Adaptive Control of a Class of Nonlinear Systems Preceded by an Unknown Backlash-Like Hysteresis

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Abstract

This paper deals with adaptive control of a class of nonlinear dynamic systems preceded by unknown backlash-like hysteresis nonlinearities, where the hysteresis is modeled by a differential equation. By exploiting solution properties of the differential equation and combining those properties with adaptive control techniques, a robust adaptive control algorithm is developed without constructing a hysteresis inverse. The new control law ensures global stability of the adaptive system and achieves both stabilization and tracking to within a desired precision. Simulations performed on a nonlinear system illustrate and clarify the approach.

1 Introduction

Hysteresis is a property of a wide range of physical systems and devices, such as electro-magnetic fields, mechanical actuators, and electronic relay circuits. Control of a system is typically challenged in the presence of hysteresis nonlinearities, since they are nodifferentiable nonlinearities and severely limit system performance such as giving rise to undesirable inaccuracy or oscillations, even leading to instability [14]. The development of control techniques to mitigate effects of unknown hystereses has been studied for decades and recently re-attracted significant attention [1-2][4][10][12-15]. Much of this interest is a consequence of the importance in applications. Interest in studying dynamic systems with hystereses is also motivated by their role as nonlinear systems with *hard-nonlinearities* for which traditional control methods are insufficient and new approaches must be developed [4].

To address such a challenge, it is important to find a model to describe their nonlinear behavior and to utilize this model for a controller design. Various models have been proposed to describe the hysteresis [7]. For example, Preisach model [8], Krasnosel'skii-Pokrovkii hysteron [6], Ishlinskii hysteresis operator [6], and Duhem hysteresis operator [7]. The most familiar and simple model perhaps is the one for a backlash hysteresis described by two parallel lines connected via horizontal line segments. However, we should mention that modeling a general type of hystereses itself is still a research topic and the reader may refer to [7] for a recent review.

However, except the backlash hysteresis model the above models are very complicated and it is still not clear how to fuse them into the controller design. Focusing on the backlash hysteresis, several adaptive control schemes have recently been proposed (see, for example, [14][15] [1] [12]) to deal with unknown backlash hysteresis. A common feature of those schemes is that they all need to construct an inverse hysteresis to mitigate the effects of the hysteresis. These results, especially [14][15], provide a theoretic framework which can serve as a base for the future research.

Inspired by the above research, in this paper a dynamic hysteresis model is defined to approximate the backlash hysteresis. Such an approximation is, therefore, called *backlash-like hysteresis*. The advantage for defining a dynamic hysteresis model is that the controller can be synthesized directly without constructing a hysteresis inverse to mitigate effects of the hysteresis. To illustrate such an idea, we propose a method of controller synthesis for a class of nonlinear systems where an unknown backlash hysteresis precedes. The proposed control law ensures global stability of the adaptive system and achieves both stabilization and tracking to within a desired precision. Simulations performed on a nonlinear system illustrate and clarify the approach. We should mention that the proposed method can be thought of as a preliminary step to fuse complicated general hysteresis model into the controller design.

2 Problem Statement

The controlled system consists of a nonlinear plant preceded by a backlash-like hysteresis actuator, that is, the hysteresis is present as an input of the nonlinear plant. It is a challenging task of major practical interests to develop a control scheme for unknown backlash-like hysteresis. In this paper, we will pursue this task.

A backlash-like hysteresis nonlinearity can be denoted as an operator

$$w(t) = P[v](t) \tag{1}$$

with $v(t)$ as input $v(t)$ and $w(t)$ as output $w(t)$. The notation $[\cdot](t)$ represents the fact that the operator in $[\cdot]$ is dependent on the trajectory, $v \in C^o[0,t]$, not an instantaneous value $v(t)$. The operator $P(v(t))$ will be discussed in details in the subsequent section. The nonlinear dynamic system preceded by the above hysteresis is described in the canonical form:

$$x^{(n)}(t) + \sum_{i=1}^{r} a_i Y_i(x(t), \dot{x}(t), ..., x^{(n-1)}(t)) = bw(t) \tag{2}$$

where Y_i are known continuous, linear or nonlinear function, parameters a_i and control gain b are unknown but constant. It is a common assumption that the sign of b is known. From now onward, without losing generality, we shall assume $b > 0$. It should be noted that more general classes of nonlinear systems can be transformed into this structure [5].

