The Problem of Optimal Robust Sensor Scheduling *

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Abstract

This paper considers the sensor scheduling problem which consists of estimating the state of an uncertain process based on measurements obtained by switching a given set of noisy sensors. The noise and uncertainty models considered in this paper are assumed to be unknown deterministic functions which satisfy an energy type constraint known as an integral quadratic constraint. The problem of optimal robust sensor scheduling is formulated and solution to this problem is given in terms of the existence of suitable solutions to a Riccati differential equation of the game type and a dynamic programming equation. Furthermore, a real time implementable method for sensor scheduling is also presented.

1 Introduction

Classical estimation theory deals with the problem of forming an estimate of a process given measurements produced by sensors *observing* the process; e.g., see [1]. A standard solution is to compute the posterior density of the process state conditioned on all the available measurements. A more difficult class of estimation problem arises in applications such as robotics, command and control and networked systems where an estimator is given dynamic control over the measurements. These sensor scheduling problems occur, for example, when a *flexible* or *intelligent* sensor is able to operate in one of several different measurement modes and the estimator can dynamically switch the sensor mode. Alternatively several sensors may be remotely linked to the estimator via a low bandwidth communication channel and only one sensor can send measurement data during any measurement interval. Again the estimator can dynamically select which sensor uses the channel. Finally, sensor scheduling problems arise when measurements from a large number of sensors is available to the estimator but the computational power is such that only data from a small selection of the sensors can be processed at any given time hence forcing the estimator to dynamically select which sensor data is important for the task at hand. Sensor scheduling has been addressed for stochastic systems in [2, 7, 11] where it is assumed that the process is generated by a known linear systems with Gaussian input noise. It is shown that the optimal sensor schedule can be computed a priori and this schedule is independent of the observed data. In particular a sufficient statistic for a linear zero mean Gaussian processes with linear sensors and a minimum variance estimation objective is given by the estimation error covariance matrix which can be determined by the solution to a Riccati differential equation. This matrix depends on the sequence of sensors used but is independent of the actual observed measurements. Hence for any given sequence of sensors the estimation error covariance can be determined before the experiment has commenced. As a consequence the optimal sensor sequence can be determined a priori and is given by the sequence which minimizes, under some measure (such as the trace of the error covariance matrix at the final time) the precomputable solution to a Riccati differential equation.

In practice, however, it often occurs that the system model is not precisely known and standard stochastic models cannot be readily applied. Many of the recent advances in the area of robust control system design assume that the system to be controlled is modeled as an *uncertain system*; e.g., see [5]. In this paper, we follow the approach of [14] were the uncertainties are modeled by unknown functions that satisfy an energy constraint (also known as an integral quadratic constraint). This class of uncertain systems originated in the work of Yakubovich [15] and is a particularly rich uncertainty class allowing for nonlinear, time-varying, dynamic uncertainties. Furthermore, a number of new robust control system design methodologies have recently been developed for uncertain systems with integral quadratic constraints; e.g., see [10, 12, 13]. It should be pointed out, that Reference [14] builds on

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the deterministic interpretation of Kalman filtering presented in [3]. In this framework, the estimation problem is one of characterizing the set of possible states which could have given rise to the observed measurements. This approach to estimation was considered in the research monograph [9]. In Section 2, we introduce the concept of uniform robust observability for linear uncertain systems and give a necessary and sufficient condition for this requirement to be satisfied. This necessary and sufficient condition is given in terms of a pair of coupled Riccati differential equations. In Section 3, the measurement process is defined by a collection of given sensors which we call *basic sensors*. Without loss of generality it is assumed that only one of the basic sensors can be used at any time. Hence, our sensor schedule is a rule for switching from one basic sensor to another. The objective is to ensure robust observability and an optimal estimate of the system state. We show that the optimal switching rule can be computed by solving a set of Riccati differential equations of the game type and a dynamic programming procedure. It is shown that for the framework considered here the optimal sensor sequence depends on the past history of measurements. This is unlike sensor scheduling problems with Gaussian noise models and mean-square estimation criteria where the sensor schedule is independent of the observed measurements and can be computed a priori using only the statistical structure of the measurement and process noise. Finally, in Section 4, we apply ideas of model predictive or finite horizon control (e.g., see [4]) to derive a non-optimal but implementable in real time method for sensor scheduling.

