Suboptimal Static Output Feedback Control

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Abstract

This paper is concerned with the problem of designing suboptimal H_2 (LQ, H_∞) static output feedback control for linear time-invariant systems. Necessary and sufficient conditions for the solvability of the problem under consideration are presented in terms of a set of matrix inequalities. An iterative LMI algorithm is given to obtain the solution, which is illustrated by examples.

Keywords: Linear systems; static output feedback; H_2 performance; LQ control; H_∞ control; LMI.

1 Introduction

The static output feedback problem is to find a static output feedback such that the closed-loop system has some desirable characteristics, or to determine that such a feedback does not exist. The problem has been extensively studied in recent years, and many important advances have been achieved, see [1], [2], [4], [5], [6], [9]-[12], [17]-[19], and [20]. In particular, Iwasaki et al. [12] have shown that LQ suboptimal static output feedback controller is related to solutions to a set of Lyapunov inequalities coupled by the constraint that one Lyapunov matrix is the inverse of another. Based on the LMI (Linear Matrix Inequality) approach [3] [8], Geromel et al. [9] [11] proposed a min/max algorithm to design suboptimal H_2 static output feedback controller. A modified min/max algorithm was developed to design suboptimal LQ static output feedback controller [12]. More recently, an LMI-based algorithm for the static and reduced-order output feedback synthesis problems is proposed in [7]. The static output feedback H_∞ control problem is addressed in [17], [5] and [1], and necessary and sufficient conditions for the solvability of the problem are presented in terms of solvability of matrix inequalities, and [5] gives an iterative LMI algorithm.

This paper will be concerned with the suboptimal H_2 (LQ, H_∞) static output feedback control problem for linear time-invariant systems. New necessary and sufficient conditions for the solvability of the considered problem are presented. A convergent iterative LMI algorithm is proposed to design suboptimal controller, and is illustrated by examples.

2 Problem statement and preliminaries

Consider a linear time invariant system Σ described by equations

$$\dot{x} = Ax + B_1 w + B_2 u \tag{1}$$
$$z = C_1 x + D_{12} u \tag{2}$$
$$y = C_2 x \tag{3}$$

where $x \in R^n$ is the state, $u \in R^m$ is the control input, $w \in R^r$ is the disturbance input, $z \in R^p$ is the output to be regulated, and $y \in R^q$ is the measured output. When a static output feedback control $u = Ky$ is applied to the system Σ, the resulting closed-loop system is given by

$$\dot{x} = (A + B_2 K C_2)x + B_1 w$$
$$z = (C_1 + D_{12} K C_2)x$$

Denote

$$T(s,K) = (C_1 + D_{12}KC_2)(sI - A - B_2KC_2)^{-1}B_1 \tag{4}$$

The H_2 and H_∞ norm of $T(s,K)$ are, respectively, defined by

$$\| T(s,K) \|_2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} Trace[T(j\omega,K)^H T(j\omega,K)]d\omega \tag{5}$$

$$\| T(s,K) \|_\infty = sup_{\omega \in R} \| T(j\omega,K) \| \tag{6}$$

where the norm $\| E \|$ of a complex matrix E is defined as the largest singular value of E. From [16], the LQ cost function $J(K)$ of the closed-loop system is defined by

$$J(K) = max_{\|w_0\|=1} J_w(w_0, K) \tag{7}$$

where

$$J_w(w_0,K) = \int_0^\infty \| z(t) \|^2 \, dt$$

with w_0 the intensity of the impulsive disturbance, and $w(t) = w_0\delta(t)$ ($\delta(t)$ is the Dirac delta function). Then the problems under consideration in this paper are as follows.

Suboptimal H_2 static output feedback control problem: Let $v > 0$ be a given constant. If possible, to find a static output feedback gain K such that $A + B_2KC_2$ is asymptotically stable and $\| T(s,K) \|_2 < v$.

Suboptimal LQ static output feedback control problem: Let $\beta > 0$ be a given constant. If possible, to find a static output feedback gain K such that $A + B_2KC_2$ is asymptotically stable and $J(K) < \beta$.

Suboptimal H_∞ static output feedback control problem: Let $\gamma > 0$ be a given constant. If possible, to find a static output feedback gain K such that $A + B_2KC_2$ is asymptotically stable and $\| T(s,K) \|_\infty < \gamma$.

