - Y_r\bar{\Theta} sat(Y_r^T S_r), \quad (15)$$

where $\bar{\Theta} > \|\Theta\|$, $K_d = K_d^T \in R^T n \times n$ is a positive definite matrix, and $sat(x) = \frac{x}{|x|+\epsilon}$ is a saturation function for $\epsilon > 0$. Parameter $j = \frac{j_n}{j_d}$, $j_n, j_d \in Z_+$, $j_n < j_d$, $\frac{1}{2} < j < 1$, j_n, j_d odd. We now have the following result.

Theorem 1 *Consider robot dynamics (1) in closed-loop with (15). Then, a singularity-free closed-loop dynamics arises with FTT if K_i is chosen as given in the proof. Furthermore, a dynamic sliding mode is enforced for all time and for any initial conditions with continuous control input.*

Proof.- The proof has been organized in seven parts.

A1. *Bounded trajectories* $S_r, \Delta\Theta$. Equation (15) into (13) gives rise to the following closed-loop error dynamics

$$\begin{aligned} H(q)\dot{S}_r &= -\{B_0 + C(q,\dot{q})\} S_r - K_d S_r^j - Y_r\Theta \\ &\quad -Y_r\bar{\Theta} sat(Y_r^T S_r) - H(q)K_i Z, \end{aligned} \qquad (16)$$

A passivity analysis sugests the following Lyapunov function

$$V = \frac{1}{2} S_r^T H(q) S_r. \qquad (17)$$

If we compute the total derivative of (17) along its solution (16) we obtain

$$\begin{aligned} \dot{V} &= -S_r^T B_0 S_r - S_r^T K_d S_r^j - S_r^T Y_r \bar{\Theta} sat(Y_r^T S_r) \\ &\quad -S_r^T Y_r \Theta + S_r^T H(q) K_i Z, \\ &\leq -S_r^T K_d S_r^j + S_r^T H(q) K_i Z + \bar{\Theta}\epsilon \\ &< -K_i V^\eta + S_r^T H(q) K_i Z + \bar{\Theta}\epsilon. \end{aligned} \qquad (18)$$

where $\eta = \frac{1+j}{2}$ and $K_i \equiv \lambda_m(K_d)\left\{\frac{2}{\lambda_m[H(q)]}\right\}^\eta$. Note that $\|S_r^T H(q) K_i\| > \|S_r^T H(q) K_i Z\|$ is a function of S_r. Then, equation (18) becomes

$$\begin{aligned} \dot{V} &\leq -K_i V^\eta + \|H(q)\|\|K_i\|\|S_r\| + \bar{\Theta}\epsilon, \\ &\leq -K_i V^\eta + \eta\|S_r\| + \bar{\Theta}\epsilon \end{aligned} \qquad (19)$$

where $\eta = \|H(q)\| * \|K_i\|$[1] Then according to (10), that is, $S_r \to \delta_0$ where δ_0 is a hyperball with radii $r_0 > 0$. Thus, we can conclude the asymptotic properties of the following variables

$$S_r \to \delta_1, \quad \dot{S}_r \to \delta_2, \quad \text{and} \quad \|\dot{S}_r\| < \delta_3 \qquad (20)$$

where $\delta_i > 0$ are bounded, for $i = 1, ..., 3$. This establishes the boundedness of all signals of the closed-loop system.

A2. *Sliding mode and FTC for* S_q. Note that the state of the closed-loop system is S_r, then we now show that from the dynamical system defined by the equation (11)

$$\dot{S}_r = \dot{S}_q + K_i sgn(S_q),$$

that is

$$\dot{S}_q = -K_i sgn(S_q) + \dot{S}_r, \qquad (21)$$

a sliding mode dynamics at $S_q = 0$ is established. To see this, consider the following Lyapunov function

$$V_q = \frac{1}{2} S_q^T S_q. \qquad (22)$$

[1]Norms $\|X\|$ stands for the Euclidian norm of vector $X \in R^n$, $\|A\| = \sqrt{\lambda_M(A^T A)}$ and $\lambda_M(A)$ stand for the induced Frobenius norms and the maximum eigenvalue of a $A \in R^{n \times n}$ matrix, respectively.