The proofs of all the results will be given in the full version of the paper.

2 Robust Observability of Uncertain Linear Systems

Consider the time-varying uncertain system:

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)w(t); \\ z(t) &= K(t)x(t); \\ y(t) &= C(t)x(t) + v(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the *state*, $w(t) \in \mathbf{R}^p$ and $v(t) \in \mathbf{R}^l$ are the *uncertainty inputs*, $z(t) \in \mathbf{R}^q$ is the *uncertainty output* and $y(t) \in \mathbf{R}^l$ is the *measured output*, $A(\cdot), B(\cdot), K(\cdot)$ and $C(\cdot)$ are bounded piecewise continuous matrix functions.

System Uncertainty The uncertainty in the above system is required to satisfy the following Integral Quadratic Constraint; e.g., see [9,10,12–15]. Let $X_0 = X_0' > 0$ be a given matrix, $x_0 \in \mathbf{R}^n$ be a given vector, $d > 0$ be a given constant. For a given finite time interval $[0, s]$, we will consider the uncertainty inputs $w(\cdot)$ and $v(\cdot)$ and initial conditions $x(0)$ such that

$$\begin{aligned} &(x(0) - x_0)'X_0(x(0) - x_0) + \\ &\int_0^s (\|w(t)\|^2 + \|v(t)\|^2)dt \leq \\ &\qquad d + \int_0^s \|z(t)\|^2 dt. \end{aligned} \tag{2}$$

Here $\|\cdot\|$ denotes the standard Euclidean norm.

Note that the above uncertainty description allows for uncertainties in which the uncertainty inputs $w(\cdot)$ and $v(\cdot)$ depend dynamically on the uncertainty output $z(\cdot)$. In this case, the constant d may be interpreted as a measure of the size of the initial conditions on the nominal system and uncertainty dynamics.

It is clear that the uncertain system (1), (2) allows for uncertainty satisfying a standard norm bound constraint. In this case, the uncertain system would be described by the state equations

$$\begin{aligned} \dot{x}(t) &= [A(t) + B(t)\Delta_1(t)K(t)]x(t); \\ y(t) &= [C(t) + \Delta_2(t)K(t)]x(t); \\ &\|\Delta_1(t)' \ \Delta_2(t)'\| \leq 1 \end{aligned} \tag{3}$$

where $\Delta_1(t)$ and $\Delta_2(t)$ are the uncertainty matrices and $\|\cdot\|$ denotes the standard induced matrix norm. Also, the initial conditions would be required to satisfy the inequality $(x(0) - x_0)'X_0(x(0) - x_0) \leq d$. To verify that such uncertainty is admissible for the uncertain system (1), (2), let $w(t) = \Delta_1(t)z(t)$, $v(t) = \Delta_2(t)z(t)$ where $\|\Delta_1(t)' \ \Delta_2(t)'\| \leq 1$ for all $t \in [0, T]$. Then condition (2) is satisfied.

Notation Let $y(t) = y_0(t)$ be a fixed measured output of the uncertain system (1), and let the finite time interval $[0, s]$ be given. Then, $X_s[x_0, y_0(\cdot)|_0^s, d]$ denotes the set of all possible states $x(s)$ at time s for the uncertain system (1) with uncertainty inputs and initial conditions satisfying the constraint (2).

Definition 2.1 *The uncertain system (1), (2) is said to be* robustly observable *on* $[0, T]$*, if for any vector* $x_0 \in \mathbf{R}^n$*, any time* $s \in (0, T]$*, any constant* $d > 0$*, and any fixed measured output* $y(t) = y_0(t)$*, the set* $X_s[x_0, y_0(\cdot)|_0^s, d]$ *is bounded.*

Our necessary and sufficient condition for robust observability involves the following Riccati differential equation

$$\begin{aligned} &\dot{P}(t) = A(t)P(t) + P(t)A(t)' + \\ &P(t)[K(t)'K(t) - C(t)'C(t)]P(t) + B(t)B(t)'. \end{aligned} \tag{4}$$

Also, we consider a set of state equations of the form

$$\begin{aligned} \dot{\hat{x}}(t) = [A(t) + P(t)[K(t)'K(t) - C(t)'C(t)]]\,\hat{x}(t) \\ +P(t)C(t)'y_0(t). \end{aligned} \tag{5}$$

The following theorem gives a necessary and sufficient condition for robust observability. This result is close to the main result of [14].