The above three problems have been addressed by [11], [12], [5], [1] and [17], respectively, and some algorithms have been proposed to obtain the solutions to the suboptimal control problems. In this paper, we will present new necessary and sufficient conditions for the solvability of the suboptimal control problems, and give a convergent iterative LMI algorithm. The comparisons with the existing algorithms are illustrated by examples.

The following lemmas will be used in the sequel.

Lemma 2.1 ([21]): Let $v > 0$ be a given constant. Then $K \in R^{m\times q}$ solves the suboptimal H_2 static output feedback control problem if and only if there exists a positive-definite matrix P such that

$$P(A+B_2KC_2) + (A+B_2KC_2)^T P + (C_1 + D_{12}KC_2)^T(C_1 + D_{12}KC_2) < 0 \quad (8)$$

and

$$Trace(B_1^T PB_1) < v^2 \quad (9)$$

Lemma 2.2 ([16]): Let $\beta > 0$ be a given constant. Then $K \in R^{m\times q}$ solves the suboptimal LQ static output feedback control problem if and only if there exists a positive-definite matrix P such that the inequalities (8) and

$$B_1^T PB_1 < \beta I \quad (10)$$

Lemma 2.3 ([21]): Let $\gamma > 0$ be a given constant. Then $K \in R^{m\times q}$ solves the suboptimal H_∞ static output feedback control problem if and only if there exists a positive-definite matrix P such that

$$P(A+B_2KC_2) + (A+B_2KC_2)^T P + \frac{1}{\gamma^2} PB_1^T B_1 P + (C_1 + D_{12}KC_2)^T(C_1 + D_{12}KC_2) < 0 \quad (11)$$

In this paper, we assume that

$$R_2 = C_2C_2^T > 0 \quad (12)$$

$$R_1 = D_{12}^T D_{12} > 0 \quad (13)$$

and $C_2^\perp \in R^{(n-q)\times n}$ is a given matrix such that

$$C_2^\perp (C_2^\perp)^T = I_{n-q}, \qquad C_2(C_2^\perp)^T = 0 \quad (14)$$

3 Suboptimal static output feedback controllers

For the H_2 suboptimal static output feedback control problem, we have the following theorem.

Theorem 3.1: Let $v > 0$ be a given constant. Then there exists a static output feedback gain $K \in R^{m\times q}$ which solves the suboptimal H_2 static output feedback control problem, if and only if there exist positive-definite matrices P and P_0, a matrix $K_s \in R^{m\times n}$, and constants $\epsilon > 0$ and $\delta > 0$ such that (9) and the following inequalities hold.

$$\Delta_2(P,P_0) = \begin{bmatrix} M(P,P_0) + \delta I & \Delta_{212} \\ \Delta_{212}^T & -R_1^{-1} \end{bmatrix} < 0 \quad (15)$$

$$\begin{bmatrix} -\epsilon I & K_s(C_2^\perp)^T \\ C_2^\perp K_s^T & -I \end{bmatrix} < 0 \quad (16)$$

$$\epsilon \| R_1 \| + 2\epsilon^{\frac{1}{2}} \| PB_2 + C_1^T D_{12} \| \le \delta \quad (17)$$

where

$$\begin{aligned} M(P,P_0) = {} & P(A - B_2R_1^{-1}D_{12}^T C_1) \\ & +(A - B_2R_1^{-1}D_{12}^T C_1)^T P \\ & +C_1^T(I - D_{12}R_1^{-1}D_{12}^T)C_1 \\ & -PB_2R_1^{-1}B_2^T P_0 - P_0B_2R_1^{-1}B_2^T P \\ & +P_0B_2R_1^{-1}B_2^T P_0 \qquad (18) \\ \Delta_{212} = {} & K_s^T + (PB_2 + C_1D_{12}^T)R_1^{-1} \qquad (19) \end{aligned}$$

When the conditions (9) and (15)-(17) hold,

$$K = K_sC_2^T R_2^{-1} \quad (20)$$

is a solution to the suboptimal H_2 static output feedback control problem.