The control objective is to design a control law for v(t) in (1) to force the plant state vector, $\mathbf{x} = [x, \dot{x}, ..., x^{(n-1)}]^T$, to follow a specified desired trajectory, $\mathbf{x}_d = [x_d, \dot{x}_d, ..., x_d^{(n-1)}]^T$, i.e., $\mathbf{x} \to \mathbf{x}_d$ as $t \to \infty$.

3 Backlash-Like Hysteresis Model and Its Properties

Traditionally, a backlash hysteresis nonlinearity can be described by $w(t) = P(v(t))$

$$= \begin{cases} c(v(t) - B) & \text{if } \dot{v}(t) > 0 \text{ and } w(t) = c(v(t) - B) \\ c(v(t) + B) & \text{if } \dot{v}(t) < 0 \text{ and } w(t) = c(v(t) + B) \\ w(t_-) & \text{otherwise} \end{cases} \tag{3}$$

where $c > 0$ is the slope of the lines and $B > 0$ is the backlash distance. This model is discontinuous itself and may not be amenable to controller design for the nonlinear systems (2).

Instead of using above model, in this paper we define a continuous-time dynamic model to describe a class of backlash-like hysteresis, as is given by following equation

$$\frac{dw}{dt} = \alpha \left|\frac{dv}{dt}\right| (cv - w) + B_1 \frac{dv}{dt} \tag{4}$$

where α and B_1 are constants, satisfying $c > B_1$.

Remark: Other dynamic models for hystereses are provided, for example, by [3]. Generally, Modelling hysteresis noninearities is still a research topic and the reader may refer to [7] for a recent review.

Let us examine the properties of dynamic model (4), which is crucial for the controller design. The equation (4) can be solved explicitly for v piecewise monotone:

$$w(t) = cv(t) + d(v) \quad \text{with} \tag{5}$$

$$\begin{aligned} d(v) &= [w_o - cv_o]e^{-\alpha(v-v_o)sgn\dot{v}} \\ &\quad + e^{-\alpha v sgn\dot{v}} \int_{v_o}^{v} [B_1 - c]e^{\alpha\zeta(sgn\dot{v})} d\zeta \end{aligned}$$

for $\dot{v}$ constant, $w(v_o) = w_0$. Analyzing the solution (5), we see that the solution is composed of a line with the slope c, together with a term $d(v)$. For $d(v)$, it can easily be shown that if $w(v; v_o, w_o)$ is the solution of (5) with initial values (v_o, w_o), then: if $\dot{v} > 0$ ($\dot{v} < 0$) and $v \to +\infty$ ($-\infty$), one has

$$\lim_{v\to\infty} d(v) = \lim_{v\to\infty} [w(v; v_o, w_o) - f(v)] = -\frac{c - B_1}{\alpha}, \tag{6}$$

$$(\lim_{v\to-\infty} d(v) = \lim_{v\to-\infty} [w(v; v_o, w_o) - f(v)] = \frac{c - B_1}{\alpha}). \tag{7}$$

It should be noticed that the above convergence is exponential at the rate of α. Solution (5) and properties (6) and (7) show that $w(t)$ eventually satisfies the first and second conditions of (3). Furthermore, setting $\dot{v} = 0$ results in $\dot{w} = 0$ which satisfies the last condition of (3). This implies that the dynamic equation (4) can be utilized to model a class of backlash-like hystereses and is an approximation of backlash hysteresis (3).

