

Continuous-Time Adaptive Observer for Linear System with Unknown Time Delay

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Abstract

Although many objectives involving the time-delay system exist, the time-delay systems are often dealt as a finite dimensional system by using Pade approximation, etc. This paper presents a design of a continuous-time adaptive observer for the linear system with the unknown time delay without using such a approximation. To our best knowledge, no research result of an adaptive observer for the unknown time-delay system designed without the approximations is proposed until now.

Keywords: adaptive observer, time-delay system, convex/concave

1 INTRODUCTION

The state observer, which can estimate unknown parameters and state variables at the same time from the information of the input and the output, is useful when parameters of the plant are unknown and only a part of state variables is measurable. In the case of the adaptive estimation, the needed input signal can be selected freely by a designer, however in the case of the adaptive control, the input signal can not be selected freely [1]. So far, a number of designs of an adaptive observer are proposed (Lüders-Narendra's and Kreisselmeier's types, and so on), however, only a few designs consider the time-delay of the plant. In this paper, we propose the Continuous-time Adaptive observer for the unknown time-delay system. The study of the time-delay system is one of the significant problems which confront science and technology.

Our research in this paper is to estimate the unknown time-delay in input and unknown parameters of the plant based on Lüders-Narendra's adaptive observer. Another research is to design the estimation laws without the approximations even though an unknown time-delay is an unknown non-linear parameter, and to verify that the designed adaptive system is stable.

2 PROBLEM STATEMENT

We consider the following single-input single-output continuous-time linear system with unknown time-

delay in input.

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{b}u(t - \ell), \quad \mathbf{x}(0) = \mathbf{x}_0 \\ y(t) &= \mathbf{c}^T \mathbf{x}(t)\end{aligned}\quad (1)$$

where

$$\begin{aligned}\mathbf{A} &= \left[\mathbf{a} \mid \frac{\mathbf{g}^T}{\mathbf{K}} \right], \\ \mathbf{b} &= [b_1 \ b_2 \ \cdots \ b_n]^T, \\ \mathbf{c} &= [1 \ 0 \ \cdots \ 0]^T.\end{aligned}$$

$\ell \geq 0$ is an unknown time-delay, $\mathbf{x}(t) \in \mathbf{R}^n$ is the plant state, and $y(t) \in \mathbf{R}$ is the output. We make the following assumptions.

A1 *The input delay ℓ is an unknown value, but belongs to a known closed interval $\mathcal{L} \triangleq [\ell_{\min}, \ell_{\max}] \subset \mathbf{R}$.*

A2 *The input $u(\cdot) \in \mathbf{R}$ is a known scalar function, time-continuous and bounded with respect to time and ℓ . Moreover, $u(t - \ell)$ is either convex or concave with respect to ℓ on a known closed interval \mathcal{L} .*

A3 *The plant is asymptotically stable and completely controllable, completely observable, and its order n is known.*

A4 *The sign and the maximum (or minimum) of b_1 are known.*

Definition1 *A function $f(\ell)$ is said to be (i)convex on \mathcal{L} if it satisfies the inequality*

$$f(\phi\ell_1 + (1 - \phi)\ell_2) \leq \phi f(\ell_1) + (1 - \phi)f(\ell_2), \quad \forall \ell_1, \ell_2 \in \mathcal{L} \quad (2)$$

and (ii)concave if it satisfies the inequality

$$f(\phi\ell_1 + (1 - \phi)\ell_2) \geq \phi f(\ell_1) + (1 - \phi)f(\ell_2), \quad \forall \ell_1, \ell_2 \in \mathcal{L} \quad (3)$$

where $0 \leq \phi \leq 1$. A useful property of these function is their relation to the gradient. When $f(\ell)$ is (i)convex on \mathcal{L} , then it can be shown that

$$f(\ell) - f(\ell_0) \geq \nabla f_{\ell_0}(\ell - \ell_0), \quad \forall \ell, \ell_0 \in \mathcal{L} \quad (4)$$

and when $f(\ell)$ is (ii)concave on \mathcal{L} , then

$$f(\ell) - f(\ell_0) \leq \nabla f_{\ell_0}(\ell - \ell_0), \quad \forall \ell, \ell_0 \in \mathcal{L} \quad (5)$$

where $\nabla f_{\ell_0} = (\frac{\partial f}{\partial \ell})|_{\ell_0}$.