Proof: Necessity. Suppose that K is a solution to the H_2 suboptimal static output feedback control problem. By Lemma 2.1, it follows that there exists a positive-definite matrix P such that (8) and (9) hold. Let $K_s = KC_2$. Then, from (13), (18) and direct algebraic manipulations, we have

$$\begin{aligned} & P(A+B_2KC_2) + (A+B_2KC_2)^T P \\ & \quad +(C_1 + D_{12}KC_2)^T(C_1 + D_{12}KC_2) \\ & = M(P,P) + [K_s^T + (PB_2 + C_1D_{12}^T)R_1^{-1}]R_1 \\ & \quad \times [K_s^T + (PB_2 + C_1D_{12}^T)R_1^{-1}]^T \qquad (21) \end{aligned}$$

From (8), (21) and the Schur complement, it follows that (15) holds for some $\delta > 0$ and $P_0 = P$. For the $\delta > 0$, let

$$\epsilon^{\frac{1}{2}} = \frac{d - \| PB_2 + C_1^T D_{12} \|}{\| R_1 \|}$$

with $d = (\| PB_2 + C_1^T D_{12} \|^2 + \delta \| R_1 \|)^{\frac{1}{2}}$, then (17) holds. (16) is immediate from $K_s = KC_2$.
Sufficiency. Suppose that the inequalities (9), (15)-(17)

hold for $P > 0$, $P_0 > 0$, $\delta > 0$, $\epsilon > 0$, and $K_s \in R^{m\times n}$. Let K be defined by (20), and $K_0 = K_s(C_2^{\perp})^T$. Then from (14) and (16), we have

$$K_s = KC_2 + K_0 C_2^{\perp} \tag{22}$$

and

$$\| K_0 \| < \epsilon^{\frac{1}{2}} \tag{23}$$

By (15) and (18), it follows

$$\Delta_2(P,P) \leq \Delta_2(P,P_0) < 0 \tag{24}$$

Combining (24), (22), (23) and (17), we have

$$\begin{aligned}
0 > & P(A+B_2KC_2) + (A+B_2KC_2)^T P \\
& +(C_1+D_{12}KC_2)^T(C_1+D_{12}KC_2) + \delta I \\
& +(C_2^{\perp})^T K_0^T R_1 K_0 C_2^{\perp} + (PB_2 + C_1^T D_{12})K_0 C_2^{\perp} \\
& +(C_2^{\perp})^T K_0^T (PB_2 + C_1^T D_{12})^T \\
\geq & P(A+B_2KC_2) + (A+B_2KC_2)^T P \\
& +(C_1+D_{12}KC_2)^T(C_1+D_{12}KC_2) + \\
& +(\delta - 2 \| PB_2 + C_1^T D_{12} \| \| K_0 \| \\
& - \| R_1 \| \| K_0 \|^2) I \\
\geq & P(A+B_2KC_2) + (A+B_2KC_2)^T P \\
& +(C_1+D_{12}KC_2)^T(C_1+D_{12}KC_2)
\end{aligned}$$

Thus, the proof is completed by Lemma 2.1.

Remark 3.2: Theorem 3.1 presents necessary and sufficient conditions for the solvability of the suboptimal H_2 static output feedback control problem in terms of the matrix inequalities (9), (15) and (16), and a nonlinear inequality (17). The result essentially gives the conditions under which the existence of a state feedback suboptimal H_2 control implies the existence of a static output feedback suboptimal H_2 control. If P_0 is given, then (9), (15) and (16) are LMIs, which is the basis of the algorithm in the sequel. When $K_s = KC_2$ and $C_1 D_{12}^T = 0$, the inequality (15) is reduced to that used in [14] for the problem of simultaneous linear quadratic optimal control problem.

For the LQ and H_∞ suboptimal control problems, we have

Theorem 3.3: Let $\beta > 0$ be a given constant. Then there exists a static output feedback gain $K \in R^{m\times q}$ which solves the suboptimal LQ static output feedback control problem, if and only if there exist positive-definite matrices P and P_0, a matrix $K_s \in R^{m\times n}$, and constants $\epsilon > 0$ and $\delta > 0$ such that the inequalities (10) and (15)-(17) hold. When (10) and (15)-(17) hold, K given by (20) is a solution to the suboptimal LQ static output feedback control problem.

