Internal Model-Based Robust Iterative Learning Control for Uncertain LTI Systems

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Abstract

This paper investigates the combination of an iterative learning control (ILC) with an internal model control (IMC) for uncertain linear time-invariant (LTI) systems. The convergence of the iterative process is investigated and reformulated as a general robust control problem. For a certain choice of the IMC and ILC filters, we prove that the condition of convergence to zero of the iterative process is nothing but the robust performance condition of the IMC structure. Using the general robust control formulation, we propose a design procedure for the ILC-IMC filters using the μ-synthesis approach.

1 Introduction

Iterative learning control, which is well known for its performance in the control of repetitive systems with partially unknown and time varying parameters, has recently generated a considerable amount of interest in the automatic control community. This approach was originally proposed for robot manipulator control as an intelligent mechanism for progressively improving tracking performances [1]. It consists in finding an adequate iterative rule, which allows the controller to learn from the errors of the previous trials and perform progressively better with every new trial in order to increase the tracking accuracy as the number of trials increases. The ILC control scheme was initially developed as a feedforward action applied directly to the open-loop system (see, for example, [2], [3] and [7]. Although theoretically correct, this control schemes may generate harmful effects when an inappropriate initial control law is chosen. To overcome this drawback, several feedback-based iterative learning controllers have been proposed in the literature (see for example, [5] and [8]. In [8] a classical feedback control structure is used together with an iterative learning control, which is updated according to the control variable given by the feedback controller. Based on this feedback structure the iterative learning problem is reformulated as a general robust control problem.

On other hand, the well-known internal model control scheme is based on a particular structure, in which the controller explicitly includes a model of the plant to be controlled. This structure has been and continues to be very popular in process control applications since it allows to the designer to tune a single parameter to achieve an interesting tradeoff between closed loop performance and robustness to model inaccuracies [10]. Furthermore, this structure is particularly appropriate for open-loop stable plant, the type of plant found in many engineering applications. As already mentioned, the controller in the IMC structure requires a model of the plant, and however the performances to be achieved will be seriously affected by the parameter inaccuracies. Hence, several IMC extensions have been proposed in the literature such as the adaptive IMC scheme proposed in [4].

In this paper we propose an iterative learning controller for repetitive SISO-LTI systems in an IMC structure. The IMC part guarantees robust performance when it performs alone. The ILC part is introduced to iteratively improve the transient behavior of the control system, particularly in the presence of regular repetitive output disturbances (i.e. unchanged disturbances at each trial). First of all, sufficient conditions for the convergence of the iterative process are given. Thereafter, we prove that, for a certain choice of the IMC and the ILC filters, the condition of convergence to zero of the tracking error is nothing but the robust performance condition for the IMC structure. Finally, we propose a design procedure for the ILC-IMC filters using the μ-synthesis approach.

2 Internal model-based iterative scheme

Consider the SISO iterative scheme shown in Figure 1, where $G(s)$ is the plant, $G_m(s)$ the model, $C(s)$ the controller, $u_k(s)$ the feedback control variable, $v_k(s)$ is an additional iterative control variable, $y_k(s)$ the output to be controlled, $y_d(s)$ is the desired trajectory, and $d_k(s)$ is the output disturbance.
We assume that the system operates repeatedly. To distinguish between the signals at each operation or iteration, we introduce an additional subscript k. In

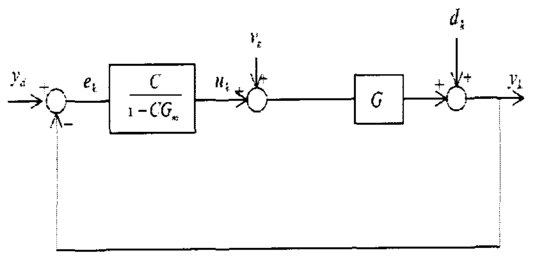

Figure 1: ILC-IMC scheme

the sequel the Laplace variable s will be omitted where there is no matter to confusion. From Figure 1, the tracking error at the k-th iteration is given by

$$\begin{aligned} e_k = y_d - y_k &= \frac{1-CG_m}{1+C(G-G_m)}(y_d - y_k) \\ &- \frac{G(1-CG_m)}{1+C(G-G_m)}v_k. \end{aligned} \tag{1}$$