Now, in equation (1), \mathbf{a} and \mathbf{b} are unknown constant vectors, \mathbf{g} is an $(n-1)$ -constant vector, \mathbf{K} is an $(n-1) \times (n-1)$ constant parameter matrix, and $(\mathbf{g}^T, \mathbf{K})$ is a known observable set.

Based on Lüders-Narendra's adaptive observer [2], we choose the observable set $(\mathbf{g}^T, \mathbf{K})$ as follows:

$$\mathbf{g}^T = [1 \ 1 \ \cdots \ 1], \quad (6)$$

$$\mathbf{K} = \begin{bmatrix} -\lambda_2 & 0 & \cdots & 0 \\ 0 & -\lambda_3 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & -\lambda_n \end{bmatrix} \quad (7)$$

where $\lambda_i > 0$, $\lambda_i \neq \lambda_j$, $i \neq j$ ($i, j = 2, 3, \dots, n$).

By using an $n \times n$ stable-matrix \mathbf{F} , we can transform the plant (1) into the following equation.

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{F}\mathbf{x}(t) + (\mathbf{a} - \mathbf{f})y(t) + \mathbf{b}u(t - \ell), \\ \mathbf{x}(0) &= \mathbf{x}_0 \end{aligned} \quad (8)$$

where

$$\mathbf{F} = \left[\mathbf{f} \left| \frac{\mathbf{g}^T}{\mathbf{K}} \right. \right], \quad \mathbf{f}^T = [f_1 \ f_2 \ \cdots \ f_n]. \quad (9)$$

3 DESIGN OF THE ADAPTIVE OBSERVER

3.1 Structure of the Adaptive Observer

In equation (9), by the choice of $\mathbf{f}^T = [-\lambda_1 \ 0 \ \cdots \ 0]$, $\lambda_1 > 0$, equation (8) becomes below:

$$\begin{aligned} \dot{y}(t) &= -\lambda_1 y(t) + \sum_{i=2}^n x_i(t) \\ &+ (a_1 + \lambda_1)y(t) + b_1 u(t - \ell), \end{aligned} \quad (10)$$

$$\dot{x}_i(t) = -\lambda_i x_i(t) + a_i y(t) + b_i u(t - \ell), \quad i = 2, \dots, n \quad (11)$$

Using the measurable input $u(t)$ and output $y(t)$, we define $(n-1)$ -state variable filters below:

$$\dot{f}_{y_i}(t) = -\lambda_i f_{y_i}(t) + y(t), \quad f_{y_i}(0) = 0, \quad (12)$$

$$\dot{f}_{u_i}(t) = -\lambda_i f_{u_i}(t) + u(t - \ell), \quad f_{u_i}(0) = 0, \quad (13)$$

where $i = 1, 2, \dots, n$. From (12) and (13), we can construct the plant state $x_i(t)$ as follows:

$$x_i(t) = a_i f_{y_i}(t) + b_i f_{u_i}(t) + \exp(-\lambda_i t) x_{0_i}, \quad (14)$$

where $i = 2, 3, \dots, n$. By substituting (14) into (10), we obtain the following equation.

$$\begin{aligned} \dot{y}(t) &= -\lambda_1 y(t) + \sum_{i=2}^n (a_i(t) f_{y_i}(t) \\ &+ b_i(t) f_{u_i}(t) + \exp(-\lambda_i t) x_{0_i}) \\ &+ (a_1 + \lambda_1)y(t) + b_1 u(t - \ell), \end{aligned} \quad (15)$$

where x_{0_i} is an initial value. Here, we make reference to the equation (15) and produce the output estimation $\hat{y}(t)$ by introducing the variable a^* , which is defined later, and the term $-a^* \text{sat}(\frac{\epsilon}{\epsilon})$.