Proof: By Lemma 2.2 and the proof of Theorem 3.1, it is immediate.

Theorem 3.4: Let $\gamma > 0$ be a given constant. Then there exists a static output feedback gain $K \in R^{m\times q}$ which solves the suboptimal H_∞ static output feedback control problem, if and only if there exist positive-definite matrices P and P_0, a matrix $K_s \in R^{m\times n}$, and constants $\epsilon > 0$ and $\delta > 0$ such that the inequalities (16), (17) and

$$\begin{bmatrix} M(P,P_0)+\delta I & M_{12} & PB_1 \\ M_{12}^T & -R_1^{-1} & 0 \\ B_1^T P & 0 & -\gamma^2 I \end{bmatrix} < 0 \tag{25}$$

hold, where $M(P,P_0)$ is defined by (18), and $M_{12} = K_s^T + (PB_2 + C_1 D_{12}^T)R_1^{-1}$. When (16), (17) and (25) hold, K defined by (20) is a solution to the suboptimal H_∞ static output feedback control problem.

Proof: By Lemma 2.3, it is similar to that of Theorem 3.1, and the details are omitted.

Based on the Theorem 3.1, the following algorithm is proposed to search a feasible solution to the suboptimal H_2 static output feedback control problem.

Algorithm 3.5:
Step 1: Choose a small constant $\delta > 0$. Minimize $Trace(Q)$ subject to the following LMI constraints

$$\begin{bmatrix} \Delta_{11} & XC_1^T + Y^T D_{12}^T & \delta^{\frac{1}{2}} I \\ C_1 X + D_{12} Y & -I & 0 \\ \delta^{\frac{1}{2}} I & 0 & -I \end{bmatrix} < 0 \tag{26}$$

with $\Delta_{11} = AX + XA^T + B_2 Y + Y^T B_2^T$, and

$$\begin{bmatrix} -Q & B_1^T \\ B_1 & -X \end{bmatrix} < 0 \tag{27}$$

Choose v such that $v^2 > Trace(Q_{opt})$, and denote $P_0^1 = X_{opt}^{-1}$.
Step 2: Minimize ϵ^j subject to the LMI constraints $Trace(B_1^T P^j B_1) < v^2$, $\Delta_2(P^j, P_0^j) < 0$ and

$$\begin{bmatrix} -\epsilon^j I & K_s^j (C_2^{\perp})^T \\ C_2^{\perp}(K_s^j)^T & -I \end{bmatrix} < 0 \tag{28}$$

where $\Delta_2(P^j, P_0^j)$ is defined by (15), $P_0^j = P_{opt}^{j-1}$, $j = 2, 3\cdots$, and P_{opt}^{j-1} and ϵ_{opt}^{j-1} denote the solutions of the $(j-1)$th optimization. When ϵ_{opt}^j converges to a constant $(\epsilon_{v\delta})$, go to step 3.
Step 3: Check if $\epsilon = \epsilon_{v\delta}$ satisfies the inequality (17). If yes, reduce δ and v, and go to step 1. If not, denote P_{opt} corresponding to $\epsilon_{v\delta}$, increase v and set $P_0^1 = P_{opt}$, and then go to step 2.

Lemma 3.6: Let ϵ_{opt}^j $(j = 1, 2, \cdots)$ be defined in Step 2 of Algorithm 3.5. Then the sequence $\{\epsilon_{opt}^j\}_{j=1}^{\infty}$ is convergent.

Proof: For a given integer $j \geq 2$, P_{opt}^{j-1} and ϵ_{opt}^{j-1} are feasible solutions for the j-th optimization in Step 2, and P^j_{opt} and ϵ^j_{opt} are the optimal solutions. So $0 < \epsilon^j_{opt} \le \epsilon^{j-1}_{opt}$. Thus, the sequence $\{\epsilon^j_{opt}\}_{j=1}^{\infty}$ must be convergent.