Hence, the tracking error at the $(k+1)$-th iteration is given by

$$\begin{aligned} e_{k+1} &= A - \frac{1-CG_m}{1+C(G-G_m)}d_{k+1} \\ &- \frac{G(1-CG_m)}{1+C(G-G_m)}v_{k+1}, \end{aligned} \tag{2}$$

where $A = \dfrac{1-CG_m}{1+C(G-G_m)}y_d$.
Consider the following updating law for the signal v_k in the frequency domain:

$$v_{k+1}(s) = F_1(s)v_k(s) + \frac{F_2(s)}{1-C(s)G_m(s)}e_k(s), \tag{3}$$

where $F_1(s)$ and $F_2(s)$ are two transfert functions to be designed later.
From (1), (2) and (3), we obtain the following result:

$$e_{k+1} = \left(F_1 - \frac{GF_2}{1+C(G-G_m)}\right)e_k - \frac{1-CG_m}{1+C(G-G_m)}(d_{k+1} - F_1 d_k) + (1-F_1)A. \tag{4}$$

Assuming that $d_{k+1}(s) = d_k(s) = d(s)$, we obtain

$$\begin{aligned} e_{k+1} &= \left(F_1 - \frac{GF_2}{1+C(G-G_m)}\right)e_k \\ &+ \frac{(1-F_1)(1-CG_m)}{1+C(G-G_m)}(y_d - d), \end{aligned} \tag{5}$$

which implies that

$$\begin{aligned} e_k &= \left(F_1 - \frac{GF_2}{1+C(G-G_m)}\right)e_{k-1} \\ &+ \frac{(1-F_1)(1-CG_m)}{1+C(G-G_m)}(y_d - d). \end{aligned} \tag{6}$$

From (5) and (6), one has

$$e_{k+1} - e_k = \left(F_1 - \frac{GF_2}{1+C(G-G_m)}\right)(e_k - e_{k-1}). \tag{7}$$

Hence, if

$$\left\| F_1 - \frac{GF_2}{1+C(G-G_m)} \right\|_\infty < 1, \tag{8}$$

the convergence to a fixed point is guaranteed (i.e., $\lim_{k\to\infty} e_k = e^*$). The limit e^* satisfies the following equality

$$\begin{aligned} e^* &= \left(F_1 - \frac{GF_2}{1+C(G-G_m)}\right)e^* \\ &+ \frac{(1-F_1)(1-CG_m)}{1+C(G-G_m)}(y_d - d), \end{aligned} \tag{9}$$

which gives

$$e^* = \frac{(1-F_1)(1-CG_m)}{(1-F_1)(1+C(G-G_m)) + GF_2}(y_d - d). \tag{10}$$

If $F_1 = 1$, from (10), one can conclude that the tracking error converges to zero. In this case the condition of convergence becomes

$$\left\| 1 - \frac{GF_2}{1+C(G-G_m)} \right\|_\infty < 1. \tag{11}$$

At first glance, it seems obvious from (11) that, for any plant G, a perfect tracking will be achieved at the first iteration by choosing $C = F_2 = G_m^{-1}$. Unfortunately, this is not possible since the controller in Figure 1 is parameterized as $\frac{C}{1 - CG_m}$, which becomes infinity if $C = G_m^{-1}$.

3 General robust control problem framework

Let M be a complex matrix partitioned as follows

$$M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$$

and let Δ be a complex matrix and $\mathcal{F}_l(M, \Delta)$ the lower fractional transformation (LFT) with respect to Δ defined as follows

$$\mathcal{F}_l(M, \Delta) = M_{11} + M_{12}\Delta(I - M_{22}\Delta)^{-1}M_{21}.$$

The LFT is well posed if $(I - M_{22}\Delta)^{-1}$ exists. The structured singular value $\mu_{\underline{\Delta}}(M)$ is defined as

$$\mu_{\underline{\Delta}}(M) = \frac{1}{min\{\overline{\sigma}(\Delta) : \Delta \in \underline{\Delta},\ det(I - M\Delta) = 0\}},$$

unless no $\Delta \in \underline{\Delta}$ makes $I - M\Delta$ singular, in which case $\mu_{\underline{\Delta}}(M) = 0$.
$\overline{\sigma}(\Delta)$ denotes the largest singular value of Δ, and $\underline{\Delta}$ denotes a prescribed set of block diagonal matrices [11]. For further development we need the following Lemma.