$$\begin{aligned} \dot{\hat{y}}(t) &= -\lambda_1 \hat{y}(t) + \sum_{i=2}^n (\hat{a}_i(t) f_{y_i}(t) \\ &+ \hat{b}_i(t) \hat{f}_{u_i}(t) + \exp(-\lambda_i t) \hat{x}_{0_i}) \\ &+ (\hat{a}_1(t) + \lambda_1) \hat{y}(t) + \hat{b}_1(t) u(t - \hat{\ell}(t)) \\ &- a^* \text{sat}\left(\frac{\epsilon}{\epsilon}\right), \end{aligned} \quad (16)$$

$$\hat{f}_{u_i}(t) = -\lambda_i \hat{f}_{u_i}(t) + u(t - \hat{\ell}(t)), \quad \hat{f}_{u_i}(0) = 0, \quad (17)$$

where $\hat{a}_i(t)$, $\hat{b}_i(t)$ are adjustable parameters with respect to unknown parameters a_i , b_i , respectively. Furthermore, we define the identification error $e(t)$ by

$$e(t) = \hat{y}(t) - y(t). \quad (18)$$

From (10) and (16), we have the error equation as follows:

$$\begin{aligned} \dot{e}(t) &= -\lambda_1 e(t) + \sum_{i=1}^n (\hat{a}_i(t) - a_i) f_{y_i}(t) \\ &+ \sum_{i=2}^n (\hat{b}_i(t) - b_i) \hat{f}_{u_i}(t) + \sum_{i=2}^n \exp(-\lambda_i t) (\hat{x}_{0_i} - x_{0_i}) \\ &+ \hat{b}_1(t) u(t - \hat{\ell}(t)) - b_1 u(t - \ell) \\ &+ \sum_{i=2}^n (\hat{b}_i(t) \hat{f}_{u_i}(t) - b_i(t) f_{u_i}(t)) - a^* \text{sat}\left(\frac{\epsilon}{\epsilon}\right), \end{aligned} \quad (19)$$

where $f_{y_1}(t) = y(t)$. The term $\sum_{i=2}^n \exp(-\lambda_i t) (\hat{x}_{0_i} - x_{0_i})$ in (19) converges to zero, then, this term can be ignored.

Moreover, the term $\sum_{i=1}^n (\hat{a}_i(t) - a_i) f_{y_i}(t) + \sum_{i=2}^n (\hat{b}_i(t) - b_i) \hat{f}_{u_i}(t)$, which involves adjustable parameters, is rewritten as $\tilde{\boldsymbol{\alpha}}^T \mathbf{X}$ by using

$$\tilde{\boldsymbol{\alpha}} \triangleq \hat{\boldsymbol{\alpha}} - \boldsymbol{\alpha} = \begin{bmatrix} \hat{a}_1(t) - a_1 \\ \vdots \\ \hat{a}_n(t) - a_n \\ \hat{b}_2(t) - b_2 \\ \vdots \\ \hat{b}_n(t) - b_n \end{bmatrix}, \quad \mathbf{X} \triangleq \begin{bmatrix} f_{y_1}(t) (= y(t)) \\ \vdots \\ f_{y_n}(t) \\ \hat{f}_{u_2}(t) \\ \vdots \\ \hat{f}_{u_n}(t) \end{bmatrix}.$$

Finally, by using the simple transformation,

$$\begin{aligned} &\hat{b}_1(t) u(t - \hat{\ell}(t)) - b_1 u(t - \ell) \\ &= \hat{b}_1(t) u(t - \hat{\ell}(t)) - b_1 u(t - \ell) + b_1 u(t - \hat{\ell}(t)) - b_1 u(t - \hat{\ell}(t)) \\ &= b_1 \left\{ u(t - \hat{\ell}(t)) - u(t - \ell) \right\} + (\hat{b}_1(t) - b_1) u(t - \hat{\ell}(t)), \end{aligned}$$

$\bar{\ell} > \ell_{\max}$, (iii) $\bar{\ell} < \ell_{\min}$.

(i) $\bar{\ell} \in \mathcal{L}$.

In this case, $\hat{\ell}(t) = \bar{\ell}$ and hence $m(\hat{\ell}(t), \bar{\ell}) = 0$.