Remark 3.7: Lemma 3.6 shows that Algorithm 3.5 is convergent. However, the convergence of $\{\epsilon^j_{opt}\}_{j=1}^{\infty}$ does not in general imply the convergence of $\{P^j_{opt}\}_{j=1}^{\infty}$. If $\{P^j_{opt}\}_{j=1}^{\infty}$ does not converge, then the algorithm only can achieve suboptimality. Otherwise, the algorithm may achieve local or global optimality. In fact, in the algorithm, denote

$$v_{opt} = min\{v : (v, \delta, \epsilon_{v\delta}) \text{ is feasible for steps 1-3}\}$$

where $\epsilon_{v\delta} = \lim_{j\to\infty} \epsilon^j_{opt}$. Let δ and $\epsilon_{v_{opt}\delta}$ corresponding to v_{opt} be denoted by δ_{opt} and ϵ_{opt}, respectively. If $\delta_{opt} = \epsilon_{opt} = 0$, then v_{opt} is the optimal H_2 performance achieved by the algorithm. The optimal controller gain K can be calculated by (20). So we may directly set $\delta = 0$ in step 1 to check if $\epsilon_{v\delta} \to 0$ in step 2 for achieving optimal H_2 performance. Moreover, denote $K_s = [K_{sij}]_{m\times n}$ and $C_2^T R_2^{-1} = [C_{sjk}]_{n\times q}$, then the following element-wise constraints on the controller gain $K = K_s C_2^T R_2^{-1} = \left[\sum_{j=1}^n K_{sij} C_{sjk}\right]_{m\times q}$ can be imposed

$$\underline{g}_{ik} < \sum_{j=1}^{n} K_{sij} C_{sjk} < \bar{g}_{ik}, \quad i = 1, \cdots, m; \quad k = 1, \cdots, q \tag{29}$$

where $\underline{g}_{ik}$ and $\bar{g}_{ik}$ are given upper and lower bounds, respectively. For the suboptimal LQ and H_∞ static output feedback control problems, the similar algorithms can be given to search suboptimal solutions. The details are omitted.

Remark 3.8: Comparison with the algorithm proposed in [5] and [14], Algorithm 3.5 provides a method by which a suboptimal static output feedback gain can be iteratively computed by using a suboptimal state feedback gain, while the algorithm proposed in [5] and [14] is performed by searching a feasible solution for the quadratic inequalities and then optimizing the performance. Both of the two algoriths are based on the quadratic inequalities. It should be pointed out that it remain open that if the algorithm would always give a feasible solution when the suboptimal H_2 static output feedback control problem is solvable.

Remark 3.9: By the results [12] and [7], the suboptimal H_2 (LQ) static output feedback control problem can be converted to the following rank minimization problem: $\min_{P,Q}\ rank \begin{bmatrix} P & I \\ I & Q \end{bmatrix}$ subject to

$$[B_2^T \;\; D_{12}^T]^\perp \begin{bmatrix} AQ + QA^T & QC_1^T \\ C_1 Q & -I \end{bmatrix} [B_2^T \;\; D_{12}^T]^{\perp T} < 0$$

$$C_2^\perp (PA + A^T P + C_1^T C_1) C_2^{\perp T} < 0$$

$$Trace(B_1^T P B_1) < v^2 \quad \text{or} \quad B_1^T P B_1 < \beta I$$

where $[B_2^T \;\; D_{12}^T]^\perp$ and $C_2^\perp$ are defined as (14). The problem can be approached by using the algorithm proposed in [7]. The performance v (or β) can be optimized by using bisection search. The resulting algorithm is denoted by IE. In next section, we will compare Algorithm 3.5 with the existing algorithms including the IE algorithm [12, 7].

4 Examples

In this section, we will present some examples to illustrate the design method proposed in the paper.

Example 4.1: Consider the classical example provided in [15]. The model is given by

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad C_2^T = B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

From [15], the optimal solution to the static output feedback H_2 problem is $K = -0.8165$ and $\| T(s, K) \|_2^2 = 2.4495$. By Algorithm 3.5 with $\delta = 0$, after 15 iterations in steps, we obtain $\epsilon_{opt} = 2.4375 \times 10^{-12}$, $K_{sopt} = [0 \quad -0.8165]$ and

$$P_{opt} = \begin{bmatrix} 1.4289 & 0.5000 \\ 0.5000 & 1.0206 \end{bmatrix}$$

So $K_{opt} = -0.8165$ and $\| T(s, K_{opt}) \|_2^2 = Trace(B_1^T P_{opt} B_1) = 2.4495$, which means that the solution given by Algorithm 3.5 also is optimal. Table 1 presents the results by using the algorithms in [11] and [14], and the algorithm IE, it shows that for the example, Algorithm 3.5, the algorithm IE and that in [5] can obtain the optimal solution, but the solution given in [11] is suboptimal.