Let us use an example for specified initial data to show the switching mechanism for the dynamic model (4) when $\dot{v}$ changes the direction. We note that when $\dot{v} > 0$ on $w(0) = 0$ and $v(0) = 0$, the solution (5) gives

$$w(t) = cv(t) - \frac{c - B_1}{\alpha}(1 - e^{-\alpha v(t)}) \text{for } v(t) \geq 0 \;\&\; \dot{v} > 0. \tag{8}$$

Let v_s be a positive value of v and consider now a specimen that v is increasing along the initial curve (8) until a time t_s at which v reaches the level v_s, and suppose that from the time t_s the signal v is decreased. In this case, w is given by

$$\begin{aligned} w(t) &= cv(t) + \frac{c - B_1}{\alpha}[1 - (2e^{-\alpha v_s} - e^{-2\alpha v_s})e^{\alpha v(t)}] \\ &\quad \text{for } \dot{v} < 0. \end{aligned} \tag{9}$$

where $v < v_s$. Equations (8) and (9) indeed show that w switches exponentially from the line $cv(t) - \frac{c-B_1}{\alpha}$ to $cv(t) + \frac{c-B_1}{\alpha}$ to generate backlash-like hysteresis curves.

To confirm the above analysis, the solutions of (4) can be obtained by numerical integration with v as the independent variable. Fig. 1 shows that model (4) indeed generates backlash-like hysteresis curves, which matches the above analysis. The details are described in the section of simulation studies. We should mention that the parameter α determines the rate for $w(t)$ to switch between $-\frac{c-B_1}{\alpha}$ and $\frac{c-B_1}{\alpha}$. The larger the parameter α is, the faster the transition in $w(t)$ is going to be. However, the backlask distance is determined by $\frac{c-B_1}{\alpha}$ and the parameter must satisfy $c > B_1$; hence, the parameter α cannot be chosen freely. A compromise should be made for choosing a suitable parameter set $\{\alpha, c, B_1\}$ to model the required shape of backlash-like hysteresis. If the values of the backlash slope and distance are not exactly known, as will be clear later, then adaptations will be used to estimate them.

4 Adaptive Controller Design

In this section, we shall propose a controller for plants of the form in (2) preceded by the hysteresis described in (4), that leads to global stability and yields tracking to within a desired precision.

Using the solution expression (5), the system (2) becomes

$$x^{(n)}(t) + \sum_{i=1}^{r} a_i Y_i(x, \dot{x}, ..., x^{(n-1)}) = bcv(t) + bd(v(t)) \tag{10}$$

which becomes linear to the input signal $v(t)$. It is very important to note that the equation (6) or (7) implies that there exists a uniform bound η such that

$$||d(v)|| \leq \rho. \tag{11}$$

Remark: Thanks to the solution structure (5), which makes it possible to seek a robust adaptive controller for the system (1-2), since the signal $w(t)$ is expressed as a linear function of input signal $v(t)$ plus a bounded term. In this case, the currently available robust adaptive control techniques can be utilized for the controller design. This gives a reason for using the dynamic hysteresis model (4).

For the development of control law, the following assumptions regarding the plant and hysteresis are made.

(A1) There exist known constants b_{min} and b_{max} such that the control gain b in (2) satisfies $b \in [b_{min}, b_{max}]$.

(A2) There exist known constants c_{min} and c_{max} such that the slope c in (3) satisfies $c \in [c_{min}, c_{max}]$.

(A3) Define $\theta \triangleq [\frac{a_1}{bc}, ..., \frac{a_r}{bc}]^T \in R^r$, then

$$\theta \in \Omega_\theta \triangleq \{\theta : \theta_{imin} \leq \theta_i \leq \theta_{imax}, \forall i \in \{1, r\}\}$$

where θ_{imin} and θ_{imax} are some known real numbers.

(A4) The bound η for the relation $||d(v)|| \leq \rho$ is known.

(A5) The desired trajectory, $\mathbf{x}_d = [x_d, \dot{x}_d, ..., x_d^{(n-1)}]^T$ is continuous and available. Furthermore $[\mathbf{x}_d^T, x_d^{(n)}]^T \in \Omega_d \subset R^{n+1}$ with Ω_d a compact set.

Remark: Assumption (A1) is common for the nonlinear system described by (2) [11]. Assumption (A2) assumes the slope range of a backlash hysteresis nonlinearity, which is reasonable. Based on Assumptions (A1)-(A2), it naturally leads to Assumption (A3), which characterizes the nature of the parameters for the plant. It is a reasonable assumption about the prior knowledge of the system and hysteresis. Assumption (A4) requires an upper bound of size of the hysteresis loop, which is quite reasonable and practical. Assumption (A5) poses a restriction on the types of reference signals which may be used.

