On the design of reduced-order H^∞ controllers for nonaffine nonlinear systems

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Abstract

Sufficient conditions are proposed for the existence of reduced-order (fixed-order) controllers solving the H^∞ control problem for nonaffine nonlinear systems. State-space formulas for such reduced-order H^∞ controllers are also derived in terms of the solutions to two standard Hamilton-Jacobi-Isaacs inequalities.

Keywords: H^∞ control, Controller reduction, Hamilton-Jacobi-Isaacs inequality, Nonaffine nonlinear systems

1 Introduction

It has been shown that full-order H^∞ controllers can be constructed from two algebraic Riccati equations for linear systems or two Hamilton-Jacobi-Isaacs inequalities for nonlinear systems. The controllers thus obtained have a state dimension not less than that of the generalized plant (Doyle *et al.*, 1989; Petersen *et al.*, 1991; Ball *et al.*, 1993; Isidori, 1994; Isidori and Kang, 1995; Lu and Doyle, 1994; Yung *et al.*, 1996; Yung *et al.*, 1998). Since the generalized plant is built from the physical plant and some weighting functions that are used to reflect performance and robustness requirements, the order of generalized plant may be very high. In this case, the full-order controllers may be of limited use in practical applications.

Recently, a number of papers have appeared that deal with reduced-order (or fixed-order) H^∞ controller design for linear systems (see, e.g., DeShetler and Ridgely, 1992; Gahinet and Apkarian, 1994; Gu *et al.*, 1993; Haddad and Bernstein, 1990; Hsu *et al.*, 1994; Hyland and Bernstein, 1984; Iwasaki and Skelton, 1993 and 1994; Juang *et al.*, 1996; Li and Chang, 1993; Pensar and Toivonen, 1993; Stoorvogel *et al.*, 1991; Sweriduk and Calise, 1993; Xin *et al.*, 1996; Yeh *et al..*, 1993; Yung, 2000).Most recently, the reduced-order H^∞ controller design problem for affine nonlinear systems has been addressed and extensively studied by Yung(1999). In terms of the two standard Hamilton-Jacobi-Isaacs inequalities (Isidori, 1994), sufficient conditions for the existence of reduced-order (fixed-order) nonlinear H^∞ controllers have been derived, and state-space formulas for such reduced-order nonlinear H^∞ controllers have also been provided (Yung 1999).

The purpose of the present paper is to extend the results of (Yung, 1999) to general nonaffine nonlinear systems. Sufficient conditions will be established for the existence of reduced-order (fixed-order) controllers solving the H^∞ controll problem for nonaffine nonlinear systems. State-space formulas for such reduced-order H^∞ controllers will also be derived.

2 Problem Formulation and Preliminaries

Consider a smooth (i.e. C^∞) nonaffine nonlinear system described by the state equations

$$\dot{x} = X(x,w,u) \quad (1a)$$
$$z = Z(x,w,u) \quad (1b)$$
$$y = Y(x,w,u), \quad (1c)$$

where x represents the state defined on a neighborhood of the origin in $\mathbb{R}^n$, $u \in \mathbb{R}^{m_2}$ is the control input, $w \in \mathbb{R}^{m_1}$ represents a set of exogenous inputs which includes disturbances to be rejected and/or reference commands to be tracked, $z \in \mathbb{R}^{p_1}$ is the controlled variable, and $y \in \mathbb{R}^{p_2}$ is the measured variable. It is assumed throghout that $X(0,0,0) = 0, Z(0,0,0) = 0$ and $Y(0,0,0) = 0$. In this paper, we restrict ourselves to the consideration of systems satisfying the following assumptions; see also Isidori and Kang(1995) and Yung *et al.*(1998).

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Assumption (A1): The matrices D_{12} has rank m_2 and the matrix $D_{11}^T D_{11} - \gamma^2 I$ is negative definite, where $D_{12} = (\frac{\partial Z}{\partial u})_{(x,w,u)=(0,0,0)}$ and $D_{11} = (\frac{\partial Z}{\partial w})_{(x,w,u)=(0,0,0)}$.