(ii) $\bar{\ell} > \ell_{\max}$.

In this case, $\hat{\ell}(t) = \ell_{\max}$ (constant) and hence $\dot{\hat{\ell}}(t) = 0$. By the choice of $\hat{\ell}(t)$ and $\bar{\ell}$ and since $\ell \in \mathcal{L}$; ($\ell_{\min} \leq \ell \leq \ell_{\max}$), we have $\bar{\ell} - \hat{\ell}(t) > 0$, $\hat{\ell}(t) - \ell \geq 0$ when $\bar{\ell} > \ell_{\max}$. Therefore, we obtain $m(\hat{\ell}(t), \bar{\ell}) \leq 0$.

(iii) $\bar{\ell} < \ell_{\min}$.

In this case, $\hat{\ell}(t) = \ell_{\min}$ (constant) and hence $\dot{\hat{\ell}}(t) = 0$. By the choice of $\hat{\ell}(t)$ and $\bar{\ell}$ and since $\ell \in \mathcal{L}$; ($\ell_{\min} \leq \ell \leq \ell_{\max}$), we have $\bar{\ell} - \hat{\ell}(t) < 0$, $\hat{\ell}(t) - \ell \leq 0$ when $\bar{\ell} < \ell_{\min}$. Therefore, we obtain $m(\hat{\ell}(t), \bar{\ell}) \leq 0$.

Therefore, we have $m(\hat{\ell}(t), \bar{\ell}) \leq 0$ for all $\hat{\ell}(t)$ and $\bar{\ell}$.

Since $m(\hat{\ell}(t), \bar{\ell}) \leq 0$ is verified, the time derivative (30) can be dealt as follows from here.

$$\begin{aligned} \dot{V} = & -\lambda_1 e e_\varepsilon + e_\varepsilon \left[b_1 \left\{ u(t - \hat{\ell}(t)) - u(t - \ell) \right\} \right. \\ & \left. + \tilde{\ell} \omega^* + \sum_{i=2}^n b_i (\hat{f}_{u_i}(t) - f_{u_i}(t)) - a^* \text{sat} \left(\frac{e}{\varepsilon} \right) \right] \end{aligned} \quad (32)$$

Let $y = e_\varepsilon^2$. Since the discontinuity at $|e| = \varepsilon$ is of the first kind, and since $|e_\varepsilon| = 0$ because of $\text{sat} \left(\frac{e}{\varepsilon} \right) = \frac{e}{\varepsilon}$, when $|e| \leq \varepsilon$, it follows that the derivative \dot{V} exists for all e , and we have the result below.

$$\dot{V} = 0 \quad \text{when } |e| \leq \varepsilon \quad (33)$$

when $|e| > \varepsilon$,

$$\begin{aligned} \dot{V} = & -\lambda_1 e e_\varepsilon + e_\varepsilon \left[b_1 \left\{ u(t - \hat{\ell}(t)) - u(t - \ell) \right\} \right. \\ & \left. + \tilde{\ell} \omega^* + \sum_{i=2}^n b_i (\hat{f}_{u_i}(t) - f_{u_i}(t)) - a^* \text{sat} \left(\frac{e}{\varepsilon} \right) \right] \end{aligned} \quad (34)$$

From the definition in (21), we have $e \geq e_\varepsilon$, and by replacing e with e_ε , the above equation can be simplified as

$$\begin{aligned} \dot{V} \leq & -\lambda_1 e e_\varepsilon + e_\varepsilon \left[b_1 \left\{ u(t - \hat{\ell}(t)) - u(t - \ell) \right\} \right. \\ & \left. + \tilde{\ell} \omega^* + \sum_{i=2}^n b_i (\hat{f}_{u_i}(t) - f_{u_i}(t)) - a^* \text{sat} \left(\frac{e}{\varepsilon} \right) \right] \end{aligned} \quad (35)$$

We now consider two distinct cases, (a) $e > \varepsilon$ and (b) $e < -\varepsilon$, and show that $\dot{V} \leq 0$ in both cases.

(a) $e > \varepsilon$.