In presenting the developed robust adaptive control law, the following definitions are required:

$$\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_d \quad \tilde{\theta} = \hat{\theta} - \theta \quad \tilde{\phi} = \hat{\phi} - \phi \tag{12}$$

where $\tilde{\mathbf{x}}$ presents the tracking error vector, $\hat{\theta}$ is an estimate of θ defined in Assumption (A2), and $\hat{\phi}$ is an estimate of ϕ defined as $\phi \triangleq (bc)^{-1}$.

A filtered tracking error is defined as

$$s(t) = (\frac{d}{dt} + \lambda)^{(n-1)} \tilde{x}(t) \quad with \quad \lambda > 0 \tag{13}$$

which can be rewritten as $s(t) = \Lambda^T \tilde{\mathbf{x}}(t)$ with $\Lambda^T = [\lambda^{(n-1)}, (n-1)\lambda^{(n-2)}, ..., 1]$.

Remark: It has been shown in [11] that the definition (13) has the following properties: (i) the equation $s(t) = 0$ defines a time-varying hyperplane in R^n on which the tracking error vector $\tilde{\mathbf{x}}(t)$ decays exponentially to zero, (ii) if $\tilde{\mathbf{x}}(0) = 0$ and $|s(t)| \leq \epsilon$ with constant ϵ, then $\tilde{\mathbf{x}}(t) \in \Omega_\epsilon \triangleq \{\tilde{\mathbf{x}}(t) \mid |\tilde{\mathbf{x}}_i| \leq 2^{i-1}\lambda^{i-n}\epsilon, i = 1, ..., n\}$ for $\forall t \geq 0$, and (iii) if $\tilde{\mathbf{x}}(0) \neq 0$ and $|s(t)| \leq \epsilon$, then $\tilde{\mathbf{x}}(t)$ will converge to Ω_ϵ within a time-constant $(n-1)/\lambda$.

Rather than driving the adaptive law with the filtered error, $s(t)$, we prefer to introduce a tuning error, s_ϵ, as follows:

$$s_\epsilon = s - \epsilon sat(\frac{s}{\epsilon}) \tag{14}$$

where ϵ is an arbitrary positive constant and $sat(\cdot)$ is the saturation function.

Remark: The tuning error, s_ϵ, disappears when the filtered error, s, is less than ϵ, which shall be the equivalent of creating an adaptation deadband.

Given the plant and hysteresis models subject to the assumption described above, the following control and adaptation laws are presented:

$$v(t) = -k_d s(t) + \hat{\phi} u_{fd}(t) + Y^T(\mathbf{x})\hat{\theta} - k^* sat(\frac{s}{\epsilon}) \quad (15)$$

$$u_{fd}(t) = x_d^{(n)}(t) - \Lambda_v^T \tilde{\mathbf{x}}(t) \quad (16)$$

$$\dot{\hat{\theta}} = Proj(\hat{\theta}, -\gamma Y(\mathbf{x}) s_\epsilon) \quad (17)$$

$$\dot{\hat{\phi}} = Proj(\hat{\phi}, -\eta u_{fd} s_\epsilon) \quad (18)$$

where $Y \triangleq [Y_1, ..., Y_r]^T \in R^r$; $\Lambda_v^T = [0, \lambda^{(n-1)}, (n-1)\lambda^{(n-2)}, ..., (n-1)\lambda]$; k^* is a control gain, satisfying $k^* \geq \rho / c_{min}$, therein, ρ is defined in (11); γ and η are positive constants, determining the rates of adaptations; $Proj(\cdot,\cdot)$ is a projection operator, which is constructed as follows: $\{Proj(\hat{\theta}, -\gamma Y s_\epsilon)\}_i$

$$= \begin{cases} 0 & \text{if } \hat{\theta}_i = \theta_{imax} \text{ and } \gamma(Y s_\epsilon)_i < 0 \\ -\gamma(Y s_\epsilon)_i & \text{if } [\theta_{imin} < \hat{\theta}_i < \theta_{imax}] \\ & \text{or } [\hat{\theta}_i = \theta_{imax} \text{ and } \gamma(Y s_\epsilon)_i \geq 0] \\ & \text{or } [\hat{\theta}_i = \theta_{imin} \text{ and } \gamma(Y s_\epsilon)_i \leq 0] \\ 0 & \text{if } \hat{\theta}_i = \theta_{imin} \text{ and } \gamma(Y s_\epsilon)_i > 0 \end{cases} \quad (19)$$

