

# Adaptive $L_2$ Disturbance Attenuation of Hamiltonian Systems with Parametric Perturbation and Application to Power Systems<sup>1</sup>

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**Abstract.** This paper deals with the problem of  $L_2$  disturbance attenuation for Hamiltonian systems. We first show that the  $L_2$  gain from the disturbance to a penalty signal may be reduced to any given level if the penalty signal is defined properly. Then, an adaptive version of the controller will be presented to compensate the parameter perturbation. An adaptive  $L_2$  controller for the power system is designed using the proposed method and a simulation result with the proposed controller is given.

**Keywords.** Hamiltonian system,  $L_2$  disturbance attenuation, adaptive control, power system.

## 1 Introduction

A powerful design technique for stabilization of nonlinear systems is passivity-based control (PBC)<sup>[1]</sup>. In the PBC framework, the controller design proceeds along two stages. The first stage is to render passive a map with a suitably defined storage function, and the second stage is to perform an output feedback. For mechanical systems the design process has a physical meaning, e.g. the first stage can be carried out by shaping the potential energy of the system in such a way that the new potential energy function has a strict local minimum at the desired equilibrium, and the second stage is nothing but damping injection. Recently, the design technology has been extended to a broader class of systems described by port-controlled Hamiltonian (PCH)

models. Indeed, the Hamiltonian function in PCH systems is the total energy, potential and kinetic energy in physical systems, and can play the role of Lyapunov function for the system. However, when the system is forced by external input such as set-point regulation, the Hamiltonian function does not necessarily have a minimum at the desired operating point. In this case, we could employ a pre-feedback and shape the Hamiltonian function such that the closed loop system has Hamiltonian structure with the modified function to ensure stability. Furthermore, under some detectability conditions, asymptotic stability is also ensured. The problem of passivity-based control for PCH systems has been investigated by [2]~[4] and several application examples have been illustrated by [5, 6].

In practical engineering, disturbance attenuation and parametric uncertainty are also important issues. PBC design method has been extended by many researchers to achieve  $\gamma$ -dissipativity<sup>[7]</sup> that not only guarantees asymptotic stability but also renders the  $L_2$ -gain from disturbance to a penalty signal less than a given level  $\gamma > 0$ . As PBC design, a key to solve the disturbance attenuation problem along this line is to construct a proper storage function that ensures the  $\gamma$ -dissipativity. Several effective methods have been reported by [8]~[10]. For nonlinear systems with unknown parameters, the PBC design methodology has been extended to include adaptive mechanism.

In this paper, we are interested in the problem of  $L_2$  disturbance attenuation for the Hamiltonian systems with parametric perturbations. It will be shown that for a given  $\gamma > 0$ ,  $\gamma$ -dissipativity can be achieved by making a sufficiently large damping injection in the second design stage only, if the penalty signal is properly defined. Then, we will consider the case when

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the Hamiltonian system involves parametric perturbations. We shall show that if the perturbations satisfy a suitable matching condition, then an adaptation mechanism can be added to the feedback controller to estimate the controller parameters corresponding to the parameter perturbations.

Another aim of this paper is to apply the proposed approach to power systems. Application examples of the PBC methodology on the power system have been shown by [11, 12]. An adaptive  $L_2$ -disturbance attenuation control has been illustrated by [13], and an Hamiltonian system view has been reported by [6]. In this paper, it will be shown that the proposed approach can be applied to the excitation control problem of the power system with electrical parameter perturbations.

## 2 Preliminaries

Consider the port-controlled Hamiltonian system with dissipation described by

$$\begin{cases} \dot{x} = [J(x) - R(x)] \frac{\partial H}{\partial x}(x) + g(x)u \\ y = g^T(x) \frac{\partial H}{\partial x}(x) \end{cases} \quad (1)$$

where  $x \in \mathcal{X}$ , an  $n$ -dimensional manifold,  $u, y \in R^m$  are the input and output, respectively;  $H : R^n \rightarrow R$  represents the total stored energy called *Hamiltonian function*,  $J(x)$  is a skew-symmetric structure matrix, and  $R(x)$  is a non-negative symmetric matrix.