Lemma 1 *(Zhou et al [11]). Let $\beta > 0$. For all $\Delta(s) \in \Re\mathcal{H}_\infty$ with $\|\Delta(s)\|_\infty < \frac{1}{\beta}$, the loop shown in Figure 2 is well-posed, internally stable, and $\|\mathcal{F}_l(M, \Delta)\|_\infty \le \beta$, if and only if*

$$\sup_{\omega \in \Re} \mu_{\underline{\Delta}_P}(M(j\omega)) \le \beta,$$

where $\underline{\Delta}_P$ is an augmented block structure defined by

$$\underline{\Delta}_P = \left\{ \begin{pmatrix} \Delta & 0 \\ 0 & \Delta_f \end{pmatrix} : \ \Delta \in \underline{\Delta},\ \Delta_f \in \Re\mathcal{H}_\infty \right\}.$$

Now, assume that the plant G is described in the following multiplicative uncertain form

$$G = (1 + \Delta W)G_m, \qquad (12)$$

where G_m is the nominal plant model, W is a known stable transfer function satisfying $\|W\|_\infty < 1$, and Δ is an unknown stable transfer function satisfying $\|\Delta\|_\infty < 1$.

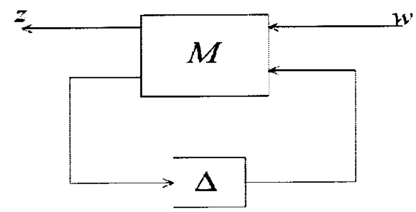

Figure 2: Block diagram

3.1 Perfect tracking case ($F_1 = 1$)

Consider the ILC-IMC control scheme in Figure 1 with the iterative rule (3).
Taking $F_1 = 1$ and $C = F_2$ in (7), and using (12), we obtain

$$e_{k+1} = \left(\frac{1 - G_mC}{1 + G_mC\Delta W} \right) e_k. \qquad (13)$$

To guarantee the convergence of the tracking error to zero, one has to satisfy the following condition

$$\left\| \frac{1 - G_mC}{1 + G_mC\Delta W} \right\|_\infty < 1. \qquad (14)$$

That is

$$\|1 + G_mC\Delta W\|_\infty > \|1 - G_mC\|_\infty. \qquad (15)$$

In other hand, it is obvious that

$$1 = \|1 - G_mC\Delta W + G_mC\Delta W\|_\infty, \qquad (16)$$

which leads to

$$1 \le \|1 + G_mC\Delta W\|_\infty + \|G_mCW\|_\infty, \qquad (17)$$

and

$$\|1 + G_mC\Delta W\|_\infty \ge 1 - \|G_mCW\|_\infty, \qquad (18)$$

From (15) and (18), it is clear that if we have

$$1 - \|G_mCW\|_\infty \ge \|1 - G_mC\|_\infty, \qquad (19)$$

then (14) is fulfilled.
From (19), one has

$$\|1 - G_mC\|_\infty + \|G_mCW\|_\infty < 1. \qquad (20)$$

Finally, we arrive at the conclusion that the tracking error converges to zero when k tends to infinity, if condition (20) is fulfilled. Furthermore, condition (20) is also the condition of robust performance for the internal model control since $(1 - G_m(s)C(s))$ is a sensitivity function relating the reference signal and the output disturbance to the error, and $G_m(s)C(s)$ is a complementary sensitivity function relating the reference signal to the output.
According to the previous development, one can state the following theorem:

Theorem 1 *Consider the iterative control scheme in Figure 1 with the iterative rule (3), and take $F_1 = 1$ and $F_2 = C$. Assume that the output disturbances are stationary with respect to the iteration index (i.e., $d_k(s) = d_{k+1}(s)$). Assume also that the uncertain plant G is as described in (12). If there exists C such that inequality (20) is fulfilled, then*

i) The tracking error converges to zero when $k \to \infty$.

ii) Robust performance is guaranteed for the IMC at the first iteration (i.e., for $v_0 = 0$).