$Proj(\hat{\phi}, -\eta u_{fd} s_\epsilon)$

$$= \begin{cases} 0 & \text{if } \hat{\phi}_i = \phi_{max} \text{ and } \eta u_{fd} s_\epsilon < 0 \\ -\eta u_{fd} s_\epsilon & \text{if } [\phi_{min} < \hat{\phi} < \phi_{max}] \\ & \text{or } [\hat{\phi} = \phi_{max} \text{ and } \eta u_{fd} s_\epsilon \geq 0] \\ & \text{or } [\hat{\phi} = \phi_{min} \text{ and } \eta u_{fd} s_\epsilon \leq 0] \\ 0 & \text{if } \hat{\phi} = \phi_{min} \text{ and } \eta u_{fd} s_\epsilon > 0 \end{cases} \quad (20)$$

Remarks:

1) For the given set $\Omega_\theta = \{\theta : \theta_{imin} \leq \theta_i \leq \theta_{imax}, \forall i \in \{1, r\}\}$ in Assumption (A3), it can easily prove that projection operator for $\hat{\theta}$ satisfies i) $\hat{\theta}(t) \in \Omega_\theta$ if $\hat{\theta}(0) \in \Omega_\theta$; ii) $||Proj(p, y)|| \leq ||y||$; and iii) $-(p - p^*)^T \Lambda Proj(p, y) \geq -(p - p^*)^T \Lambda y$, where Λ is a positive definite symmetric matrix. Note that those three properties are also valid for projection operator defined for $\hat{\phi}$ and we omit it to save the space.

2) The choice of θ_{imin} and θ_{imam} is only related to the range of parameter changes for the projection operator and such a range in this paper is not restricted as long as the estimated parameters are bounded (required for the stability proof); hence one can always choose suitable θ_{imin} and θ_{imam}, although such a choice may be conservative.

The stability of the closed-loop system described by (2), (4) and (15)-(20) is established in the following theorem.

Theorem: For the plant in equation (2) with the hysteresis (4) at the input subject to assumptions (A1)-(A5), the robust adaptive controller specified by equations (15)-(20) ensures that if $\hat{\theta}(t_0) \in \Omega_\theta$ and $\hat{\phi}(t_0) \in \Omega_\phi$, all the closed-loop signals are bounded and the state vector $\mathbf{x}(t)$ converges to $\Omega_\epsilon = \{\mathbf{x}(t) | |\tilde{\mathbf{x}}_i| \leq 2^{i-1} \lambda^{i-n} \epsilon, i = 1, ..., n\}$ for $\forall t \geq t_0$.

Proof: Using the expression (10), the time derivative of the filtered error (13) can be written as:

$$\dot{s}(t) = -u_{fd}(t) - \sum_{i=1}^{r} a_i Y_i(\mathbf{x}(t)) + bcv(t) + bd(v) \quad (21)$$

Using the control law (15)-(20), the above equation can be rewritten as:

$$\begin{aligned} \dot{s}(t) \quad = \quad & -u_{fd}(t) - \sum_{i=1}^{r} a_i Y_i(\mathbf{x}(t)) + bc[-k_d s(t) + \hat{\phi} u_{fd}(t) \\ & + Y^T(\mathbf{x})\hat{\theta} - k^* sat(\frac{s}{\epsilon})] + bd(v) \end{aligned} \quad (22)$$

To establish global boundedness, we define a Lyapunov function candidate

$$V(t) = \frac{1}{2}[\frac{1}{bc} s_\epsilon^2 + \frac{1}{\gamma}(\hat{\theta} - \theta)^T(\hat{\theta} - \theta) + \frac{1}{\eta}(\hat{\phi} - \phi)^2] \quad (23)$$