An interesting property of this class of the systems is that the Hamiltonian function  $H(x)$  can play the role of Lyapunov function for stability analysis, i.e. if  $H(x)$  admits a strict minimum at  $x_e$ , then  $x_e$  is a stable equilibrium of the unforced systems

$$\dot{x} = [J(x) - R(x)] \frac{\partial H}{\partial x}(x) \quad (2)$$

Usually  $x_e$  corresponds to the zero operating point, therefore the control input can be used to shape the Hamiltonian function  $H(x) \rightarrow H_c(x)$  (and eventually to inject additional damping  $R(x) \rightarrow R_c(x)$ ) such that under a proper feedback  $u = \alpha(x)$ , the closed loop system has the Hamiltonian structure, i.e.

$$[J - R] \frac{\partial H}{\partial x} + g(x)\alpha(x) = [J - R_c] \frac{\partial H_c}{\partial x} \quad (3)$$

and  $H_c(x)$  admits a strict minimum at a desired equilibrium  $x_0$ . This problem has been investigated by [2].

The main objective of this paper is the disturbance attenuation. We will start with the model (1) to which we add a disturbance

$$\dot{x} = [J(x) - R(x)] \frac{\partial H}{\partial x}(x) + g(x)(u + w) \quad (4)$$

where  $w \in R^m$  is an unknown disturbance.

The  $L_2$  disturbance attenuation objective is as follows: Given a penalty signal  $z = q(x)$ , a disturbance attenuation level  $\gamma > 0$  and a desired equilibrium  $x_0 \in R^n$ . Find a control law  $u = k(x)$  and a positive storage function  $V(x)$  such that the  $\gamma$ -dissipation inequality

$$\dot{V} + Q(x) \leq \frac{1}{2}\{\gamma^2\|w\|^2 - \|z\|^2\}, \quad \forall w \quad (5)$$

holds along all trajectories of the closed loop system consist of (4) with the feedback law, where  $Q(x)$  ( $Q(x) \neq 0, \forall x \neq x_0$ ) is a given non-negative definite function.

## 3 Control Law Design

### 3.1 Known parameter case

Suppose that for the system (4) there exists such a feedback  $u = \alpha(x)$  that preserves Hamiltonian structure with a modified Hamiltonian function  $H_c(x)$  and a symmetric non-negative matrix  $R_c(x)$ , i.e. (3) holds, and  $H_c$  admits a strict minimum at the desired equilibrium  $x_0$ . A method for seeking such a feedback law  $\alpha(x)$  has been proposed by [2].

Let the penalty signal is defined as follows

$$z = h(x)g^T(x) \frac{\partial H_c}{\partial x}(x) \quad (6)$$

where  $h(x)$  is a weighting matrix.

**Theorem 1.** Consider the system (4) with the penalty signal (6). For any given  $\gamma > 0$ , the  $L_2$  disturbance attenuation objective is achieved by the following feedback control law.

$$\begin{cases} u = \alpha(x) + \beta(x) \\ \beta(x) = -\frac{1}{2} \left\{ \frac{1}{\gamma^2} I + h^T(x)h(x) \right\} g^T(x) \frac{\partial H_c}{\partial x} \end{cases} \quad (7)$$

**Proof.** Note that, under the feedback (7), the closed loop system with the modified Hamiltonian function  $H_c$  can be represented by

$$\begin{cases} \dot{x} = [J_c(x) - R_c(x)] \frac{\partial H_c}{\partial x} + g(x)(\beta(x) + w) \\ z = h(x)g^T(x) \frac{\partial H_c}{\partial x}(x) \end{cases} \quad (8)$$