Assumption (A2): The matrix $D_{21} = (\frac{\partial Y}{\partial w})_{(x,w,u)=(0,0,0)}$ has rank p_2.

Our aim in this paper is to find a smooth reduced-order (fixed-order) output feedback controller of the form

$$\begin{aligned} \dot{\xi} &= \tilde{F}(\xi, y) \\ u &= H(\xi) \end{aligned} \quad (2)$$

where $\xi \in I\!R^r$ ($r \le n$) is defined on a neighborhood of the origin, with $\tilde{F}(0,0) = 0$ and $H(0) = 0$, such that the resulting closed-loop system has a locally asymptotically stable equilibrium at the origin $(x,\xi)=(0,0)$, and has L^2-gain $\le \gamma$, or equivalently, such that there exists a neighborhood of the origin $(x,\xi) = (0,0)$ such that for all $T > 0$ and for each input $w(\cdot) \in L^2[0,T]$, the state trajectory of the closed-loop system starting from the initial state $(x(0), \xi(0)) = (0,0)$ remains in the neighborhood for all $t \in [0,T]$, and the response $z(\cdot)$ of the closed-loop system satisfies

$$\int_0^T \| z(t) \|^2 \, dt \le \gamma^2 \int_0^T \| w(t) \|^2 \, dt.$$

For details, see Van der Schaft (1992).

The following proposition follows immediatelly from the results of Isidori and Kang (1995), which provides an output feedback controller solving the problem in question.

Proposition 1 *Consider system* (1) *and suppose that Assumptions* **(A1)-(A2)** *are satisfied. Suppose that the following hypotheses hold.*

(H1) *There exists a smooth, positive definite function $V(x)$, locally defined on a neighborhood of the origin in $I\!R^n$, such that the function*

$$Y_1(x) = L(x, (\tfrac{dV}{dx})^T(x), \alpha_1(x), \alpha_2(x)) \quad (3)$$

is negative definite near $x = 0$, where the function $L : I\!R^n \times I\!R^n \times I\!R^{m_1} \times I\!R^{m_2} \to I\!R$ is defined on a neighborhood of $(x,p,w,u) = (0,0,0,0)$ as

$$L(x,p,w,u) = p^T X(x,w,u) + \|Z(x,w,u)\|^2 - \gamma^2 \|w\|^2,$$

where $\alpha_1(x) = w^*(x, (\frac{dV}{dx})^T(x))$, and $\alpha_2(x) = u^*(x, (\frac{dV}{dx})^T(x))$, and where $w^*(x,p)$ and $u^*(x,p)$ are defined on a neighborhood of $(x,p) = (0,0)$, satisfying

$$\begin{aligned} \frac{\partial L}{\partial w}(x,p,w^*(x,p),u^*(x,p)) &= 0 \\ \frac{\partial L}{\partial u}(x,p,w^*(x,p),u^*(x,p)) &= 0 \end{aligned}$$

with $w^*(0,0) = 0$ and $u^*(0,0) = 0$.

(H2) *There exists a smooth, positive definite function $W(x)$, locally defined on a neighborhood of $x = 0$, such that the function*

$$Y_2(x) = K(x, (\tfrac{dW}{dx})^T(x), \hat{w}(x, (\tfrac{dW}{dx})^T(x), \hat{y}(x, (\tfrac{dW}{dx})^T(x))), \hat{y}(x, (\tfrac{dW}{dx})^T(x))) - Y_1(x)$$

is negative definite near $x = 0$, and its Hessian matrix is nonsingular at $x = 0$, where the function $K : I\!R^n \times I\!R^n \times I\!R^{m_1} \times I\!R^{p_2} \to I\!R$ is defined on a neighborhood of $(x,p,w,y) = (0,0,0,0)$ as

$$K(x,p,w,y) = p^T X(x,w,0) - y^T Y(x,w,0) + \|Z(x,w,0)\|^2 - \gamma^2 \|w\|^2,$$

the function $\hat{w}(x,p,y)$, defined on a neighborhood of $(0,0,0)$, satisfies

$$(\frac{\partial K(x,p,w,y)}{\partial w})_{w=\hat{w}(x,p,y)} = 0, \quad \hat{w}(0,0,0) = 0,$$

and the function $\hat{y}(x,p)$, defined on a neighborhood of $(0,0)$, is such that

$$(\frac{\partial K(x,p,\hat{w}(x,p,y),y)}{\partial y})_{y=\hat{y}(x,p)} = 0, \quad \hat{y}(0,0) = 0.$$