Table 1:

Algorithm 3.5	GSS [11]	CLS [5]	IE [12, 7]
2.4495	2.4525	2.4495	2.4495

Example 4.2: Consider the example presented in [13] about the control of the nominal linearized model of a VTOL helicopter. The model is given by

$$A = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.0100 & 0.0024 & -4.0208 \\ 0.1002 & 0.3681 & -0.7070 & 1.4200 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ -5.5200 & 4.4900 \\ 0 & 0 \end{bmatrix}$$

$$C_2 = [0\ 1\ 0\ 0],\quad B_1 = I_4,\quad C_1 = \begin{bmatrix} I_4 \\ 0 \end{bmatrix},\quad D_{12} = \begin{bmatrix} 0 \\ I_2 \end{bmatrix}$$

By the direct calculation in [11],

$$min_{K\in[-5,5]\times[-5,5]} \parallel T(s,K) \parallel_2^2 = 13.3428$$

Combining Algorithm 3.5 and (29) with the constraint

$$K \in [-5,\ 5] \times [-5,\ 5] \qquad (30)$$

and $\delta = 0$ and $v^2 = 13.3428$, after 500 iterative computations in step 2, we obtain $\epsilon_{opt} = 7.1997 \times 10^{-10}$, $K_{sopt} = \begin{bmatrix} 0 & -1.3976 & 0 & 0 \\ 0 & 4.9998 & 0 & 0 \end{bmatrix}$ and

$$P_{opt} = \begin{bmatrix} 6.2565 & -1.3719 & -2.0423 & -1.9550 \\ -1.3719 & 1.4602 & 1.6558 & 0.7844 \\ -2.0433 & 1.6558 & 2.4149 & 1.2418 \\ -1.9550 & 0.7844 & 1.2418 & 3.2113 \end{bmatrix}$$

Thus, by (20), $K_{opt} = \begin{bmatrix} -1.3976 \\ 4.9998 \end{bmatrix}$ and $\parallel T(s,K_{opt}) \parallel_2^2 = Trace(B_1^T P_{opt} B_1) = 13.3428$, and the solution is optimal. The comparison with those by [11] and [5] is shown in Table 2.

Table 2:

	Algorithm 3.5	GSS [11]	CLS [5]
$\parallel T(s,K_{opt}) \parallel_2^2$	13.3428	15.3191	13.3530

Without the constraint (30), by using Algorithm 3.5 with $\delta = 0$ and $v^2 = 13.3055$, after 600 iterative computations in step 2, we obtain $\epsilon_{opt} = 2.655795 \times 10^{-9}$, $K_{sopt} = \begin{bmatrix} 0 & -1.6965 & 0 & 0 \\ 0 & 6.5166 & 0 & 0 \end{bmatrix}$ and

$$P_{opt} = \begin{bmatrix} 6.1948 & -1.3718 & -2.0625 & -1.9688 \\ -1.3718 & 1.5354 & 1.6453 & 0.7865 \\ -2.0625 & 1.6453 & 2.4167 & 1.2435 \\ -1.9688 & 0.7865 & 1.2435 & 3.1586 \end{bmatrix}$$

Thus, by (20), $K_{opt} = \begin{bmatrix} -1.6965 \\ 6.5166 \end{bmatrix}$ and $\parallel T(s,K_{opt}) \parallel_2^2 = Trace(B_1^T P_{opt} B_1) = 13.3055$. The comparison with those by [11], [5] and the algorithm *IE* is shown in Table 3.