Along any trajectory of this system, a straightforward calculation gets

$$\begin{aligned} \dot{H}_c &= -\frac{\partial^T H_c}{\partial x} R_c \frac{\partial H_c}{\partial x} + \frac{\partial^T H_c}{\partial x}(x)g(x) \\ &\quad \left\{ \beta(x) + \frac{1}{2} \left[ \frac{1}{\gamma^2} I + h^T(x)h(x) \right] g^T(x) \frac{\partial H_c}{\partial x} \right\} \\ &\quad - \frac{1}{2} \left\| \gamma w - \frac{1}{\gamma} g^T \frac{\partial H_c}{\partial x} \right\|^2 + \frac{1}{2} \{ \gamma^2 \|w\|^2 - \|z\|^2 \} \end{aligned} \quad (9)$$

Hence, by substituting  $\beta(x)$  into the right side and setting the non-negative definite function  $Q(x) = \frac{\partial^T H_c}{\partial x}(x)R_c(x)\frac{\partial H_c}{\partial x}(x)$ , we have

$$\dot{H}_c + Q(x) \leq \frac{1}{2}\{\gamma^2\|w\|^2 - \|z\|^2\}, \quad \forall w \quad (10)$$

This means the Hamiltonian function  $H_c$  serves as the storage function for the closed loop system. ■

**Remark 1.** The PBC design mentioned in Section 1 allows us to calculate the pre-feedback  $\alpha(x)$  which shapes the energy and adds damping, that is,  $H(x) \rightarrow H_c(x)$ ,  $R(x) \rightarrow R_c(x)$  in such a way that the unforced system, i.e.  $w = 0$ , is stable at the desired equilibrium  $x_0$ . Theorem 1 shows that in order to furthermore render the closed loop system  $\gamma$ -dissipative, we only need to inject additional damping in the second stage  $R_c(x) \rightarrow R_c(x) + g(x)\frac{1}{2}\left\{\frac{1}{\gamma^2}I + h^T(x)h(x)\right\}g^T(x)$ .

### 3.2 Adaptive case

We now consider the case when the model of system (4) involves parameter perturbations. Let the parameter perturbations be represented by a constant vector  $p$  whose nominal value is zero. Suppose that the system (4) is represented by

$$\dot{x} = [J(x, p) - R(x, p)]\frac{\partial H}{\partial x}(x, p) + g(x)(u + w) \quad (11)$$

For simplicity, we denote  $J(x, 0) = J(x)$ ,  $R(x, 0) = R(x)$  and  $H(x, 0) = H(x)$ .

In this case, the modified Hamiltonian function  $H_c$ ,  $J_c$ ,  $R_c$  and the pre-feedback law  $\alpha$  will involve the perturbed parameter vector  $p$ , i.e. under the state feedback

$$u = \alpha(x, p) + v \quad (12)$$

the closed loop system can be represented as follows

$$\dot{x} = [J_c(x, p) - R_c(x, p)]\frac{\partial H_c}{\partial x}(x, p) + g(x)(v + w) \quad (13)$$

Decompose all functions related to the perturbed parameters  $p$  as follows:

$$\frac{\partial H_c}{\partial x} = \frac{\partial H_c}{\partial x} + \Delta_h(x, p), \quad R_c = R_c(x) + \Delta_R(x, p),$$

$$J_c(x, p) = J_c(x) + \Delta_J(x, p), \quad \alpha(x, p) = \alpha(x) + \Delta_\alpha(x, p)$$

where  $\Delta_i(x, 0) = 0$  ( $i = H, R, J, \alpha$ ). It should be noted that we denote the corresponding nominal functions as  $H_c(x) = H_c(x, 0)$ ,  $J_c(x) = J_c(x, 0)$ ,  $R_c(x) = R_c(x, 0)$  and  $\alpha(x) = \alpha(x, 0)$  for simplicity.