In this case, since $e_\varepsilon > 0$, we have $\text{sat} \left(\frac{e}{\varepsilon} \right) = 1$. It follows that $\dot{V} \leq 0$ if

$$\begin{aligned} a^* \geq & b_{1c} \left\{ u(t - \hat{\ell}(t)) - u(t - \ell) \right\} + \tilde{\ell} \omega^* \\ & + \sum_{i=2}^n b_i (\hat{f}_{u_i}(t) - f_{u_i}(t)) \quad \forall \ell \in \mathcal{L}. \end{aligned} \quad (36)$$

Therefore, we choose

$$\begin{aligned} a^* = & \max_{\ell \in \mathcal{L}} b_{1c} \left\{ u(t - \hat{\ell}(t)) - u(t - \ell) \right\} + \omega^* (\hat{\ell}(t) - \ell) \\ & + \sum_{i=2}^n b_i (\hat{f}_{u_i}(t) - f_{u_i}(t)) \quad \text{for any } \omega^*. \end{aligned} \quad (37)$$

Since the form of the controller suggests that the quantity a^* is like a gain [3], we seek to find an ω^* so that a^* is minimized. Hence our goal is to choose a^* as

$$\begin{aligned} a^* = & \min_{\omega \in R} \max_{\ell \in \mathcal{L}} b_{1c} \left\{ u(t - \hat{\ell}(t)) - u(t - \ell) \right\} + \omega^* (\hat{\ell}(t) - \ell) \\ & + \sum_{i=2}^n b_i (\hat{f}_{u_i}(t) - f_{u_i}(t)). \end{aligned} \quad (38)$$

The above choice of a^* and ω^* coincides with our selection of a^* and ω^* in the estimation law for $e_\varepsilon > 0$.

(b) $e < -\varepsilon$.

In this case, since $e_\varepsilon < 0$ here, we have $\text{sat} \left(\frac{e}{\varepsilon} \right) = -1$.

It follows that $\dot{V} \leq 0$ if

$$\begin{aligned} a^* \geq & b_{1c} \left\{ u(t - \ell) - u(t - \hat{\ell}(t)) \right\} - \tilde{\ell} \omega^* \\ & - \sum_{i=2}^n b_i (\hat{f}_{u_i}(t) - f_{u_i}(t)) \quad \forall \ell \in \mathcal{L}. \end{aligned} \quad (39)$$

Following along the same lines as in case (a), we choose

$$\begin{aligned} a^* = & \min_{\omega \in R} \max_{\ell \in \mathcal{L}} b_{1c} \left\{ u(t - \ell) - u(t - \hat{\ell}(t)) \right\} - \omega^* (\hat{\ell}(t) - \ell) \\ & - \sum_{i=2}^n b_i (\hat{f}_{u_i}(t) - f_{u_i}(t)). \end{aligned} \quad (40)$$

Also, the above choice of a^* and ω^* coincides with our selection of a^* and ω^* in the estimation law for $e_\varepsilon < 0$. Therefore, we have the result as follows.

$$\dot{V} \leq 0 \quad \forall |e| > \varepsilon \quad (41)$$

(33) and (41) imply that V is a Lyapunov function which leads to global boundedness of e_ε , $\tilde{\alpha}$, \tilde{b}_1 and $\tilde{\ell}$. From the definition of e_ε , e is bounded. As a result, $y \in L^\infty \cap L^1$ is bounded, hence $y \in L^2$ as well. Since \dot{e}_ε has a discontinuity of the first kind when $|e| = \varepsilon$, it follows that $\dot{y} \in L^\infty$. Hence $\lim_{t \rightarrow \infty} y(t) = 0$ and as a result, $\lim_{t \rightarrow \infty} e_\varepsilon(t) = 0$. This guarantees convergence of e to within a prescribed precision ε [3].

5 SIMULATION RESULTS

By using the estimation laws suggested in section 3, we show the numerical simulations and clarify the effectiveness of the adaptive observer with the time-delay estimation. In the numerical simulations, we consider the structure of the proposed adaptive observer for equation (1) of its order $n = 2$.