Theorem 2.1 *Let $X_0 = X_0' > 0$ be a given matrix. Consider the uncertain system (1), (2). Then the following statements hold:*

(i) The system (1), (2) is robustly observable on $[0,T]$ if and only if the solution $P(\cdot)$ to the Riccati equation (4) with initial condition $P(0) = X_0^{-1}$ is defined and positive definite on the interval $[0,T]$.

(ii) Suppose the system (1), (2) is robustly observable on $[0,T]$. Also, let $s \in (0,T]$ be given and let $x_0 \in \mathbf{R}^n$ be a given vector, $d > 0$ be a given constant, and $y_0(t)$ be a given vector function defined on $[0,s]$. Then, the set $X_s[x_0, y_0(\cdot)|_0^s, d]$ is given by

$$X_s[x_0, y_0(\cdot)|_0^s, d] = \left\{ \begin{array}{l} x_s \in \mathbf{R}^n : (x_s - \hat{x}(s))'P(s)^{-1}(x_s - \hat{x}(s)) \\ \leq d + \rho_s[y_0(\cdot)] \end{array} \right\} \quad (6)$$

where

$$\rho_s[y_0(\cdot)] := \int_0^s \left[\|K(t)\hat{x}(t)\|^2 - \|(C(t)\hat{x}(t) - y_0(t))\|^2 \right] dt \quad (7)$$

and $\hat{x}(\cdot)$ is defined by the equation (5) with initial condition $\hat{x}(0) = x_0$.

Remark It is of interest to note that equations (4) and (5) define a state estimator which is closely related to the state estimator which occurs in the output feedback H^∞ control problem; e.g., see [6,8].

Let $\mathcal{A}(S)$ be some measure of the size of a bounded convex set S. Let $M = M' > 0$ be a square matrix, $a \in \mathbf{R}^n$ be a vector, and $d > 0$ be a number. Then

$$\mathcal{E}(M,a,d) := \{x \in \mathbf{R}^n : (x-a)'M(x-a) \leq d\}. \quad (8)$$

We suppose that the following assumptions hold

Assumption 2.1 For all a_1, a_2, $\mathcal{A}(\mathcal{E}(M,a_1,d)) = \mathcal{A}(\mathcal{E}(M,a_2,d))$.

Assumption 2.2 If $d_1 > d_2$ then $\mathcal{A}(\mathcal{E}(M,a,d_1)) > \mathcal{A}(\mathcal{E}(M,a,d_2))$.

Assumption 2.3 $\mathcal{A}(\mathcal{E}(M,a,d)) \to \infty$ as $d \to \infty$.

Notation We will use the notation $\mathcal{A}(H,d)$ for the number $\mathcal{A}(\mathcal{E}(H,a,d))$ where $\mathcal{E}(H,a,d)$ is defined by (8) (according to Assumption 2.1 this number does not depend on a).

Definition 2.2 *The uncertain system (1), (2) is said to be* uniformly robustly observable *on $[0,T]$, if it is robustly observable and for any vector $x_0 \in \mathbf{R}^n$, any constant $d > 0$, the following condition holds*

$$c[x_0,d] := \sup_{y_0(\cdot)} \mathcal{A}(X_T[x_0, y_0(\cdot)|_0^T, d]) < \infty \quad (9)$$

where the supremum is taken over all fixed measured outputs $y_0(t)$.

Our goal in this section is to give an effective necessary and sufficient condition of uniform robust observability and determine the upper bound $c[x_0,d]$ from Definition 2.2.