Then if $W(x) - V(x) > 0$ for all $x \neq 0$, and if the equation

$$(\frac{dW}{dx}(x) - \frac{dV}{dx}(x))\hat{G}(x) = \hat{y}^T(x, (\frac{dW}{dx})^T(x))$$

has a smooth solution $\hat{G}(x)$ near $x = 0$, the nonlinear H^∞ output feedback control problem is solved by the output feedback

$$\begin{aligned} \dot{\hat{x}} &= \hat{F}(\hat{x}) + \hat{G}(\hat{x}) y \\ u &= \hat{H}(\hat{x}) \end{aligned} \quad (4)$$

where $\hat{x} \in I\!R^n$ is defined on a neighborhood of the origin,

$$\hat{F}(\hat{x}) = X(\hat{x}, \alpha_1(\hat{x}), \alpha_2(\hat{x})) - \hat{G}(\hat{x}) Y(\hat{x}, \alpha_1(\hat{x}), \alpha_2(\hat{x})),$$

and

$$\hat{H}(\hat{x}) = \alpha_2(\hat{x}).$$

3 Main Results

In this section, we will propose a reduced-order controller of the form (2) that locally asymptotically stabilizes the resulting closed-loop system and renders its L^2-gain $\leq \gamma$. For this purpose, suppose that the hypotheses (H1) and (H2) of Proposition 1 hold. We also assume that there exists a smooth function $\phi : I\!R^n \to I\!R^r$ defined around the origin $x = 0$ in $I\!R^n$ with $\phi(0) = 0$ and rank$\frac{d\phi}{dx}(0) = r$. The rank condition implies that the restriction of ϕ to some neighborhood Ω of $x = 0$ is a surjection. Then we make a change of variables

$$\hat{\xi} = -\phi(x) + \xi \tag{5}$$

where $\hat{\xi} \in I\!R^r$ and $\xi \in I\!R^r$ are defined on a neighborhood of the origin. In terms of these variables the resulting closed-loop system is

$$\begin{array}{rcl} \dot{x_e} & = & F_e(x_e, w) \\ z & = & H_e(x_e, w) \end{array} \tag{6}$$

where $x_e = col(x, \hat{\xi})$,

$$F_e(x_e, w) =$$
$$\begin{bmatrix} X(x,w,H(\hat{\xi}+\phi(x))) \\ -\frac{d\phi}{dx}(x)X(x,w,H(\hat{\xi}+\phi(x)))+\tilde{F}(\hat{\xi}+\phi(x),Y(x,w,H(\hat{\xi}+\phi(x))) \end{bmatrix},$$

and

$$H_e(x_e, w) = Z(x, w, H(\hat{\xi} + \phi(x))).$$

The problem of choosing a control law (2) in such a way that the L^2-gain of the closed-loop system (6) from the exogenous input w to the penalty output z is less than or equal to γ can be viewed as a game problem of rendering the so-called Hamiltonian function $J : I\!R^{(n+r)} \times I\!R^{(n+r)} \times I\!R^{m_1} \to I\!R$ defined as

$$J(x_e, p, w) = p^T F_e(x_e, w) + \|H_e(x_e, w)\|^2 - \gamma^2\|w\|^2$$

nonpositive for each x_e and each (p, w). It is easy to see by the implicit function theorem that, on a neighborhood of the point $(x_e, p, w) = (0, 0, 0)$, the Hamiltonian function can be rewritten as

$$\begin{array}{rl} J(x_e, p, w) = & J(x_e, p, w^{**}(x_e, p)) + \|w - w^{**}(x_e, p)\|^2_{\beta(x)} \\ & +o(\|w - w^{**}(x_e, p)\|^3), \end{array} \tag{7}$$