Since the discontinuity at $|s| = \epsilon$ is of the first kind and since $s_\epsilon = 0$ when $|s| \leq \epsilon$, it follows that the derivative $\dot{V}$ exists for all s, which is

$$\dot{V}(t) = 0 \quad \text{when } |s| \leq \epsilon \quad (24)$$

When $|s| > \epsilon$, using (22) and the fact $s_\epsilon \dot{s}_\epsilon = s_\epsilon \dot{s}$, one has

$$\begin{aligned} \dot{V}(t) \quad = \quad & \frac{1}{bc} s_\epsilon \dot{s} + \frac{1}{\gamma}(\hat{\theta} - \theta)^T \dot{\hat{\theta}} + \frac{1}{\eta}(\hat{\phi} - \phi)\dot{\hat{\phi}} \\ = \quad & -k_d s_\epsilon s + s_\epsilon[\hat{\phi} u_{fd}(t) + Y^T(\mathbf{x})\hat{\theta} - k^* sat(\frac{s}{\epsilon})] \\ & + \frac{1}{bc} s_\epsilon[-u_{fd}(t) - \sum_{i=1}^{r} a_i Y_i(\mathbf{x}(t)) + bd(v)] \\ & + \frac{1}{\gamma}(\hat{\theta} - \theta)^T \dot{\hat{\theta}} + \frac{1}{\eta}(\hat{\phi} - \phi)\dot{\hat{\phi}} \\ = \quad & -k_d s_\epsilon s + s_\epsilon[\hat{\phi} u_{fd}(t) + Y^T(\mathbf{x})\hat{\theta} - k^* sat(\frac{s}{\epsilon})] \\ & + s_\epsilon[-\phi u_{fd}(t) - Y^T\theta + d(v)/c] \\ & + \frac{1}{\gamma}(\hat{\theta} - \theta)^T \dot{\hat{\theta}} + \frac{1}{\eta}(\hat{\phi} - \phi)\dot{\hat{\phi}} \end{aligned} \quad (25)$$

The above equation can be simplified, by the choice of s_ϵ, as

$$\begin{aligned} \dot{V}(t) \quad \leq \quad & -k_d s_\epsilon^2 + s_\epsilon[\hat{\phi} u_{fd}(t) + Y^T(\mathbf{x})\hat{\theta} - k^* sat(\frac{s}{\epsilon})] \\ & + s_\epsilon[-\phi u_{fd}(t) - Y^T\theta + d(v)/c] \\ & + \frac{1}{\gamma}(\hat{\theta} - \theta)^T \dot{\hat{\theta}} + \frac{1}{\eta}(\hat{\phi} - \phi)\dot{\hat{\phi}} \end{aligned} \quad (26)$$

By using adaptive laws (17), (18), and the properties $\frac{1}{\gamma}(\hat{\theta}-\theta)^T Proj(\hat{\theta}, -\gamma Y s_\epsilon) \leq -(\hat{\theta}-\theta)^T Y s_\epsilon$ and $\frac{1}{\eta}(\hat{\phi}-\phi)Proj(\hat{\phi}, -\eta u_{fd} s_\epsilon) \leq -(\hat{\phi}-\phi)u_{fd}s_\epsilon$, one obtains

$$\begin{aligned} \dot{V}(t) &\leq -k_d s_\epsilon^2 + s_\epsilon[\hat{\phi}u_{fd}(t) + Y^T(\mathbf{x})\hat{\theta} - k^* sat(\frac{s}{\epsilon})] \\ &\quad + s_\epsilon[-\phi u_{fd}(t) - Y^T\theta + d(v)/c] \\ &\quad -(\hat{\theta}-\theta)^T Y s_\epsilon - (\hat{\phi}-\phi)u_{fd}s_\epsilon \\ &= -k_d s_\epsilon^2 - k^* s_\epsilon sat(\frac{s}{\epsilon}) + \frac{d(v)}{c}s_\epsilon \end{aligned} \tag{27}$$

Since $|s_\epsilon| = s_\epsilon sat(\frac{s}{\epsilon})$ for $|s| > \epsilon$, the above becomes

$$\begin{aligned} \dot{V}(t) &\leq -k_d s_\epsilon^2 - k^*|s_\epsilon| + \frac{d(v)}{c}s_\epsilon \\ &\leq -k_d s_\epsilon^2 - k^*|s_\epsilon| + \frac{\rho}{c_{min}}|s_\epsilon| \\ &\leq -k_d s_\epsilon^2 \quad \forall |s| > \epsilon \end{aligned} \tag{28}$$