Table 3:

Algorithm 3.5	GSS [11]	CLS [5]	*IE* [12, 7]
13.3055	15.3191	13.3173	13.3121

Example 4.3: The model is described by Example 4.2 by resetting $C_1 = \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$ and $D_{12} = \begin{bmatrix} 0 \\ I_2 \end{bmatrix}$. The suboptimal *LQ* static output feedback control problem was considered by [12] and the suboptimal *LQ* cost $J(K) = 7.8758$. In Algorithm 3.5, if Q is replaced by βI, then the algorithm is suitable for solving suboptimal *LQ* control problem. By using the algorithm with $\delta = 0$ and β being equal to $3, 4, 5, 5.5, 6.5, 7, 7.5, 7.6, 7.7, 7.75, 7.76, 7.763, 7.764, 7.765$ by order (for $\beta = 3$, iterate 100 times. 50 times for others), we obtain $\epsilon_{opt} = 1.250910 \times 10^{-9}$, $K_{sopt} = \begin{bmatrix} 0 & -1.0034 & 0 & 0 \\ 0 & 5.5293 & 0 & 0 \end{bmatrix}$ and

$$P_{opt} = \begin{bmatrix} 5.7648 & -1.3885 & -2.2129 & -1.90091 \\ -1.3885 & 0.8430 & 0.7713 & 0.5323 \\ -2.2129 & 0.7713 & 1.2094 & 0.8948 \\ -1.9091 & 0.5328 & 0.8948 & 1.0430 \end{bmatrix}$$

Thus, $K_{opt} = \begin{bmatrix} -1.0034 \\ 5.5293 \end{bmatrix}$ and $J(K_{opt}) = max[eig(B_1^T P_{opt} B_1)] = 7.765$. The comparison with those by [12] and the algorithm *IE* is shown in Table 4.

Table 4:

	Algorithm 3.5	ISG [12]	*IE* [12, 7]
$J(K_{opt})$	7.765	7.8758	7.7701

Example 4.4: The system under consideration is described by (1)-(3) with

$$A = \begin{bmatrix} -3 & 1 & 1.5 & 1 \\ 1 & -2 & 1 & 1 \\ 0.5 & 1 & -5 & 1 \\ 1 & 2 & 3 & -4 \end{bmatrix},\quad B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},\quad B_1 = I_4$$

$$C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},\quad C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D_{12} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T$$

By using Algorithm 3.5 with $\delta = 0$ and $v^2 = 2.79815$, after 10 iterative computations in step 2, we obtain $\epsilon_{opt} = 1.301421 \times 10^{-11}$, $K_{sopt} = \begin{bmatrix} -0.6182 & -0.9321 & 0 & 0 \\ -0.4930 & -0.7506 & 0 & 0 \end{bmatrix}$ and

$$P_{opt} = \begin{bmatrix} 0.5481 & 0.7078 & 0.4340 & 0.3456 \\ 0.7078 & 1.3654 & 0.7503 & 0.6050 \\ 0.4340 & 0.7053 & 0.5391 & 0.4315 \\ 0.3456 & 0.6050 & 0.4315 & 0.3455 \end{bmatrix}$$

Thus, by (20), $K_{opt} = \begin{bmatrix} -0.6182 & -0.9321 \\ -0.4930 & -0.7506 \end{bmatrix}$ and $\parallel T(s,K_{opt}) \parallel_2^2 = Trace(B_1^T P_{opt} B_1) = 2.79815$. The

Table 5:

	Algorithm 3.5	CLS [5]	*IE* [12, 7]
$\| T(s, K_{opt}) \|_2^2$	2.79815	2.79815	5.3

comparison with those by [5] and the algorithm *IE* is shown in Table 5.

The above examples show that Algorithm 3.5 can give good solutions to the suboptimal static output feedback control problems, which provides an alternative algorithm to the existing ones.

5 Conclusions

In this paper, we have investigated the problem of designing suboptimal H_2 (LQ, H_∞) static output feedback control for linear time-invariant systems. New necessary and sufficient conditions for the solvability of the considered problem are presented in terms of a set of matrix inequalities. A convergent iterative LMI algorithm is given to obtain the solution. The numerical examples have shown the advantages of the proposed algorithm.

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References

[1] Barabanov, N.E., "On the static H_∞ control problem", *Systems and Control Letters*, Vol. 35, pp. 13-18, 1998.

[2] Benton Jr, R.E. and D. Smith, "Static output feedback stabilization with prescribed degree of stability", *IEEE Transactions on Automatic Control*, Vol. AC-43, no.10, pp. 1493-1496, 1998.