Since the parameter perturbation vector  $p$  is unknown, we substitute the nominal function  $\alpha(x)$  with the nominal parameter for the previous feedback  $\alpha(x, p)$ , and design an adaptive controller

$$\begin{cases} u = \alpha(x) + \beta_a(x, \hat{\theta}) \\ \dot{\hat{\theta}} = \phi(x) \end{cases} \quad (14)$$

where  $\hat{\theta} \in R^q$  is a parameter estimate vector

Therefore, our goal is to seek the functions  $\beta_a(x, \hat{\theta})$ ,  $\phi(x)$  and to modify the energy function  $H_c(x)$  in such a way that the  $\gamma$ -dissipation inequality

$$\dot{U} + Q(x) \leq \frac{1}{2}\{\gamma^2\|w\|^2 - \|z\|^2\} \quad (15)$$

holds for a properly constructed storage function  $U(x, \hat{\theta})$ .

**Theorem 2.** Consider the system (11) with the penalty signal (6). Assume that there exists a function  $\Psi(x)$  such that

$$\{J_c(x, p) - R_c(x, p)\}\Delta_h(x, p) - g(x)\Delta_\alpha(x) = g(x)\Psi^T(x)\theta \quad (16)$$

hold for all  $x$ . Then, for any given  $\gamma > 0$ , the adaptive  $L_2$  disturbance attenuation problem is solved by

$$\begin{cases} \beta_a = -\frac{1}{2}\left\{\frac{1}{\gamma^2}I + h^T h\right\}g^T\frac{\partial H_c}{\partial x} - \Psi^T(x)\hat{\theta} \\ \phi = \Gamma\Psi(x)g^T(x)\frac{\partial H_c}{\partial x}(x) \end{cases} \quad (17)$$

where  $\theta \in R^q$  denotes a constant parameter vector,  $\hat{\theta}$  is an estimation of  $\theta$  and  $\Gamma = \text{diag}\{\rho_1, \rho_2, \dots, \rho_q\}$ ,  $\rho_i > 0$  ( $i = 1, 2, \dots, q$ ).

**Proof.** For system (11), we perform the feedback (14) with functions given by (17). Then, using the condition (16), the closed loop system can be presented by

$$\begin{aligned} \dot{x} &= [J_c(x, p) - R_c(x, p)]\frac{\partial H_c}{\partial x}(x) \\ &+ g(x)\{\Psi^T(x)\theta + \beta_a(x, \hat{\theta}) + w\} \end{aligned} \quad (18)$$

For this system, we modify the energy function  $H_c$  to generate the desired storage function  $U$  as follows:

$$U(x, \hat{\theta}) = H_c(x) + \frac{1}{2}(\theta - \hat{\theta}(t))^T\Gamma^{-1}(\theta - \hat{\theta}(t)) \quad (19)$$

Then, along any trajectories of the system, we have

$$\begin{aligned} \dot{U} &= -\frac{\partial^T H_c}{\partial x}(x)R_c(x, p)\frac{\partial H_c}{\partial x}(x) \\ &+ \frac{\partial^T H_c}{\partial x}\left\{-\frac{1}{2}\left[\frac{1}{\gamma^2}I + h^T h\right]g^T\frac{\partial H_c}{\partial x} + w\right\} \\ &+ \frac{\partial^T H_c}{\partial x}(x)g(x)\Psi^T(x)(\theta - \hat{\theta}) - \dot{\hat{\theta}}\Gamma^{-1}(\theta - \hat{\theta}) \end{aligned} \quad (20)$$

Note that, the parameter adaptation law is given by

$$\dot{\hat{\theta}} = \phi(x) = \Gamma\Psi(x)g^T(x)\frac{\partial H_c}{\partial x}(x) \quad (21)$$

Thus, by the same techniques used in the proof of Theorem 1, we obtained

$$\dot{U} + Q(x) \leq \frac{1}{2}\left\{\frac{1}{\gamma^2}\|w\|^2 - \|z\|^2\right\}, \quad \forall w \quad (22)$$