Our necessary and sufficient condition for uniform robust observability involves the following Riccati differential equation

$$\begin{aligned} -\dot{Y}(t) &= [A(t) + P(t)K(t)'K(t)]'Y(t) + \\ &Y(t)[A(t) + P(t)K(t)'K(t)] + K(t)'K(t) \\ &+Y(t)P(t)K(t)'K(t)K(t)'K(t)P(t)Y(t). \end{aligned} \quad (10)$$

Now we are in a position to present a necessary and sufficient condition of uniform robust observability.

Theorem 2.2 *Let $X_0 = X_0' > 0$ be a given matrix. Consider the uncertain system (1), (2). Then the system (1), (2) is robustly observable on $[0,T]$ if and only if the following two statements hold:*

(i) The solution $P(\cdot)$ to the Riccati equation (4) with initial condition $P(0) = X_0^{-1}$ is defined and positive definite on the interval $[0,T]$.

(ii) The solution $Y(\cdot)$ to the Riccati equation (10) with boundary condition $Y(T) = 0$ is defined and nonnegative definite on the interval $[0,T]$.

Furthermore, if conditions (i) and (ii) hold, then the upper bound (9) is defined by

$$c[x_0,d] = \mathcal{A}(P(T)^{-1}, d + x_0'Y(0)x_0). \quad (11)$$

3 Optimal Robust Sensor Scheduling

Consider the time varying uncertain system defined on the finite interval $[0,T]$

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)w(t) \\ z(t) &= K(t)x(t) \\ y^*(t) &= C^*(t)x(t) + \nu^*(t) \end{aligned} \quad (12)$$

where $x(t) \in R^n$ is the state, $w(t) \in R^p$ and $\nu^*(t) \in R^l$ are the uncertain inputs, $z \in R^q$ is the uncertainty output and $y^*(\cdot)$ is the continuously measured output. Here $A(\cdot), B(\cdot), K(\cdot)$ are given piecewise continuous matrix functions. The matrix function $C^*(\cdot)$ is defined by a particular sensor schedule used to measure the system state. Note that the dimension of $C^*(\cdot)$ can change with time depending on the sensors used. Let N be a given positive integer and let

$$0 = t_0 < t_1 < t_2 < \ldots < t_N = T$$

denote the permissible sensor switching times.

Sensor Scheduling Suppose we have the following collection of measured outputs which are called basic sensors

$$\begin{aligned} y^1(\cdot) &= C_1(\cdot)x(\cdot) + \nu^1(\cdot) \\ y^2(\cdot) &= C_2(\cdot)x(\cdot) + \nu^2(\cdot) \\ &\vdots \\ y^k(\cdot) &= C_k(\cdot)x(\cdot) + \nu^k(\cdot) \end{aligned} \quad (13)$$

where $C_1(\cdot), C_2(\cdot), \ldots, C_k(\cdot)$ are given matrix functions. Let $I_j(\cdot)$ be a function which maps the set of past measurements $\{y^*(\cdot)\mid_{t_0}^{t_j}\}$ to the set of symbols $\{1, 2, \ldots, k\}$. Then for any sequence of functions $\{I_j\}_{j=0}^{N-1}$, we consider the following dynamic sensor schedule

$$\begin{aligned} &\forall j \in \{0, 1, \ldots, N\}, \quad y^*(t) = y^{i_j}(t) \\ &\forall t \in [t_j, t_{j+1}] \text{ where } i_j = I_j(y^*(\cdot)\mid_{t_0}^{t_j}). \end{aligned} \quad (14)$$

Hence the sensor schedule is a rule for sequencing the basic sensors and constructs a sequence of symbols $\{i_j\}_{j=0}^{N}$ from the past measurements. Let $\mathcal{L}$ denote the class of all sensor schedules of the form (13), (14).

As in the previous section, the uncertainty in the system (12) is required to satisfy the following Integral Quadratic Constraint.