The total derivative of (22) along its solution (21) gives rise to

$$\begin{aligned} \dot{V}_q &= -S_q^T K_i sgn(S_q) + S_q^T \dot{S}_r \\ &\leq -K_i|S_q| + |S_q^T||\dot{S}_r| \\ &\leq -K_i|S_q| + \delta_3|S_q| \\ &\leq -\mu|S_q|, \end{aligned} \qquad (23)$$

where we have used (20), and $\mu = K_i - \delta_3$. Thus, in order to prove that $S_q \to 0$ in finite time, we can always choose

$$K_i > \delta_3, \qquad (24)$$

in such a way that a $\mu > 0$ guarantees the existence of a sliding mode since equation (23) is equivalent to the sliding mode condition [(13)]. This indicates that a sliding mode is established in finite time $t_q \leq \frac{|S_q(t_0)|}{\mu}$, and since for any initial condition $S_q(t_0) = 0$, then a sliding mode in $S_q(t) = 0$ is enforced for all time without reaching phase, thus $t_q \equiv 0$.

A3. *FTC of* S. We have show that $S_q(t) = 0$ is enforced for all time, then we have that since $S_d(t_0) = S(t_0)$, and since $S_d(t) = 0$ then

$$S = S_d \quad \forall t \geq t_a > 0 \quad \Longrightarrow S(t) = 0 t \geq t_b > 0$$

A4. *Terminal sliding mode and FTC of* Δq. Surface $S_q(t) = 0$ implies

$$\Delta\dot{q} = -\alpha\Delta q^p + S_d \quad \forall t. \qquad (25)$$

Now consider the following Lyapunov function

$$V_t = \frac{1}{2}\Delta q^T \Delta q. \qquad (26)$$

The total derivative of (26) along its solution (25) gives rise to

$$\begin{aligned} \dot{V}_t &= -\alpha\Delta q^T \Delta q^p + \Delta q^T S_d \\ &= -\alpha\sum_{i=1}^{n}(\Delta q_i^2)^\eta + \Delta q^T S_d \\ &= -2^\eta\alpha\left\{\frac{1}{2}\sum_{i=1}^{n}\Delta q_i^2\right\}^\eta + \Delta q^T S_d \\ &\leq -2^\eta\lambda_m(\alpha)\left\{\frac{1}{2}\sum_{i=1}^{n}\Delta q_i^2\right\}^\eta + \Delta q^T S_d \\ &\leq -2^\eta\lambda_m(\alpha)V_t^\eta + \Delta q^T S_d, \end{aligned} \qquad (27)$$

where $\eta = \frac{1+p}{2}$. If we design $S_d(t)$ such that it achieves FTC at time $t = t_d$ (3)

$$V_t(t) = 0 \quad \forall \quad t \geq t_x \qquad (28)$$

where $t_x < t_d + \frac{V_t^{1-\eta}(t_0)}{2^\eta\lambda_m(\alpha)(1-\eta)}$, which implies that

$$[\Delta q(t), \Delta\dot{q}(t)] = (0,0) \quad \forall t \geq t_x > 0 \qquad (29)$$

regardless of system parameters.

A5. *Desired transient response.* Since tracking errors obey the dynamics dictated by S_d

$$\Delta\dot{q} + \alpha\Delta q^p = S_d, \tag{30}$$

we can then shape arbitrarily the transient response of tracking errors for $t < t_1$ as long as S_d fulfills the four conditions conditions stated above equation (10). For instance consider $S_d = S_(t_0)exp^{-t\kappa}$, for $\kappa \gg 1$ fulfills this requirement *numerically.*