**Remark 2.** The basis of the feedback control law  $\beta_a(x, \hat{\theta})$  is the control law given in Theorem 1, i.e. the first term in  $\beta_a$  is the damping injection to achieve  $\gamma$ -dissipativity for the nominal system. The second term  $\psi^T(x)\hat{\theta}$  is constructed based on the parameter estimation for compensation of the perturbed parameter. In view of condition (22), boundedness of parameter estimation  $\hat{\theta}$  as well as convergence of  $x(t)$  towards a minimum of  $H_c(x)$  is guaranteed. Notice, however, that this point does not correspond to the minimum of the actual energy function  $H_c(x, p)$ , but to one that results from our *a priori* estimate. Therefore, the presence of parameter uncertainty will induce a shift in the achieved equilibrium. This bias will, of course, disappear if the estimate error  $\theta - \hat{\theta}$  converges to zero, which requires some persistency of excitation requirements.

## 4 Application to Power Systems

In this Section, we will apply the proposed controller to the problem of excitation of power systems. The problem has been addressed by many researchers. A non-linear control approach based on the exact linearization method has been proposed by [14], while PBC have been investigated by [11, 6, 12].  $L_2$  disturbance attenuation problem has been also studied by [13, ?] for the power system. The design proposed in [?] is based on  $\gamma$ -dissipativity, however, the physical energy was not taken account in constructing the storage function. The controller obtained by constructing the storage function recursively has a rather complex structure. We will show that the power system forced by a constant excitation signal which is for set-point regulation has the Hamiltonian structure. Therefore, applying the proposed design approach, the  $\gamma$ -dissipativity can be achieved by simple feedback, injection additional damping only, and the controller can be easily extended to the adaptive version.

### 4.1 System description

A simplified single-machine infinite bus power system with silicon-controlled rectifier (SRC) direct excitor is as shown in Fig.1. A model for excitation control of this system can be written as follows:<sup>[14]</sup>

$$\dot{\delta} = \omega(t) - \omega_0 \quad (23)$$

$$\dot{\omega} = -\frac{D}{M}\{\omega(t) - \omega_0\} + \frac{\omega_0}{M}\{P_m - P_e(t)\} \quad (24)$$

$$\dot{E}'_q = -\frac{1}{T'_d}E'_q + \frac{1}{T_{d0}}\frac{x_d - x'_d}{x'_\Sigma}V_s \cos \delta + \frac{1}{T_{d0}}V_f + w \quad (25)$$

where  $P_e(t) = \frac{E'_q(t)V_s}{x'_\Sigma} \sin \delta$  is the active electrical power,  $\delta(t)$  and  $\omega(t)$  are angle and speed of the rotor,

respectively.  $E'_q(t)$  is the transient EMF in the quadrature axis of the generator.  $V_f$  denotes the control input of the SCR amplifier of the generator, and  $w$  denotes the unknown disturbance which caused by faults in the line or loads level variation, etc.<sup>1</sup>

We only consider the excitation control loop. Hence, we assume that the mechanical input power  $P_m$  and the speed of synchronous machine  $\omega_0$  are constants. Define the state variable by

$$x_1 = \delta, \quad x_2 = \omega - \omega_0, \quad x_3 = E'_q \quad (26)$$

Then, dynamics of the system can be represented by the following state space model:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -D_M x_2 - b_L x_3 \sin x_1 + P \\ \dot{x}_3 = c_L \cos x_1 - c_T x_3 + V + w \end{cases} \quad (27)$$

where the control input  $V(t) = \frac{1}{T'_d}V_f(t)$ , and the parameters are defined by

$$D_M = \frac{D}{M}, \quad b_L = \frac{\omega_0 V_s}{M x'_\Sigma}, \quad P = \frac{\omega_0}{M}P_m, \quad c_L = \frac{x_d - x'_d}{T_{d0} x'_\Sigma}V_s, \quad c_T = \frac{1}{T'_d}$$