where $w^{**}(x_e, p)$ is a smooth function, defined on a neighborhood of $(0, 0)$, satisfying

$$\frac{\partial J}{\partial w}(x_e, p, w^{**}(x_e, p)) = 0, w^{**}(0, 0) = 0, \tag{8}$$

and is such that $\beta(x) = \frac{1}{2}\frac{\partial^2 J}{\partial w^2}(x_e, p, w^{**}(x_e, p))$ with $\beta(0) = D_{11}^T D_{11} - \gamma^2 I < 0$. Here the notation $\|v\|^2_M$ stands for $v^T M v$.

A preliminary lemma will be needed in the sequel.

Lemma 2 *Consider* (1), (2) *and* (5). *Suppose that Assumptions* **(A1)** *and* **(A2)** *are satisfied. Suppose also that there exists a smooth, positive definite function* $P(x_e)$, *locally defined on a neighborhood of the origin in* $I\!R^{n+r}$, *such that the function* $J^*(x_e) = J^*(x, \hat{\xi}) := J(x_e, (\frac{dP}{dx_e})^T(x_e), \alpha_3(x_e))$ *is negative for all nonzero* x_e *around* $x_e = 0$, *where* $\alpha_3(x_e) := w^{**}(x_e, (\frac{dP}{dx_e})^T(x_e))$. *Then the controller* (2) *locally asymptotically stabilizes the resulting closed-loop system* (6) *and renders its* L^2*-gain* $\leq \gamma$.

Proof:

Consider the candidate Lyapunov function $P(x_e)$. It is easy to see that, along the trajectories of the closed-loop system,

$$J(x_e, (\frac{dP}{dx_e})^T(x_e), w) = \frac{dP}{dt} + \|z\|^2 - \gamma^2\|w\|^2 \tag{9}$$

Setting $w = 0$ in the above equality shows that $\frac{dP}{dt}$ is negative definite near $x_e = 0$. This proves that the equilibrium $x_e = 0$ of the closed-loop system is locally asymptotically stable. Furthermore, since $J^*(x_e) \leq 0$ for all x_e near $x_e = 0$ by hypothesis, (7) and (9) together imply that

$$\frac{dP}{dt} + \|z\|^2 - \gamma^2\|w\|^2 \leq 0$$

from which we conclude that the closed-loop system (6) has L^2-gain $\leq \gamma$. This completes the proof. □

The remaining question now is how the condition in Lemma 2 can be met; that is, how can we make the function $J^*(x, \hat{\xi})$ negative for all nonzero x_e near $x_e = 0$. Furthermore, we also want to characterize the unknown parameters in (2), namely $\tilde{F}$ and H.

First, we apply Taylor expansion theorem to $J^*(x, \hat{\xi})$ around $\hat{\xi} = 0$, obtaining

$$J^*(x, \hat{\xi}) = J^*(x, 0) + \frac{\partial J^*}{\partial \hat{\xi}}(x, 0)\hat{\xi} + \frac{1}{2}\hat{\xi}^T\frac{\partial^2 J^*}{\partial \hat{\xi}^2}(x, 0)\hat{\xi} + h.o.t., \tag{10}$$

where "$h.o.t.$" means higher order terms. Then, we observe that in order to make $J^*(x, \hat{\xi}) \leq 0$ for all nonzero $(x, \hat{\xi})$, it suffices to have (a)$J^*(x, 0) \leq 0$, (b)$\frac{\partial J^*}{\partial \hat{\xi}}(x, 0) = 0$, and (c)$\frac{\partial^2 J^*}{\partial \hat{\xi}^2}(x, 0) < 0$. In what follows, we shall make it clear how the three conditions can be met.