System Uncertainty Given $X_0 = X_0^{'}$, $x_0 \in R^n$, $d > 0$ and a finite time interval $[0, s]$, $s \leq T$, we consider the class of uncertain inputs $\{w(\cdot), v^*(\cdot)\} \in \mathbf{L}_2[0, s]$ and initial conditions $x(0)$ such that

$$\begin{aligned} &(x(0) - x_0)^{'} X_0 (x(0) - x_0) + \\ &\int_0^s (\|w(t)\|^2 + \|\nu^*(t)\|^2) dt \leq \\ &\qquad d + \int_0^s \|z(t)\|^2 dt. \end{aligned} \quad (15)$$

Notation Let $\mathcal{M} \in \mathcal{L}$ be a given sensor schedule and $y^*(\cdot)$ be the corresponding realised measured output. Then for the finite time interval $[0, s]$, $s \leq T$, $X_s[x_0, y^*(\cdot)\mid_0^s, d, \mathcal{M}]$ is the set of all possible states $x(s)$ at time s for the uncertain system (12) with sensor schedule $\mathcal{M}$, uncertain inputs $w(\cdot)$ and $\nu^*(\cdot)$, and initial conditions satisfying the integral quadratic constraint (15).

Definition 3.1 *Let $\mathcal{M} \in \mathcal{L}$ be a given sensor schedule. The system (12), (15) is said to be* robustly observable *with the sensor schedule $\mathcal{M}$ on the interval $[0, T]$, if for any vector $x_0 \in R^n$, any time $s \in [0, T]$, any constant $d > 0$ and any realised measured output $y^*(\cdot)$ the set $X_s[x_0, y^*(\cdot)\mid_0^s, d, \mathcal{M}]$ is bounded.*

Let $\mathcal{A}$ be some measure of the size of a convex set satisfying Assumptions 2.1–2.3.

Definition 3.2 *Let $\mathcal{M} \in \mathcal{L}$ be a given sensor schedule. The uncertain system (12), (15) is said to be* uniformly robustly observable *with the sensor schedule $\mathcal{M}$ on $[0, T]$, if it is robustly observable with this sensor schedule and for any vector $x_0 \in \mathbf{R}^n$, any constant $d > 0$, the following condition holds*

$$c[x_0, d, \mathcal{M}] := \sup_{y^*(\cdot)} \mathcal{A}(X_T[x_0, y_0(\cdot)|_0^T, d, \mathcal{M}]) < \infty \quad (16)$$

where the supremum is taken over all fixed realised measured outputs $y^*(t)$.

Notation Let $\mathcal{N}_0 \subset \mathcal{L}$ denote the set of all sensor schedules such that the system (12),(15) is uniformly robustly observable.

Definition 3.3 *The uncertain system (12), (15) is said to be* uniformly robustly observable via sensor switching *with the basic sensors (13) on $[0, T]$ if the set $\mathcal{N}_0$ is non-empty. In other words, if there exists a sensor schedule such that the system (12), (15) is uniformly robustly observable with this schedule.*

Definition 3.4 *Assume that the uncertain system (12), (15) is uniformly robustly observable via sensor switching with the basic sensors (13) on $[0, T]$. Let x_0 be a given vector and $d > 0$ be a given number. A sensor schedule $\mathcal{M}^0$ is said to be* optimal *for the parameters x_0 and d if*

$$c[x_0, d, \mathcal{M}^0] = \inf_{\mathcal{M} \in \mathcal{N}_0} c[x_0, d, \mathcal{M}]$$

where $c[x_0, d, \mathcal{M}]$ is defined by (16).

Let $m := [i_0, i_1, \ldots, i_{N-1}]$ where $1 \leq i_j \leq k$ be an index sequence defining a sensor schedule. Our solution to the optimal sensor scheduling problem with continuous time measurements involves the following Riccati differential equations associated with the sequence m

$$\begin{aligned} \dot{P}^m(t) = A(t)P^m(t) + P^m(t)A^{'}(t) + \\ P^m(t)[K^{'}(t)K(t) - C_m^{*'}(t)C_m^*(t)]P^m(t) + B(t)B^{'}(t) \\ P^m(0) = X_0^{-1} \end{aligned} \quad (17)$$

and the following set of state estimator equations

$$\dot{\hat{x}}(t) = [A(t) + P^m(t)[K^{'}(t)K(t) - C_m^{*'}(t)C_m(t)]]\hat{x}(t) + P^m(t)C_m^{*'}(t)y^*(t)$$
$$\hat{x}(0) = x_0. \quad (18)$$