Equations (24) and (28) imply that V is a Lyapunov function which leads to global boundedness of s_ϵ, $(\hat{\theta}-\theta)$ and $(\hat{\phi}-\phi)$. From the definition of s_ϵ, $s(t)$ is bounded. It is easily shown that if $\tilde{\mathbf{x}}(0)$ is bounded, then $\tilde{\mathbf{x}}(t)$ is also bounded for all t, and since $\mathbf{x}_d(t)$ is bounded by design, $\mathbf{x}(t)$ must also be bounded. To complete the proof and establish asymptotic convergence of the tracking error, it is necessary to show that $s_\epsilon \to 0$ as $t \to \infty$. This is accomplished by applying Barbalat's Lemma [9] to the continuous, non-negative function:

$$\begin{aligned} V_1(t) &= V(t) - \int_0^t (\dot{V}(\tau) + k_d s_\epsilon^2(\tau))d\tau \text{ with} \\ \dot{V}_1(t) &= -k_d s_\epsilon^2(t) \end{aligned} \tag{29}$$

It can easily be shown that every term in (22) is bounded, hence $\dot{s}$, and $\dot{s}_\epsilon$ included, is bounded This implies that $\dot{V}_1(t)$ is a uniformly continuous function of time. Since V_1 is bounded below by 0, and $\dot{V}_1(t) \leq 0$ for all t, use of Barbalat's lemma proves that $\dot{V}_1(t) \to 0$. Therefore, from (29) it can be demonstrated that $s_\epsilon(t) \to 0$ as $t \to \infty$. The remark following equation (13) indicates that $\tilde{\mathbf{x}}(t)$ will converge to Ω_ϵ.

5 Simulation Studies

In this section, we illustrate the above methodology on a simple nonlinear systems described as

$$\dot{x} = a\frac{1-e^{-x(t)}}{1+e^{-x(t)}} + bw(t) \tag{30}$$

where $w(t)$ is an output of a hysteresis. The actual parameter values are $b=1$ and $a=1$. Without control, i.e., $w(t)=0$, the system (30) is unstable, because of $\dot{x} = \frac{1-e^{-x(t)}}{1+e^{-x(t)}} > 0$ for $x > 0$, and $\dot{x} = \frac{1-e^{-x(t)}}{1+e^{-x(t)}} < 0$ for $x < 0$. The objective is to control the system state x to follow the desired trajectory x_d, which will be specified later.

The backlash-like hysteresis is described by

$$\frac{dw}{dt} = \alpha|\frac{dv}{dt}|[cv - w] + \frac{dv}{dt}B_1 \tag{31}$$

with parameters $\alpha = 1$, $c = 3.1635$, and $B_1 = 0.345$. Using input signal $v(t) = ksin(2.3t)$ with k=2.5, 3.5, 4.5, 5.5, 6.5, the responses of this dynamic equation with the initial condition $w(0) = 0$ are shown in Fig. 1. We should mention that using different initial values $w(0)$ and frequencies, simulation studies show the similar shapes of the hysteresis as in Fig. 1. This confirms again the dynamic model (31) can be used to model the backlash-like hysteresis and one can choose a suitable parameter set $\{\alpha, c, B_1\}$ to model the required shape of backlash-like hysteresis.