In our present research, we have the difficulty in solving the min-max problem (25)-(27). To avoid this difficulty, we regard $J(\omega, \ell)$ as $\hat{J}(\omega, \ell)$;

$$\hat{J}(\omega, \ell) = b_{1c}\{u(t - \hat{\ell}(t)) - u(t - \ell)\} + \omega(\hat{\ell}(t) - \ell). \quad (42)$$

Note that $b_{1c}u(t - \ell)$ has the negative sign in $\hat{J}(\omega, \ell)$. By solving this min-max problem, we have a^* , ω^* as follows:

If the function $b_{1c}u(t - \ell)$ is **convex** with respect to ℓ ,

$$a^* = \begin{cases} 0, & e \geq 0 \\ -b_{1c} \left[u(t - \hat{\ell}) - u(t - \ell_{\min}) \right. \\ \left. - \frac{u(t - \ell_{\max}) - u(t - \ell_{\min})}{\ell_{\max} - \ell_{\min}} (\hat{\ell} - \ell_{\min}) \right], & e < 0 \end{cases} \quad (43)$$

$$\omega^* = \begin{cases} -b_{1c} \nabla u_{\hat{\ell}}, & e \geq 0 \\ -b_{1c} \left[\frac{u(t - \ell_{\max}) - u(t - \ell_{\min})}{\ell_{\max} - \ell_{\min}} \right], & e < 0 \end{cases} \quad (44)$$

If the function $b_{1c}u(t - \ell)$ is **concave** with respect to ℓ ,

$$a^* = \begin{cases} b_{1c} \left[u(t - \hat{\ell}) - u(t - \ell_{\min}) \right. \\ \left. - \frac{u(t - \ell_{\max}) - u(t - \ell_{\min})}{\ell_{\max} - \ell_{\min}} (\hat{\ell} - \ell_{\min}) \right], & e \geq 0 \\ 0, & e < 0 \end{cases} \quad (45)$$

$$\omega^* = \begin{cases} -b_{1c} \left[\frac{u(t - \ell_{\max}) - u(t - \ell_{\min})}{\ell_{\max} - \ell_{\min}} \right], & e \geq 0 \\ -b_{1c} \nabla u_{\hat{\ell}}, & e < 0 \end{cases} \quad (46)$$

In Figure 2, 3, 4, since the function $u(t - \ell)$ is linear with regard to ℓ , the adjustment laws for a^* , ω^* in convex and those in concave are both available. We select the adjustment laws in convex in this numerical simulations.

In Figure 5, 6, 7, since the function $u(t - \ell)$ is non-linear with regard to ℓ , we must change the adjustment laws in convex to in concave, in concave to in convex.

In all simulations, the parameters of the plant are set by $a_1 = -7$, $a_2 = -9$, $b_1 = 1$, $b_{1c} = 1.5$, $b_2 = -2$, $\lambda_1 = 3$, $\lambda_2 = 7$. The lumped system of the plant in equation (1), $G(s)$ is $G(s) = \frac{s + 5}{s^2 + 14s + 58}$. The initial values of the parameters are set by $\hat{\ell}(0) = 0$, $\hat{a}_1(0) = 0$, $\hat{a}_2(0) = 0$, $\hat{b}_1(0) = 1.5$, $\hat{b}_2(0) = 0$ and the initial value of the state estimation $\hat{x}_2(t)$ is $\hat{x}_{02} = 0$.

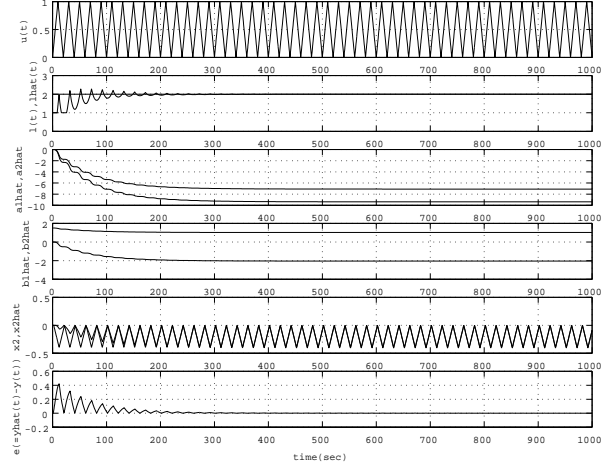


Figure 2: Time-delay ℓ ; constant, the time-delay $\ell = 2$. the parameters are set by $\ell_{\min} = 1$, $\ell_{\max} = 4$, $\gamma_{\ell} = 5$, $\gamma = 0.01$, $\varepsilon = 0.001$, $\gamma_{a1} = 7$, $\gamma_{a2} = 65$, $\gamma_{b1} = 0.04$, $\gamma_{b2} = 1.2$.