Here

$$C_m^*(t) := C_{i_j}^*(t) \quad for \quad t \in [t_j, t_{j+1}), \; j = 0, 1, \ldots, N-1. \quad (19)$$

Let $\hat{x}_0$ be a vector, and $y^*(\cdot)$ be a vector function. Introduce the following value

$$F_j^i(\hat{x}_0, y^*(\cdot)) := \int_{t_j}^{t_{j+1}} \left[\|K(t)\hat{x}(t)\|^2 - \|(C(t)\hat{x}(t) - y^*(t))\|^2\right] dt \quad (20)$$

where $\hat{x}(t)$ is the solution of (18) with $\hat{x}(t_j) = \hat{x}_0$ and $C_m^*(\cdot)$ defined by (19).

For all $\hat{x}_0 \in \mathbf{R}^n$, $j = 1, 2, \ldots, N$, and $1 \leq i_j \leq k$, introduce the functions $\hat{V}_j[\hat{x}_0, i_0, i_1, \ldots, i_{j-1}] \in \mathbf{R}^{n \times n}$ and $v_j[\hat{x}_0, i_0, i_1, \ldots, i_{j-1}] \in \mathbf{R}$ as solutions of the following dynamic programming procedure. Firstly, we define

$$v_N[\hat{x}_0, i_0, i_1, \ldots, i_{N-1}] := 0 \quad \forall \hat{x}_0, i_0, i_1, \ldots, i_{N-1},$$
$$\hat{V}_N[\hat{x}_0, i_0, i_1, \ldots, i_{N-1}] := P^m(T)^{-1}$$
$$\forall \hat{x}_0, \quad m = [i_0, i_1, \ldots, i_{N-1}]. \quad (21)$$

It should be pointed out, that we do not assume that the solution of the Riccati equation (17) exists on $[0, T]$ for any m. If the solution does not exist for some m, we take $P^m(T)^{-1} := \infty$.

Furthermore, for all $\hat{x}_0 \in \mathbf{R}^n$ and $j = 0, 1, \ldots, N-1$, let $i_j(\hat{x}_0)$ be an index for which the minimum in the following minimization problem is achieved

$$\min_{i=1,2,\ldots,k} \sup_{y^*(\cdot) \in \mathbf{L}_2[t_j, t_{j+1}]} \mathcal{A}(\hat{V}_{j+1}[\hat{x}(t_{j+1}), i_0, i_1, \ldots, i_{j-1}, i], \hat{d}) \quad (22)$$

where

$$\hat{d} := F_j^i(\hat{x}_0, y^*(\cdot)) + v_{j+1}[\hat{x}(t_{j+1}), i_0, i_1, \ldots, i_{j-1}, i].$$

Note, that this index may be non-unique. Moreover, if

$$\sup_{y^*(\cdot) \in \mathbf{L}_2[t_j, t_{j+1}]} \mathcal{A}(\hat{V}_{j+1}[\hat{x}(t_{j+1}), i_0, i_1, \ldots, i_{j-1}, i_j(\hat{x}_0)], \hat{d})$$

is finite, then there exists a matrix $M_j(\hat{x}_0) > 0$ and a number $d_j(\hat{x}_0) > 0$ such that the above supremum is equal to $\mathcal{A}(M_j(\hat{x}_0), d_j(\hat{x}_0))$. Now let

$$\hat{V}_j[\hat{x}_0, i_0, i_1, \ldots, i_{j-1}] := M_j(\hat{x}_0);$$
$$v_j[\hat{x}(t_{j+1}), i_0, i_1, \ldots, i_{j-1}] := d_j(\hat{x}_0). \quad (23)$$

Now we are in a position to present the main result of this section.

Theorem 3.1 *Consider the uncertain system (12), (15) with the basic sensors (13). Then, the following statements are equivalent:*

(i) The uncertain system (12), (15) is uniformly robustly observable via sensor switching with the basic sensors (13) on $[0, T]$.