As it is well-known (see e.g. [12] for a recent reference) if we insert a constant excitation input  $V(t) = \bar{u}$ , then the system with  $w = 0$  has a local equilibrium  $(x_{1e}, 0, x_{3e})$  solution of

$$\begin{cases} b_L x_{3e} \sin x_{1e} = P \\ c_T x_{3e} - c_L \cos x_{1e} = \bar{u} \end{cases}$$

with an energy-like function

$$H_c(x) = \frac{1}{2}x_2^2 + b_L x_3 (\cos x_{1e} - \cos x_1) - P(x_1 - x_{1e}) + \frac{1}{2} \frac{b_L c_T}{c_L} (x_3 - x_{3e})^2 \quad (28)$$

that qualifies as a Lyapunov function for the forced system (27) with the constant input

$$V(t) = \bar{u} = c_T x_{3e} - c_L \cos x_{1e} \quad (29)$$

It is easy to check that  $H_c(x)$  admits a local strict minimum at  $(x_{1e}, 0, x_{3e})$ , and the forced system (27) with the feedback control

$$V(t) = \bar{u} + v \quad (30)$$

can be represented by the following formulation with the Hamiltonian function (28)

$$\dot{x} = [J(x) - R_c(x)] \frac{\partial H_c}{\partial x}(x) + g(x)(v + w) \quad (31)$$

where  $g(x) = [0 \ 0 \ 1]^T$  and the matrices defined by

$$J = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad R_c = \begin{bmatrix} 0 & 0 & 0 \\ 0 & D_M & 0 \\ 0 & 0 & \frac{c_T}{b_L} \end{bmatrix} \quad (32)$$

<sup>1</sup>See [?] for the notation of the model parameters.

## 4.2 Control law design

Consider the penalty signal defined by (6) with the weighting function  $h(x) = [q_1 x_1 \ q_2 x_2 \ q_3 x_3]^T$ ,  $q_i \geq 0$  ( $i = 1, 2, 3$ ), i.e.

$$z = h(x) \left\{ b_L (\cos x_{1e} - \cos x_1) + \frac{b_L c_T}{c_L} (x_3 - x_{3e}) \right\} \quad (33)$$

Our goal is as follows: For given desired equilibrium  $(x_{1e}, 0, x_{3e})$  and any given disturbance attenuation level  $\gamma > 0$ , find a feedback control law such that the closed loop system consist of (27) and the control law has the property of  $\gamma$ -dissipativity.

As mentioned above, the system (27) forced by the constant input (29),  $\alpha(x) = \bar{u}$ , has the Hamiltonian structure and the total energy function  $H_c$  admit a strict minimum at the desired equilibrium. Thus, applying Theorem 1 to the system (27) gets a desired feedback law as follows:

$$\begin{aligned} u &= \bar{u} - \frac{1}{2} \left[ \frac{1}{\gamma^2} I + h^T(x) h(x) \right] g^T(x) \frac{\partial H}{\partial x}(x) \\ &= c_T x_{3e} - c_L \cos x_{1e} - \frac{1}{2} \left[ \frac{1}{\gamma^2} + \sum_{i=1}^3 q_i^2 x_i^2 \right] \\ &= \left\{ b_L (\cos x_{1e} - \cos x_1) + \frac{b_L c_T}{c_L} (x_3 - x_{3e}) \right\} \end{aligned} \quad (34)$$

We now design the adaptive controller for the power system. When a fault occurs or the structure of the network changed, the parameters of the electrical equation will change drastically, and so, the equilibrium  $x_{1e}$ ,  $x_{3e}$  will be changed. Therefore, it is reasonable in the practical power systems to consider the uncertainties in the coefficients of electrical equation  $c_L$ ,  $c_T$ ,  $x_{1e}$  and  $x_{3e}$ . We consider the parameter perturbation as follows:  $c_L \rightarrow c_L + p_1$ ,  $c_T \rightarrow c_T + p_2$ ,  $\cos x_{1e} \rightarrow \cos x_{1e} + p_3$ ,  $x_{3e} \rightarrow x_{3e} + p_4$ , where  $p_1, p_2, p_3, p_4$  are bounded unknown constants.