We begin the discussion with Condition (a). Let $U : I\!R^r \to I\!R$ be a smooth, positive definite function, locally defined on a neighborhood of the origin. With $P(x, \hat{\xi}) = V(x) + U(\hat{\xi})$ and

$$H(\phi(x)) = \alpha_2(x), \tag{11}$$

and with (8) in mind, if we first differentiate $J(x_e, (\frac{dP}{dx_e})^T(x_e), w)$ with respect to w, then substitute w for $\alpha_3(x_e)$, and finally set $\hat{\xi} = 0$, we can obtain

$$\begin{aligned}
&\tfrac{dV}{dx}(x)\tfrac{\partial X}{\partial w}(x,\alpha_3(x,0),\alpha_2(x)) \\
&+2Z^T(x,\alpha_3(x,0),\alpha_2(x))\tfrac{\partial Z}{\partial w}(x,\alpha_3(x,0),\alpha_2(x))-2\gamma^2\alpha_3^T(x,0) \\
=&\tfrac{\partial L}{\partial w}(x,(\tfrac{dV}{dx})^T(x),\alpha_3(x,0),\alpha_2(x)) \\
=&0.
\end{aligned}$$

Since $(\alpha_1(x), \alpha_2(x))$ is the unique pair satisfying

$$\frac{\partial L}{\partial w}(x, (\frac{dV}{dx})^T(x), \alpha_1(x), \alpha_2(x)) = 0$$

and

$$\frac{\partial L}{\partial u}(x, (\frac{dV}{dx})^T(x), \alpha_1(x), \alpha_2(x)) = 0,$$

(see Isidori and Kang, 1995), we conclude that $\alpha_3(x, 0) = \alpha_1(x)$. Then it is easy to show that

$$\begin{aligned}
J^*(x,0) &= \tfrac{dV}{dx}(x)X(x,\alpha_1(x),\alpha_2(x))+\|Z(x,\alpha_1(x),\alpha_2(x))\|^2-\gamma^2\|\alpha_1(x)\|^2 \\
&= Y_1(x)
\end{aligned}$$

which is negative for all nonzero x by hypothesis.

Next, consider Condition (b). It is easy to verify that

$$\begin{aligned}
&\tfrac{\partial J^*}{\partial \hat{\xi}}(x,0) \\
=&[\tfrac{dV}{dx}(x)\tfrac{\partial X}{\partial w}(x,\alpha_1(x),\alpha_2(x)) \\
&+2Z^T(x,\alpha_1(x),\alpha_2(x))\tfrac{\partial Z}{\partial w}(x,\alpha_1(x),\alpha_2(x))-2\gamma^2\alpha_1^T(x)]\tfrac{\partial \alpha_3}{\partial \hat{\xi}}(x,0) \\
&+[\tfrac{dV}{dx}(x)\tfrac{\partial X}{\partial u}(x,\alpha_1(x),\alpha_2(x)) \\
&+2Z^T(x,\alpha_1(x),\alpha_2(x))\tfrac{\partial Z}{\partial u}(x,\alpha_1(x),\alpha_2(x))]\tfrac{dH}{d\hat{\xi}}(\phi(x)) \\
&+[\tilde{F}(\phi(x),Y(x,\alpha_1(x),\alpha_2(x)))-\tfrac{d\phi}{dx}(x)X(x,\alpha_1(x),\alpha_2(x))]^T\tfrac{d^2U}{d\hat{\xi}^2}(0) \\
=&\tfrac{\partial L}{\partial w}(x,(\tfrac{dV}{dx})^T(x),\alpha_1(x),\alpha_2(x))\tfrac{\partial \alpha_3}{\partial \hat{\xi}}(x,0) \\
&+\tfrac{\partial L}{\partial u}(x,(\tfrac{dV}{dx})^T(x),\alpha_1(x),\alpha_2(x))\tfrac{dH}{d\hat{\xi}}(\phi(x)) \\
&+[\tilde{F}(\phi(x),Y(x,\alpha_1(x),\alpha_2(x)))-\tfrac{d\phi}{dx}(x)X(x,\alpha_1(x),\alpha_2(x))]^T\tfrac{d^2U}{d\hat{\xi}^2}(0) \\
=&[\tilde{F}(\phi(x),Y(x,\alpha_1(x),\alpha_2(x)))-\tfrac{d\phi}{dx}(x)X(x,\alpha_1(x),\alpha_2(x))]^T\tfrac{d^2U}{d\hat{\xi}^2}(0)
\end{aligned}$$

We now set

$$\tilde{F}(\phi(x), Y(x, \alpha_1(x), \alpha_2(x))) = \frac{d\phi}{dx}(x)X(x, \alpha_1(x), \alpha_2(x)). \tag{12}$$

Then $\frac{\partial J^*}{\partial \hat{\xi}}(x, 0) = 0$.