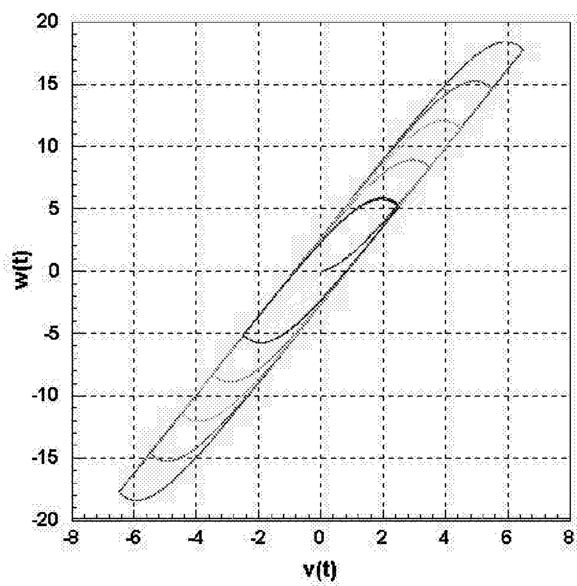

Figure 1: Hysteresis curves given by (4) or (31) with $\alpha = 1$, $c = 3.1635$, and $B_1 = 0.345$ for $v(t) = ksin(2.3t)$ with k=2.5, 3.5, 4.5, 5.5, 6.5.

In the simulations, the robust adaptive control law (15)-(20) was used, taking $k_d = 10$. Since the backlash distance is around 2.5, we can choose the upper bound ρ in (11) as $\rho = 4$ and we also choose $c_{min} = 3$, which results in $k^* = 4/3$. In the adaptation laws, we choose $\gamma = 0.5$ and $\eta = 0.5$ and the initial parameters $\theta = 1.2/3$ and $\phi = 0.8/3$. The initial state is chosen as $x(0) = 1.05$ and sample time is 0.005. In the simulation the initial value, $v(0)$, is required, which is selected as $v(0) = 0$.

Choosing the desired trajectory $x_d(t) = 12.5sin(2.3t)$, simulation results are shown in Figs. 2-3. Fig. 2 shows the tracking error for the desired trajectory and Fig. 3 shows the input control signal $v(t)$. We see from Fig. 3 that proposed robust controller clearly results in a good tracking performance.

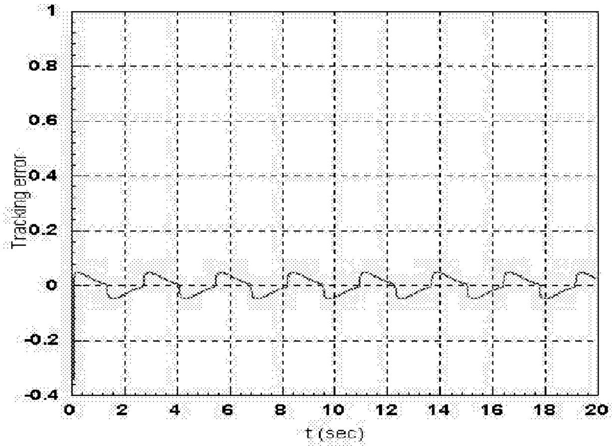

Figure 2: Tracking error of the state with backlash hysteresis

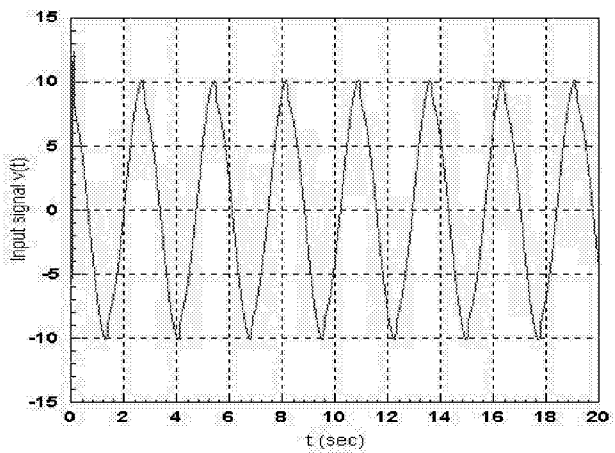

Figure 3: Control signal v(t) acting as the input of backlash hysteresis

6 Conclusion

In this paper, a robust adaptive control architecture is proposed for a class of continuous-time nonlinear dynamic systems preceded by a backlash-like hysteresis, where the backlash-like hysteresis is modeled by a dynamic equation. By showing the properties of the hysteresis model, a robust adaptive control scheme is developed without constructing the hysteresis inverse. The new adaptive control law ensures global stability of the adaptive system and achieves both stabilization and tracking to within a desired precision. Simulations performed on a simple nonlinear system illustrate and clarify the approach.

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