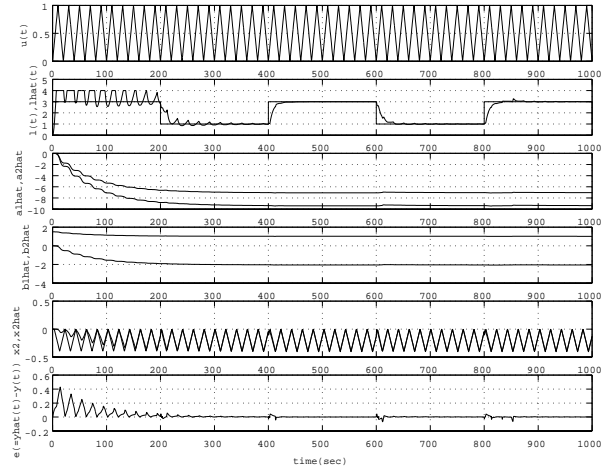


Figure 3: Time-delay ℓ ; a rectangular signal, the parameters are set by $\ell_{\min} = 0$, $\ell_{\max} = 4$, $\gamma_{\ell} = 40$, $\gamma = 0.01$, $\varepsilon = 0.001$, $\gamma_{a1} = 7$, $\gamma_{a2} = 65$, $\gamma_{b1} = 0.04$, $\gamma_{b2} = 1.2$.

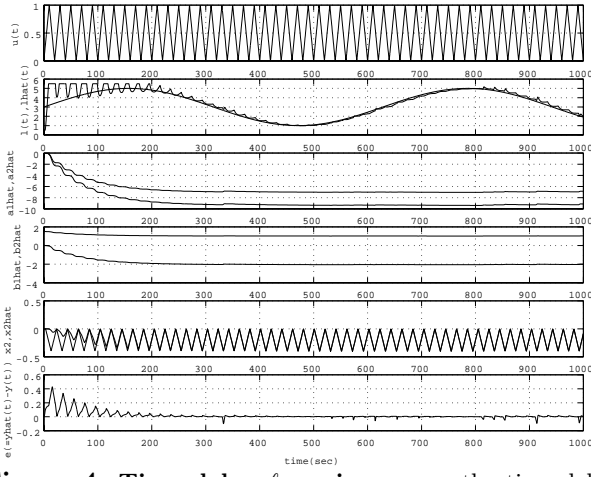


Figure 4: Time-delay ℓ ; a sine wave, the time-delay $\ell = 2 \sin(0.01t) + 3$. the parameters are set by $\ell_{\min} = 0.5$, $\ell_{\max} = 5.5$, $\gamma_{\ell} = 40$, $\gamma = 0.01$, $\varepsilon = 0.001$, $\gamma_{a1} = 7$, $\gamma_{a2} = 65$, $\gamma_{b1} = 0.04$, $\gamma_{b2} = 1.2$.