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The result in Theorem 1 is very interesting since condition (20) guarantees robust performance for the IMC when the ILC is not effective (i.e., when $v_0 = 0$). The same condition ensures also a perfect tracking (i.e., $e_k \to 0$ when $k \to \infty$). The problem of designing the IMC filter C and the ILC filter F_2 separately is transformed into a unified problem, which consists in finding a controller C solving the model-matching problem [6] by transforming (20) into the following inequality using the spectral factorization $\|T_1 - T_2CT_3\|_\infty < 1$.
Now, let us reformulate the ILC-IMC problem as a general robust control problem in order to design a unique filter $C(s)$ by means of the μ-synthesis approach.
Let us define the matrix

$$M_2 = \begin{pmatrix} 1 - CG_m & -(1 - CG_m) \\ WCG_m & -WCG_m \end{pmatrix} \quad (21)$$

such that

$$\begin{aligned}\mathcal{F}_l(M_2, \Delta) &= 1 - G_mC - \frac{(1 - CG_m)\Delta WCG_m}{1 + WCG_m\Delta} \\ &= \frac{1 - G_mC}{1 + CG_m\Delta W}.\end{aligned} \quad (22)$$

Note that the condition of convergence (14) is $\|\mathcal{F}_l(M_2, \Delta))\|_\infty < 1$.
If C is such that $\|WCG_m\|_\infty < 1$, one has $(1 + WCG_m\Delta)^{-1} \in \Re\mathcal{H}_\infty$ and the LFT is well posed.
Consider the matrix

$$M_C = \begin{pmatrix} 1 & -1 & -1 \\ 0 & 0 & W \\ G_m & -G_m & 0 \end{pmatrix} \quad (23)$$

such that

$$M_C = \mathcal{F}_l(M_C, C),$$

which has a diagram represented in Figure 3.

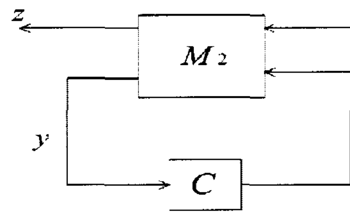

Figure 3: Block diagram

The original problem $\left\|\frac{1 - G_mC}{1 + G_mC\Delta W}\right\|_\infty < 1$ is transformed into the problem of finding a filter $C(s)$ such that $\sup_{\omega\in\Re} \mu_{\Delta_P}(M_2(j\omega)) < 1$. The solution $C(s)$ can be obtained by applying the μ-synthesis procedure called D-K iteration [11]. According to Theorem 1, the existence of such a controller C satisfying $\sup_{\omega\in\Re} \mu_{\Delta_P}(M_2(j\omega)) < 1$, guarantees the convergence of the tracking error to zero as well as the robust performance of the IMC at the first iteration(i.e., when the ILC is not effective).
The previous result can be summarized in the following theorem.

Theorem 2 *Consider the iterative control scheme in Figure 1 with the iterative rule (3), and take F_1=1 and $F_2 = C$. Assume that the output disturbances are stationary with respect to the iteration index (i.e., $d_{k+1}(s) = d_k(s)$). Assume also that the uncertain plant G is described as in (12). Then, if there exists C satisfying $\sup_{\omega\in\Re} \mu_{\Delta_P}(M_2(j\omega)) < 1$, the tracking error converges to zero when k tends to infinity and robust performance is guaranteed for the IMC structure.*

Proof: Straightforward from Theorem 1 and Lemma 1. ¤

3.2 Non-perfect tracking ($F_1 \neq 1$)

The condition $\left\|1 - \frac{GF_2}{1 + C(G - G_m)}\right\|_\infty < 1$ is required to ensure the convergence to zero of the tracking error. This condition is much restrictive for some problems because it can correspond to plant invertibility [9]. Besides, under the condition $\left\|F_1 - \frac{GF_2}{1 + C(G - G_m)}\right\|_\infty < 1$, we have more degrees of freedom, but unfortunately, the convergence to zero of the tracking error is no longer guaranteed. In this case, the idea is to choose F_1 as close as possible from one, within the tracking bandwidth, in order to minimize the tracking error. Thereafter, find the controller C according to the IMC design [10]. Finally, design the filter F_2 using the μ-synthesis procedure. Let

$$M_1 = \begin{pmatrix} F_1 - F_2 G_m & -F_2(1 - CG_m) \\ WG_m & -WCG_m \end{pmatrix} \quad (24)$$