It is possible to further simplify the expression (12) if we confine attention to an *affine* reduced-order controller, i.e., if $\tilde{F}(\xi, y)$ has the form

$$\tilde{F}(\xi, y) = F(\xi) + G(\xi)y \tag{13}$$

where F and G are some smooth functions, both locally defined on some neighborhood of the origin $\xi = 0$ in $\mathbb{R}^r$ with $F(0) = 0$. It follows from (12) and (13) that

$$\begin{aligned}
&F(\phi(x)) + G(\phi(x))Y(x, \alpha_1(x), \alpha_2(x)) = \\
&\quad \tfrac{d\phi}{dx}(x)\hat{F}(x) + \tfrac{d\phi}{dx}(x)\hat{G}(x)Y(x, \alpha_1(x).\alpha_2(x)),
\end{aligned}$$

where $\hat{F}$ and $\hat{G}$ are defined as in Proposition 1. Then it suffices to choose

$$F(\phi(x)) = \frac{d\phi}{dx}(x)\hat{F}(x) \tag{14}$$

$$\text{and} \quad G(\phi(x)) = \frac{d\phi}{dx}(x)\hat{G}(x) \tag{15}$$

to render $\frac{\partial J^*}{\partial \hat{\xi}}(x, 0) = 0$

Finally, we consider Condition (c). For our purposes we shall further make the following assumption: Suppose that ϕ and U satisfy

$$\frac{d\phi}{dx}(0)(\frac{d\phi}{dx})^T(0) = I \tag{16}$$

and

$$\frac{d^2U}{d\xi^2}(0)\frac{d\phi}{dx}(0) = \frac{d\phi}{dx}(0)(\frac{d^2W}{dx^2}(0) - \frac{d^2V}{dx^2}(0)) \tag{17}$$

Then it is straightforward but tedious to verify that

$$\frac{\partial^2 J^*}{\partial \hat{\xi}^2}(0, 0) = \frac{d\phi}{dx}(0)\frac{d^2Y_2}{dx^2}(0)(\frac{d\phi}{dx})^T(0).$$

In summary, since, by hypothesis, $Y_1(x)$ is negative definite and $\frac{d^2Y_2}{dx^2}(0)$ is also negative definite, this shows that $J^*(x, \hat{\xi})$ is negative for all nonzero x_e around $x_e = 0$. We thus have the following theorem, which is the main result of this paper.

Theorem 3 *Suppose that Assumptions* **(A1)-(A2)** *are satisfied and that Hypotheses* **(H1)** *and* **(H2)** *of Proposition 1 hold. Suppose that there exists a smooth function $\phi : \mathbb{R}^n \to \mathbb{R}^r$, locally defined on a neighborhood of the origin $x = 0$ in $\mathbb{R}^n$, satisfying $\phi(0) = 0$ and (16). Suppose also that there exists a smooth, positive definite function U, locally defined on a neighborhood of the origin $\xi = 0$ in $\mathbb{R}^r$, which satisfies (17). Then, if F, G, and H satisfy (14), (15) and respectively (11), the r-th order controller*

$$\begin{aligned}
\dot{\xi} &= F(\xi) + G(\xi)y \\
u &= H(\xi)
\end{aligned}$$

locally asymptotically stabilizes the resulting closed-loop system (6) and renders its L^2-gain $\leq \gamma$.

4 Conclusions

A method has been proposed for designing reduced-order H^∞ controllers of general nonaffine nonlinear systems. When the system is an affine nonlinear system, it can be shown that the results obtained in this paper are exactly reduced to the corresponding results given in Yung(1999).

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