A6. *Robustness.* The existence of a sliding mode theoretically guarantees *invariance* since (30) does not depend on system parameters, and thus the controller (15) yields a robust closed-loop system against bounded state dependant structured parametric uncertainty under matching conditions. On the other hand, we have considered a disturbance free robot dynamics. Now suppose that a bounded state-dependant disturbance $d = d_1 + d_2||\dot{q}|| + d_3||\dot{q}||^2 \in R^n$ is present in (1), where d_1, d_2, d_3 are bounded positive vectors. The invariance property and the fact that tracking errors converge at least exponentially convergence allows to recall for total stability arguments to assert that the closed-loop system can withstand (small) bounded structured unmodelled dynamics. It can be shown that there exist control gains such that $S_r \to \delta_6$ with $\dot{S}_r \in \varsigma$, where $\delta_6 > 0$ is bounded and ς stands for a hyper-ball centered in the equilibrium with radii $\delta_7 > 0$. The upper bound of $\dot{S}_r$ might be greater than in the disturbance free case, in which case by tuning K_i to higher value, the condition for the existence of sliding modes in (24) can be met such that a sliding mode would be enforced and FTC is still insured.

A7. *Singularity-free dynamics.* We exclude the trivial case when the system is already in the singularity $\Delta q(t_0) = 0$ at given initial conditions since any terminal attractors-based control algorithm fails in this point at $t = t_0$[2]. Then, we analyse the case of $\Delta q(t_0) \neq 0$. Note that the equation of $\bar{\ddot{q}}_r$ in (5) violates the Lipschitz condition at $\Delta q = 0$ in *open loop.* However, considering that for closed-loop dynamics $S_q = 0$ for all time, then $\Delta\dot{q} = -\alpha\Delta q^p + S_d$ for all time. Thus, equation (5) becomes

$$\begin{aligned} \bar{\ddot{q}}_r &= \ddot{q}_d - p\alpha\Delta q^{p-1}(-\alpha\Delta q^p + S_d) + \psi \\ &= \ddot{q}_d + p\alpha^2\Delta q^{2p-1} - p\alpha\Delta q^{p-1}S_d + \psi \end{aligned} \tag{31}$$

where $\psi = \dot{S}_d - K_i tanh(\lambda S_q)$. Since $p > \frac{1}{2}$ then there is not singularity in the second term of (31). The third term is singular in $\Delta q = 0$, however since we can shape the transient response of S_d independiently of system dynamics, then we can design a S_d with shorter FTC time than the FTC time of $\Delta q = 0$ in such a way that S_d tends to zero faster than Δq. Note that $\Delta\dot{q} = -\alpha\Delta q^p + S_d$ has at least exponentially convergence and no overshoot can happen, and then Δq cannot be or cross zero before the FTC of S_d happens. Thus, the third term $p\alpha\Delta q^{p-1}S_d$ tends to zero before singularity occurs.

[2]Initial position tracking error must be diferent from zero at any given initial conditions, as it is *usually* the real case.

QED

Passivity and (*terminal*) dissipativity. According to the analysis of stability, passivity arises from v_1 input to S_r output, where $v_1 = H(q)\dot{S}_r + \{C(q,\dot{q}) + B_0 + K_d\} S_r - Y_r\Delta\Theta + H(q)K_iZ$. Note that dissipativity is established from v_2 input to S_q output, where $v_2 = \dot{S}_q$. Finally see that *terminal* dissipativity arises from $\Delta\dot{q} = -\alpha\Delta q^p + S_d$ input to Δq output. On the other hand, if K_i does not meet (24), the system still preserves passivity with all signals bounded (part 1 of theorem 1).

4 Adaptive Control with Terminal Dynamic Sliding Modes

Consider the following adaptive control law (12)

$$U = -K_dS_r + Y_r\hat{\Theta}, \tag{32}$$

$$\dot{\hat{\Theta}} = -\Gamma Y_r^T S_r, \tag{33}$$

where $K_d \in R^{n\times n}$ and $\Gamma \in R^{p\times p}$ are diagonal symmetric positive definite matrices. We now have the following result. The controller (32)-(33) allows to obtain FTT with online compensation of parametric uncertainty, and then there is not need to compute $\bar{\Theta}$ of theorem 1.

Theorem 2 *Consider robot dynamics (1) in closed-loop with the control law (32)-(33). Then, closed-loop dynamics attains FTT with desired transient response and robustness to bounded unmodelled dynamics. Furthermore, a dynamic sliding mode is enforced for all time and for any initial conditions without reaching phase, and with singularity-free closed loop dynamics.*

Proof.- Similar to the proof of theorem 1. Because space limitations details are omited (see [(14)] for experimental results).