Then, it is easy to check that the Hamiltonian function  $H_c(x, p)$ , the structure matrices  $J_c(x, p)$ ,  $R_c(x, p)$  and the feedback  $\alpha(x, p)$  can be decomposed by

$$\frac{\partial H_c}{\partial x}(x, p) = \frac{\partial H_c}{\partial x} + \Delta_h(x, p), \quad R_c(x, p) = R_c + \Delta_R(x, p)$$

$$J_c(x, p) = J_c(x) + \Delta_J(x, p), \quad \alpha(x, p) = \alpha(x) + \Delta_\alpha(x, p)$$

with the perturbed functions defined by

$$\Delta_h(x, p) = [0 \ 0 \ b_L \theta'_1 + b_L \theta'_2 (x_3 - x_{3e})]^T, \quad \Delta_J(x, p) = 0,$$

$$\Delta_R(x, p) = \text{diag}\{0, 0, \frac{p_1}{b_L}\}, \quad \Delta_\alpha = \theta'_3$$

where

$$\theta'_1 = p_3 - \frac{c_T + p_2}{c_L + p_1} p_4, \quad \theta'_2 = \frac{c_T + p_2}{c_L + p_1} - \frac{c_T}{c_L},$$

$$\begin{aligned} \theta'_3 &= \{(c_T + p_2)(x_{3e} + p_4) - (c_L + p_1) \cos(x_{1e} + p_3)\} \\ &\quad - \{c_T x_{3e} - c_L \cos x_{1e}\} \end{aligned}$$

Define the parameter estimate vector  $\theta = [\theta_1 \ \theta_2]^T$ , where  $\theta_1 = \theta'_1 - \theta'_3$  and  $\theta_2 = \theta'_2$ , and let the matrix function be given by

$$\Psi^T(x) = [1 \ (x_3 - x_{3e})] \quad (35)$$

Thus, the perturbed functions will satisfy the matching condition (16). From Theorem 2, we obtained the following adaptive feedback law

$$\begin{cases} u = \bar{u} - \frac{1}{2} \left[ \frac{1}{\gamma^2} + \sum_{i=1}^3 q_i^2 x_i^2 \right] \{ b_L (\cos x_{1e} - \cos x_1) \\ \quad + \frac{b_L c_T}{c_L} (x_3 - x_{3e}) \} - \hat{\theta}_1 - (x_3 - x_{3e}) \hat{\theta}_2 \\ \dot{\hat{\theta}}_1 = \rho_1 \{ b_L (\cos x_{1e} - \cos x_1) + b_L \frac{c_T}{c_L} (x_3 - x_{3e}) \} \\ \dot{\hat{\theta}}_2 = \rho_2 \{ b_L (\cos x_{1e} - \cos x_1) + b_L \frac{c_T}{c_L} (x_3 - x_{3e}) \} \end{cases} \quad (36)$$

where  $\rho_1 > 0$  and  $\rho_2 > 0$  are the adaptation gains.

## 4.3 Simulation results

The simulation was implemented in the PSASP package which is a professional testing system for power systems designed by China Electrical Power Research Institute. A synchronous generator (100MW) against an infinite-bus with SCR excitor was chosen as the example system. Dynamical performance under a fault will be tested for the cases with different disturbance attenuation level  $\gamma$ . The fault considered in this simulation is a symmetrical three-phase short-circuit fault during the time period 0~0.2(sec.), and the fault location is at the middle of the transmission line. Also, the limitation of excitation value  $0.0(p.u.) \leq V_f(t) \leq 4.0(p.u.)$  is considered in the simulation.