(ii) The dynamic programming procedure defined by (21), (22), (23) has a finite solution

$$\hat{V}_j[\hat{x}_0, i_0, i_1, \ldots, i_{j-1}] > 0, \quad v_j[\hat{x}_0, i_0, i_1, \ldots, i_{j-1}] \geq 0$$

for $j = 0, 1, \ldots, N-1$ *for all* $\hat{x}_0 \in \mathbf{R}^n$.

Furthermore, if condition (ii) holds and $i_j(\hat{x}_0)$ *is an index defined in the above dynamic programming procedure, then the sensor schedule defined by the sequence of indexes* $i_j(\hat{x}(t_j))$ *is optimal.*

4 Model Predictive Sensor Scheduling

The solution to discrete-time dynamical equations derived in Section 3 has been the subject of much research in the field of optimal control theory. Despite the fact that several methods of obtaining numerical solutions have been proposed for specific optimal control problems, it is not easy to solve dynamic programming equations in many realistic situations. In this section, we apply ideas of model predictive control (e.g., see [4]) to give a non-optimal but implementable in real time method for sensor scheduling.

Definition 4.1 *Assume that the uncertain system (12), (15) is uniformly robustly observable via sensor switching with the basic sensors (13) on* $[0, T]$. *Let* x_0 *be a given vector and* $d > 0$ *be a given number. A sensor schedule* $\mathcal{M}^0 \in \mathcal{N}_0$ *is said to be* one-step-ahead- optimal *for the parameters* x_0 *and* d *if for any* $j = 0, 1, \ldots, N-1$ *and any schedule* $\mathcal{M}$ *such that* $\mathcal{M}$ *coincides with* $\mathcal{M}^0$ *on* $[0, t_j]$ *the following condition holds*

$$\sup_{y^*(\cdot) \in \mathbf{L}_2[0, t_j]} \mathcal{A}(X_{t_{j+1}}[x_0, y^*(\cdot)|_0^{t_{j+1}}, d, \mathcal{M}]) \geq \sup_{y^*(\cdot) \in \mathbf{L}_2[0, t_j]} \mathcal{A}(X_{t_{j+1}}[x_0, y^*(\cdot)|_0^{t_{j+1}}, d, \mathcal{M}^0]).$$

The idea of Definition 4.1 is very straightforward: we wish to design a schedule such that at any sensor switching time t_j, the upper bound of the size of the set of all possible states $X_{t_{j+1}}[x_0, y^*(\cdot)|_0^{t_{j+1}}, d, \mathcal{M}^0]$ is minimal.

Let $j \leq N-1$ and $i_0, i_1, \ldots, i_{j-1}$ be a fixed sequence of indexes ($1 \leq i_r \leq k$), and let $i = 1, 2, \ldots, k$. The result of this section involves the following k pairs of Riccati differential equations associated with the sequence

$[i_0, i_1, \ldots, i_{j-1}, i]$

$$\dot{P}^i(t) = A(t)P^i(t) + P^i(t)A^{'}(t) + P^i(t)[K^{'}(t)K(t) - C_i^{*'}(t)C_i^*(t)]P^i(t) + B(t)B^{'}(t)$$
$$P^i(0) = X_0^{-1} \quad (24)$$

$$-\dot{Y}^i(t) = [A(t) + P^i(t)K(t)'K(t)]'Y^i(t) + Y^i(t)[A(t) + P^i(t)K(t)'K(t)] + K(t)'K(t) + Y^i(t)P^i(t)K(t)'K(t)K(t)'K(t)P^i(t)Y^i(t)$$
$$Y^i(t_{j+1}) = 0. \quad (25)$$

Here

$$C_i^*(t) := C_{i_r}(t) \quad for \quad t \in [t_r, t_{r+1}), \quad r = 0, 1, \ldots, j-1;$$
$$C_i^*(t) := C_i(t) \quad for \quad t \in [t_j, t_{j+1}). \quad (26)$$

Furthermore, introduce the following values

$$c_i[x_0, d] := \mathcal{A}(P^i(t_{j+1})^{-1}, d + x_0'Y^i(0)x_0).$$

If for some i the solution to at least one of the Riccati equations does not exist on the time interval $[0, t_j]$, we take $c_i[x_0, d] := \infty$.