Hence,

$$\begin{aligned} \mathcal{F}_l(M_1, \Delta) &= F_1 - F_2 G_m - \frac{F_2(1 - CG_m)\Delta W G_m}{1 + WCG_m \Delta} \\ &= F_1 - \frac{G_m F_2 (1 + \Delta W)}{1 + WCG_m \Delta}. \end{aligned} \quad (25)$$

Note that the condition of convergence (8) is $\|\mathcal{F}_l(M_1, \Delta)\|_\infty < 1$.
If C is such that $\|WCG_m\|_\infty < 1$, one has $(1 + WCG_m\Delta)^{-1} \in \Re\mathcal{H}_\infty$ and the LFT is well posed.
Now, one can establish the following result

Theorem 3 *Assume that the output disturbances are stationary with respect to the iteration index (i.e., $d_{k+1}(s) = d_k(s)$). Assume also that the uncertain plant G is described as in (12). Then, the control scheme in Figure 1 with the iterative rule (3) is uniformly convergent if there exists C satisfying $\|WCG_m\|_\infty < 1$ and F_1, F_2 such that*

$$\sup_{\omega \in \Re} \mu_{\Delta_P}(M_1(j\omega)) < 1. \quad (26)$$

Furthermore, when $k \to \infty$, e_k tends to

$$\frac{(1 - F_1)(1 - CG_m)}{(1 - F_1)(1 + C(G - G_m)) + GF_2}(y_d - d). \quad (27)$$

Proof: The proof of the convergence is straightforward from Lemma 1, (24) and (25). The proof of (27) is in section 2. ¤

Now, to find the ILC-IMC filters satisfying (26) let us introduce the following matrix

$$M_F = \begin{pmatrix} F_1 & 0 & -1 \\ WG_m & -WCG_m & 0 \\ G_m & 1 - CG_m & 0 \end{pmatrix} \quad (28)$$

such that

$$M_1 = \mathcal{F}_l(M_F, F_2).$$

Note that the controller $C(s)$ is designed according to the IMC procedure [10] such that $\|WCG_m\|_\infty < 1$. The filter F_1 is chosen as close as possible to 1 within the tracking bandwidth in order to minimize the tracking error, which is zero for $F_1 = 1$. The convergence of the iterative process is transformed into the problem of finding a feedback controller $F_2(s)$ such that $\sup_{\omega \in \Re} \mu_{\Delta_P}(M_1(j\omega)) < 1$. The solution can be obtained by applying the μ-synthesis procedure.

4 Design Procedure

Finally, we summarize our development in the following procedure:
Consider the iterative scheme in Figure 1 with the iterative rule (3), where the uncertain plant is described as in (12). Assume that output disturbances are stationary with respect to the iteration index (i.e., $d_{k+1}(s) = d_k(s)$).

1) Set $F_1 = 1$ and $F_2 = C$.
2) Given G_m and W, solve the μ-synthesis problem [11], using $M_2 = \mathcal{F}_l(M_C, C)$ to get C. If such a solution exists the tracking error goes to zero when t tends to infinity and robust performance is guaranteed for the IMC scheme. Go to (6).
3) If the condition $\sup_{\omega \in \Re} \mu_{\Delta_P}(M_2(j\omega)) < 1$ is not fulfilled then find C according to the IMC filter design [10] such that $\|WCG_m\|_\infty < 1$, and choose an appropriate filter $F_1 \neq 1$, which is close to 1 within the tracking bandwidth.
4) Given C, F_1, G_m and W, solve the μ-synthesis problem, using $M_1 = \mathcal{F}_l(M_F, F_2)$, to get F_2.
5) If the condition $\sup_{\omega \in \Re} \mu_{\Delta_P}(M_1(j\omega)) < 1$ is not fulfilled, reduce the cutoff frequency of F_1 and return to (4).
6) End.