We consider the following operating point:  $\delta_0 = 0.7439 \text{rad.}$ ,  $\omega_0 = 1.0 p.u.$ ,  $E'_{q0} = 0.9361 p.u.$ . Hence, the desired equilibrium is given by  $(x_{1e}, 0, x_{3e}) = (0.7439, 0, 0.9361)$ . The weighting coefficients in penalty signal are chosen as  $q_1 = q_2 = q_3 = 0.1$ , and the parameter adaptation gains as  $\rho_1 = \rho_2 = 0.01$ . Using these design parameters and nominal parameters of the power system, the adaptive controller with  $L_2$  disturbance attenuation was designed by Theorem 2. A simulation results with the designed controller are as shown in Fig.1 and Fig.2.

Figure 1 shows the responses of the power angle  $\delta(t)$ , the relative speed of the generator  $\omega(t)$  and the transient EMF  $E'_q(t)$  with the adaptive controller when the disturbance attenuation level are chosen by  $\gamma = 10.0, 1.0, 0.1$ , respectively. Figure 2 shows a simulation result when the model parameter used in the controller design is perturbed 50% from the nominal values. The parameter estimation is also shown in Figure 1, and the disturbance attenuation level is chosen as  $\gamma = 0.1$ . The initial values of the parameter estimate are set as  $\hat{\theta}_1(0) = \hat{\theta}_2(0) = 0.0$  in all cases. It can be seen from

the simulation that the dynamical performance and the transient stability can be improved by reducing the disturbance attenuation level  $\gamma$ .

## 5 Conclusions

In this paper we discussed the  $L_2$  control problem for Hamiltonian systems. We have first shown that the  $L_2$  gain from disturbance to a penalty signal can be reduced to any level by injection of additional damping, if the penalty signal is defined properly. Then, we presented a methodology to introduce parameter adaptive mechanism for the  $L_2$  controller. Finally, the proposed controller was applied to a power system, and effectiveness of the proposed controller were shown by simulation implemented on a professional testing system.

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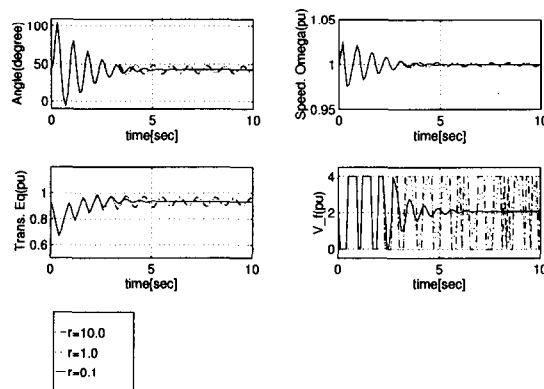


Figure 1: Responses of the adaptive controller

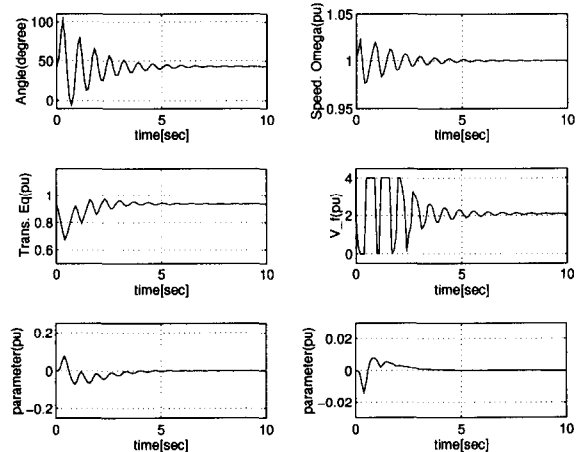


Figure 2: Responses of the system with 50% parameter perturbations.