Now we are in a position to present a method to design a one-step-ahead-optimal sensor switching strategy.

Theorem 4.1 *Consider the uncertain system (12), (15) with the basic sensors (13). A schedule* $\mathcal{M}^0$ *is one-step-ahead-optimal if and only if for any* $j = 0, 1, \ldots, N-1$*, and any sensor index sequence* $i_0, i_1, \ldots, i_j$ *associated with some realised measured output* $y^*(\cdot)|_0^{t_j}$*, the following two statements hold:*

(i) For $i = i_j$*, the solution* $P^i(\cdot)$ *to the Riccati equation (24) is defined and positive definite on the interval* $[0, t_{j+1}]$*, and the solution* $Y^i(\cdot)$ *to the Riccati equation (25) is defined and non-negative definite on the interval* $[0, t_{j+1}]$*.*

(ii) The following minimum

$$\min_{i=1,2,\ldots,k} c_i[x_0, d]$$

is achieved at $i = i_j$*.*

Remark Note that our method to design a one-step-ahead-optimal sensor switching rule requires at each step on-line solution to k pairs of Riccati differential equations and a simple look-up procedure to determine which of the basic sensors to use.

References

[1] B.D.O Anderson and J.B. Moore. *Optimal Filtering.* Prentice Hall, Englewood Cliffs, N.J., 1979.

[2] J.S. Baras and A. Bensoussan. Optimal sensor scheduling in nonlinear filtering of diffusion processes. *SIAM J. Control Optim.*, 27(4):786–813, 1989.

[3] D. P. Bertsekas and I. B. Rhodes. Recursive state estimation for a set-membership description of uncertainty. *IEEE Transactions on Automatic Control*, 16(2):117–128, 1971.

[4] E.F. Camacho and C. Bordons. *Model Predictive Control in the Process Industry.* Springer-Verlag, London, 1995.

[5] P. Dorato, R. Tempo, and G. Muscato. Bibliography on robust control. *Automatica*, 29(1):201–214, 1993.

[6] D. J. N. Limebeer, B. D. O. Anderson, P. P. Khargonekar, and M. Green. A game theoretic approach to H^∞ control for time-varying systems. *SIAM Journal on Control and Optimization*, 30:262–283, 1992.

[7] B.M. Miller and W.J. Runggaldier. Optimization of observations: a stochastic control approach. *SIAM J. Control Optim.*, 35(5):1030–1052, 1997.

[8] I. R. Petersen, B. D. O. Anderson, and E. A. Jonckheere. A first principles solution to the non-singular H^∞ control problem. *International Journal of Robust and Nonlinear Control*, 1(3):171–185, 1991.

[9] I.R. Petersen and A.V. Savkin. *Robust Kalman Filtering for Signals and Systems with Large Uncertainties.* Birkhauser, Boston, 1999.

[10] I.R. Petersen, V. Ugrinovskii, and A.V. Savkin. *Robust Control Design Using H^∞ Methods.* Springer-Verlag, to appear, London, 2000.

[11] C. Rago, P. Willet, and Y. Bar-Shalom. Censoring sensor: a low communication rate scheme for distributed detection. *IEEE Transactions on Aerospace and Electronic Systems*, 28(2):554–568, 1996.

[12] A. V. Savkin and I. R. Petersen. Minimax optimal control of uncertain systems with structured uncertainty. *International Journal of Robust and Nonlinear Control*, 5(2):119–137, 1995.

[13] A. V. Savkin and I. R. Petersen. Nonlinear versus linear control in the absolute stabilizability of uncertain linear systems with structured uncertainty. *IEEE Transactions on Automatic Control*, 40(1):122–127, 1995.

[14] A.V. Savkin and I.R. Petersen. Model validation for robust control of uncertain systems with an integral quadratic constraint. *Automatica*, 32(4):603–606, 1996.

[15] V. A. Yakubovich. Minimization of quadratic functionals under the quadratic constraints and the necessity of a frequency condition in the quadratic criterion for absolute stability of nonlinear control systems. *Soviet Mathematics Doklady*, 14:593–597, 1973.