5 Simulations

The rigid model of a 2 degrees of freedom robot arm for space applications is simulated with the parameters described in table 1. The desired task for the end-effector is to follow a circle of radius 0.4 m in

10 s. Initial conditions for both joint position errors are $[\Delta q_1(t_0), \Delta q_2(t_0)] = (+4.34, -4.53)$ deg. There are seven parameters to tune, table 1, and the parameter K_i is very critical to ensure not only tracking but also stability and a singular-free closed-loop system. Smooth control input without singularity is observed for both controllers. The performance of [(11)] delivers very high spikes in the begining, thereafter both controllers are similar, which indicates that the real effort of the dynamic terminal sliding mode control input is very small since the component of the conventional static sliding mode control input carries out the compensation of parametric uncertainty of all inertial and gravitational forces. In similar conditions and after 1 s, the controller of [(11)] delivers tracking errors bounded by a layer of $\pm 3 \times 10^{-3}$ deg, while our controllers yield trackig errors bounded by $\pm 6.24 \times 10^{-6}$ deg, with less control effort.

Simulation results with small but complex unmodelled disturbance $d = sin(20\pi t) + ||\dot{q}|| + 0.1||\dot{q}||^2$ are virtually the same in terms of tracking errors, however the control input for this case adds a small component of similar frequency and magnitude of the disturbance. In this case, feedback gain K_i is required to increase it by $+12\%$ of its nominal value of the free disturbance case in order to withstand the disturbance and ensure FTT. Results validates the all the theoretical conclusions.

6 Conclusions

The key contribution of this paper is the novel parametrizability, throughout a second order change of ccordinates, of robot dynamics which allows to merge affectively a terminal sliding surface with a dynamic sliding mode control law, wile preserving the fundamental advantages of each scheme. The result is two novel dynamic terminal sliding mode controllers, which guarantee globally finite-time convergence with continuous control inputs and singularity-free closed-loop dynamics. The adaptive version, as well as the robustness properties are outlined. Thus, this proposal stands as a well-posed control system to achieve FTT for second order systems, such as rigid robot manipulators. The controller renders better stability properties in comparision to the ill-posed class of static terminal sliding mode control [(3)]. Perfomance for both controllers is shown through a simulation study.

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Table 1: Dimension of the robot arm, and feedback gains

Robot parameters		Feedback gains	
Variable	Value	Parameter	Value
$Mass_1$	10.00	K_d	200.0
$Link_1$	1.00	α	10.0
$Link_{c1}$	0.57	λ	10.0
$Inertia_1$	0.20	η	50.0
$Mass_2$	7.00	$d_n = d_p$	$\frac{7}{9}$
$Link_2$	0.80	$j_n = j_p$	$\frac{11}{15}$
$Link_{c2}$	0.42	$\bar{\Theta}$	$1.1 * \Theta$
$Inertia_1$	0.10	K_i	1.0

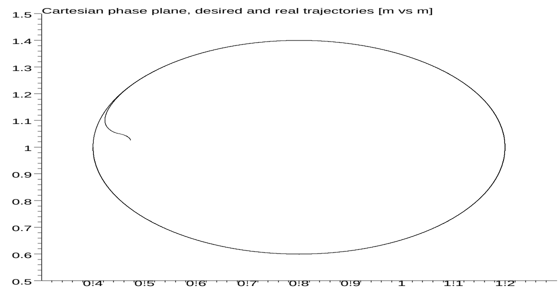

Figure 1: Phase plane in cartesian coordinates. Perfect tracking is achieved in $t < 0.8$ s.