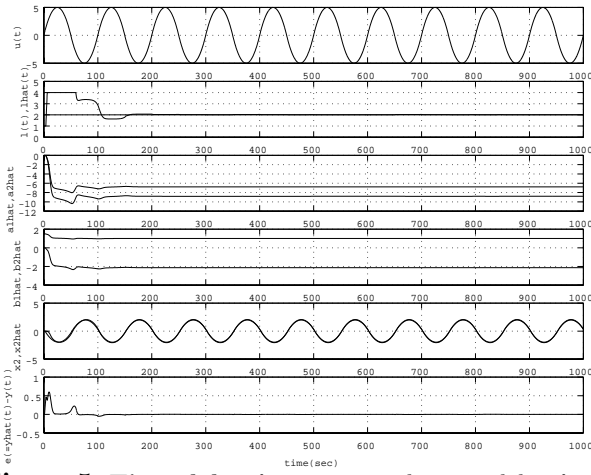


Figure 5: Time-delay ℓ ; constant, the time-delay $\ell = 2$. the parameters are set by $\ell_{\min} = 1$, $\ell_{\max} = 4$, $\gamma_{\ell} = 5$, $\gamma = 0.01$, $\varepsilon = 0.001$, $\gamma_{a1} = 6$, $\gamma_{a2} = 55$, $\gamma_{b1} = 0.04$, $\gamma_{b2} = 1.2$.

6 CONCLUSIONS

By the numerical simulations, the effectiveness of the adaptive observer can be clarified, even though the time-delay in input is unknown and time-varying.

The difficulty of selecting the appropriate input $u(t)$ emerges because it is necessary to know the function $u(t - \ell)$ with respect to ℓ on-line. Our future research is to improve this point and apply our theory to the field of control.

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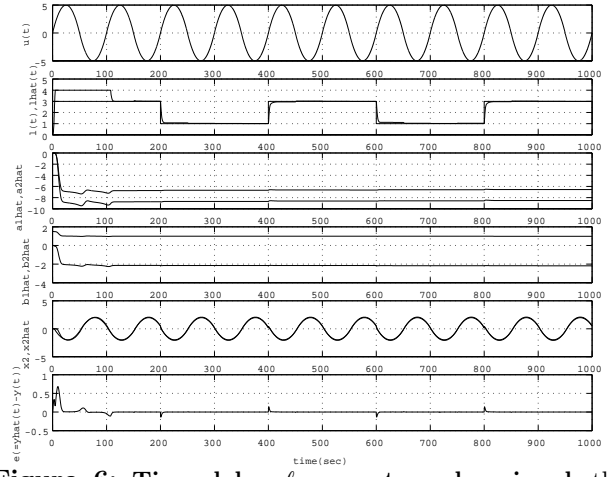


Figure 6: Time-delay ℓ ; a rectangular signal, the time-delay $\ell = 2$. the parameters are set by $\ell_{\min} = 0$, $\ell_{\max} = 4$, $\gamma_{\ell} = 10$, $\gamma = 0.01$, $\varepsilon = 0.001$, $\gamma_{a1} = 6$, $\gamma_{a2} = 55$, $\gamma_{b1} = 0.04$, $\gamma_{b2} = 1.2$.

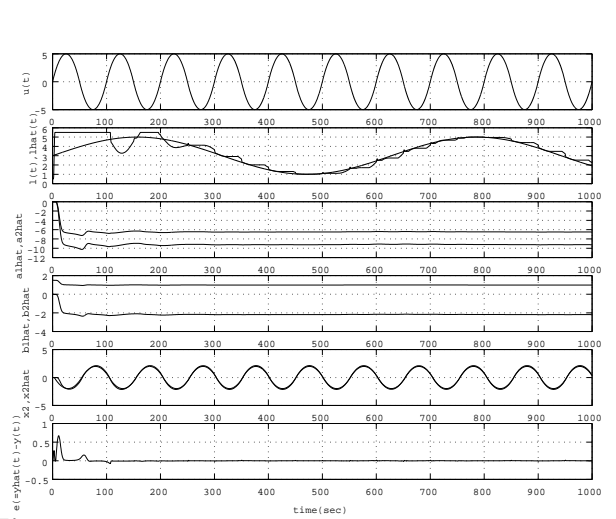


Figure 7: Time-delay ℓ ; a sine wave, the time-delay $\ell = 2 \sin(0.01t) + 3$. the parameters are set by $\ell_{\min} = 0.5$, $\ell_{\max} = 5.5$, $\gamma_{\ell} = 40$, $\gamma = 0.01$, $\varepsilon = 0.001$, $\gamma_{a1} = 5$, $\gamma_{a2} = 50$, $\gamma_{b1} = 0.04$, $\gamma_{b2} = 1.2$.

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