[3] Boyd, S., L. El Ghaoui, E. Feron and V. Balakrishan, *Linear Matrix Inequalities in Systems and Control Theory*, Philadelphia, PA: SIAM, 1994.

[4] Cao, Y.Y., Y.X. Sun, and W.J. Mao, "A new necessary and sufficient condition for static output feedback stabilizability and comments on 'stabilization via static output feedback'," *IEEE Transactions on Automatic Control*, Vol. AC-43, no. 8, pp. 1110-1111, 1998.

[5] Cao, Y. Y., J. Lam, and Y. X. Sun, "Stable output feedback stabilization: an ILMI approach," *Automatica*, Vol. 34, No. 12, pp. 1641-1645, 1998.

[6] Corrado, J.R. and W.M. Haddad, "Static output feedback controllers for systems with parametric uncertainty and controller gain variation", *Proc. 1999 Amer. Contr. Conf.*, San Diego, CA, pp. 915-919, 1999.

[7] El Ghaoui, L., F. Oustry, and M. AitRami, "A cone complementarity linearization algorithm for static output feedback and related problems," *IEEE Transactions on Automatic Control*, Vol. AC-42, No. 8, pp. 1171-1176, 1997.

[8] Gahinet, P., A. Nemirovski, A.J. Laub, and M. Chilali, LMI Control Toolbox, Natick, MA: The MathWorks, 1995.

[9] Geromel, C., C.C. de Souza, and R.E. Skelton, "LMI numerical solution for output feedback stabilization", *Proc. 1994 Amer. Contr. Conf.*, Baltimore, MD, pp. 40-44, 1994.

[10] Geromel, C., P.L.D. Peres, and C.C. de Souza, "Convex analysis of output feedback control problems: robust stability and performance", *IEEE Transactions on Automatic Control*, Vol. AC-41, no. 7, pp. 997-1003, 1996.

[11] Geromel, C., C.C. de Souza, and R.E. Skelton, "Static output feedback controllers: stability and convexity", *IEEE Transactions on Automatic Control*, Vol. AC-43, no. 1, pp. 120-125, 1998.

[12] Iwasaki, T., R.E. Skelton, and J.C. Geromel, "Linear quadratic suboptimal control with static output feedback", *Systems and Control Letters*, Vol. 23, pp. 421-430, 1994.

[13] Keel, L., S.P. Bhattacharyya, and J.W. Howze, "Robust control with structured perturbations", *IEEE Transactions on Automatic Control*, Vol. AC-33, no. 1, pp. 68-78, 1988.

[14] Lam, J. and Y.Y. Cao, "Simultaneous linear-quadratic optimal control design via static output feedback", *Int. J. Robust Nonlinear Control*, Vol. 9, pp.551-558, 1999.

[15] Levine, W.S. and M. Athans, "On the determination of the optimal constant feedback gains for linear multivariable systems", *IEEE Transactions on Automatic Control*, Vol. AC-15, no. 1, pp. 44-48, 1970.

[16] Rotea, M.A., "The generalized H_2 control problem", *Automatica*, Vol. 29, pp. 373-386, 1993.

[17] Skelton, R.E., J. Stoustrup, and T. Iwasaki, "The H_∞ control problem using static output feedback", *Int. J. Robust Nonlinear Control*, Vol. 4, pp. 449-455, 1994.

[18] Syrmos, V.L., C. Abdallah, and P. Dorato, "Static output feedback: a survey", *Proc. 33rd IEEE Conference on Decision and Control*, Lake Buena Vista, FL, pp. 837-842, 1994.

[19] Trofino-Neto, A. and V. Kucera, "Stabilization via static output feedback", *IEEE Transactions on Automatic Control*, Vol. AC-38, no. 5, pp. 764-768, 1993.

[20] Yaesh, I. and U. Shaked, "Minimum entropy static output feedback control with an H_∞-norm performance bound", *IEEE Transactions on Automatic Control*, Vol. AC-42, no. 6, pp. 853-858, 1997.

[21] Zhou, K., J.C. Doyle and K. Glover, *Robust and Optimal Control*, Prentice Hall, New Jersey, 1996.