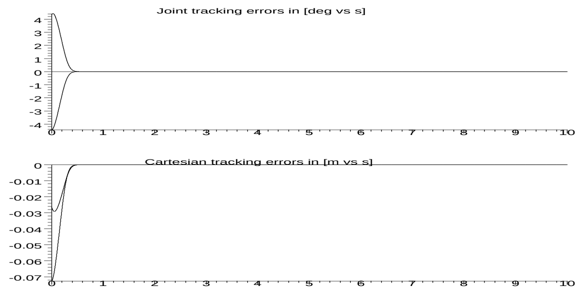

Figure 2: Joint and cartesian position tracking errors for $t0.8$ s remains around $\pm 6.24 \times 10^{-6}$ deg, and $\pm 1.09 \times 10^{-5}$ m, respectively.

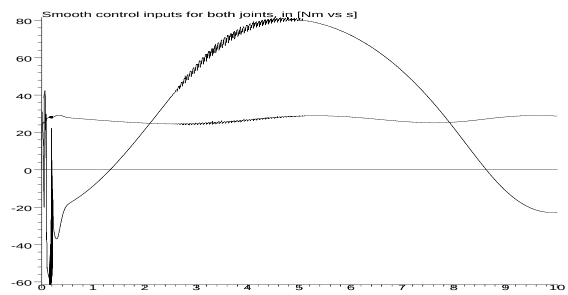

Figure 3: Control inputs.

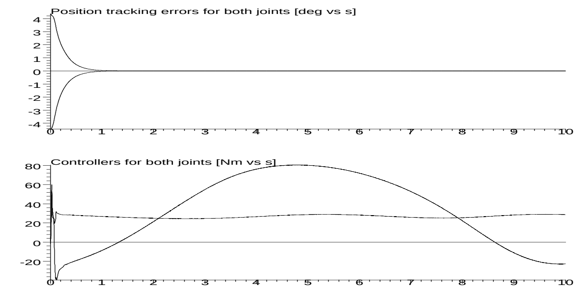

Figure 4: Joint position tracking errors for $t > 1$ s remains around $\pm 5.72 \times 10^{-7}$ rad. The spike in the beginning arises because the controller develops power to hold its own weight, afterwards a continuous and smooth control input is delivered.

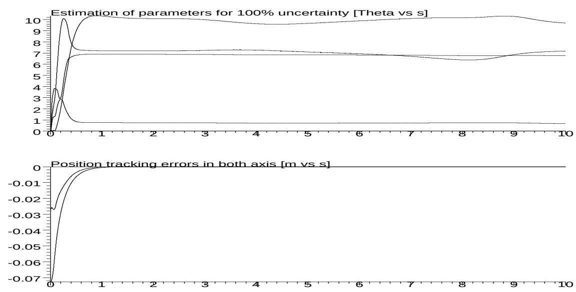

Figure 5: Estimator $\hat{\Theta}$ finds its *good* set of values in less than a second. Tracking errors in cartesian space are shown below.

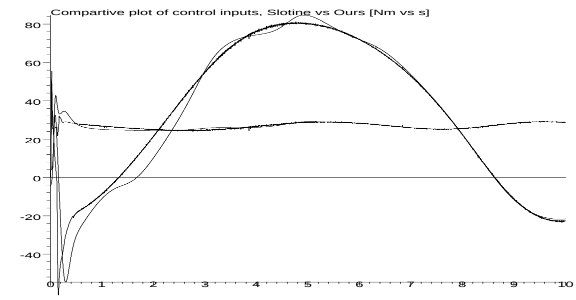

Figure 6: Comparative results of the continuous bounded control input between [(12)] and our controller under same conditions and common feedback gains ($\alpha = 10$, see figure 7).

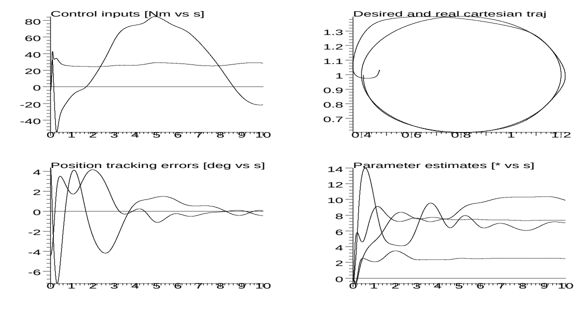

Figure 7: Performance of the baseline controller [(12)].