5 Illustrative example

Consider the nominal plant with the uncertainty weighting function used in [8]:

$$G_m(s) = \frac{25s + 80}{s^2 + 24s + 370}, \qquad W(s) = \frac{0.5s + 5}{s + 100}.$$

Using the D-K iteration , we determine a 9th order filter $C(s)$ which can be reduced to the following 3rd order

$$C(s) = \frac{8000s^3 + 9.92e5s^2 + 2.216e7s + 2.96e8}{s^3 + 250100s^2 + 2.58e7s + 8e7},$$

providing $\sup_{\omega \in \Re} \mu_{\underline{\Delta}_P}(M_2(j\omega)) \leq 0.63$.
The desired trajectory is taken as $y_d(t) = 0.01t^2(10-t)$ and the plant as in (12). Applying the iterative rule (3), with $F_1 = 1$ and $F_2 = C$, to the control scheme in Figure 1 we achieve a perfect tracking after five iterations as shown in Figure 4.

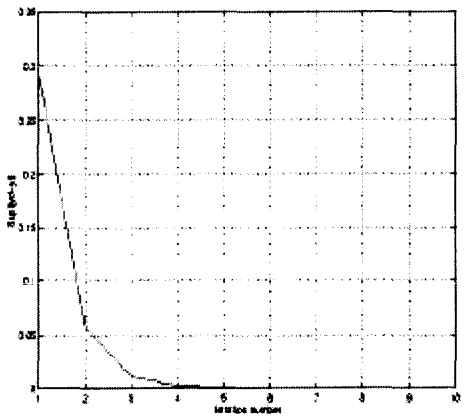

Figure 4: Tracking error versus the number of iteration

6 Conclusion

We have presented an iterative learning controller applied to an IMC structure with output disturbances. The IMC part guarantees robust performance when it performs alone at the first iteration. The ILC part is introduced to iteratively improve the transient behavior of the control system. The convergence of the iterative process has been investigated. The combined ILC-IMC problem is formulated as a general robust control problem. It is shown that the condition of convergence to zero of the tracking error is nothing but the robust performance condition for the IMC structure. Using the general robust control formulation, we have suggested a design procedure for the ILC-IMC filters using the μ-synthesis approach. The principal advantage of the ILC-IMC combination is stated in Theorem 2, i.e., the design of a unique filter C(s) by means of the μ-synthesis approach guarantees simultaneously the convergence to zero of the tracking error and robust performance of the IMC scheme when $v_k = 0$. Even though we have considered only SISO systems in this paper, the extension of the obtained results to MIMO systems is straight forward.

Acknowledgements

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References

[1] S. Arimoto, S. Kawamura, and F. Miyazaki, "Bettering operation of robots by learning," *Journal Robot. Syst.*, 1, pp. 123-140, 1984.

[2] Y. Chen, C. Wen, Z. Gong and M. Sun, "An iterative learning controller with initial state learning," *IEEE Trans. on Automatic Control*, 44(2), pp. 371-376, 1999.

[3] T.W.S. Chow, and Y. Fang, "An iterative learning control method for continuous-time systems based on 2-D system theory," *IEEE Trans. On Circuit and Systems-1: fundamental theory and applications*, 45(4), pp.683-689, 1998.

[4] A. Datta, and J. Ochoa, "Adaptive internal model control: H_2 optimization for stable plants," *Automatica*, 34(1), pp. 75-82, 1998.

[5] D. De Roover,"Synthesis of a robust iterative learning controller using an H_∞ approach," *Proc. of the 35th Conference on Decision and Control*, Kobe, Japan, pp. 3044-3049, 1996.

[6] B.A. Francis,*A course in H_∞ control theory*, Springer-verlag, 1987.

[7] J.E. Kurek and M. Zaremba,"Iterative learning control synthesis based on 2-D system theory," *IEEE Trans. on Automatic Control*, 38(1), pp. 121-125, 1993.

[8] J. H. Moon, T.Y. Doh and M. J. Chung, "A robust approach to iterative learning control design for uncertain systems," *Automatica*, 34(8), pp. 1001-1004, 1998.

[9] K.L. Moore, M. Dahleh and S. P. Bhattacharyya, "Iterative learning control: a survey and new results," *Journal of Robotic Systems*, 9(5), pp. 563-594, 1992.

[10] M. Morari and E. Zafriou, *Robust Process Control*, Prentice Hall, 1989.

[11] K. Zhou, J.C. Doyle and K. Glover, *Robust and optimal control*, Prentice Hall, Englewood Cliffs, NJ, 1996.