Stability of Receding Horizon Kalman Filter in State Estimation of Linear Time-Varying Systems [1]

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Abstract

The paper presents a state predictor for linear time-varying systems using Kalman filter with the receding horizon strategy. It can be seen as a standard Kalman filter which takes into account the most recent data, those included in a moving data window of fixed length. The main purpose here is to assure stability for this type of filter. Under standard conditions we can establish a minimum horizon length for which the closed-loop filter with the receding horizon gain is exponentially stable. The approach makes no direct reference to the properties of the underlying Riccati equation, which allow us to address more general problems that can not be coined in terms of Riccati equations.

1 Introduction

The receding horizon technique, also known as moving horizon or model predictive control, is becoming a popular strategy in control problems as it stands as a third paradigm of optimal control, nestled between the more classical problems of finite horizon and infinite horizon. It combines the stability requirement that is intrinsic to infinite horizon problems, with the numerical simplicity of finite horizon methods, providing the grounds to deal with adaptiveness in various settings, e.g. see [6], [9], [10] and [5].

In parallel, the interest on receding horizon strategies for filtering problems has increased. In this context, receding horizon usually appears in limited memory filters (FIR filters) that preclude divergence due to modeling errors. Michalska and Mayne in [14] presented receding horizon observers for nonlinear systems and studied stability; Wang and Zhang in [15] derived a recursive Kalman filter with rejection of old data; Ling and Lim in [13] propose a receding horizon deterministic least squares state observer and Kwon et al. in [11] used receding horizon strategy in a

This paper presents a receding horizon Kalman filter which consists of a one-step ahead state predictor for linear time-varying systems combined with the receding horizon strategy. The covariance matrix at each time instant is calculated using a fixed length window of data and an arbitrary initial condition. It yield the state estimator gain in a strategy also known as Kalman filter frozen at a particular iteration, when applied to time-invariant systems, see [2].

The stability of the closed-loop receding horizon Kalman filter is pursued. Stability of Kalman filters for linear time-varying systems has deserved the attention of many authors and nowadays, well-established results can be found in the current literature. Almost all results rely on monotonicity, convergence and stability properties of the time-varying Riccati difference equation. One of the most general result may be summarized as follows: uniform detectability and stabilizability are sufficient conditions for the exponential stability of the closed-loop Kalman filter in the long run; see [7], [1] and [3]. In receding horizon problems, closed loop stability is usually obtained by adopting a boundary condition (an appropriate boundary constraint or weighting matrix) [12], [4] or by choosing the horizon length [2], [13].

We show here under standard conditions, that the filter with finite horizon length is exponentially stable provided the horizon length is large enough, see Theorem 1. One of the most important aspects is that we do not rely on Riccati equation properties to derive the filter stability, which opens a road to possible extensions.

2 Receding Horizon Filter

Consider the linear time varying system

$$\Phi : \begin{cases} x(k+1) = A_k x(k) + E_k w(k) \\ y(k) = C_k x(k) + D_k v(k) \end{cases}$$

where $x \in R^n$ is the state, $y \in R^r$ is the observed variable and $w \in R^p$, $v \in R^q$ are independent sequences of random variables with

$$w(k) \sim N(0,Q), \quad v(k) \sim N(0,R)$$

and the matrices A_k, C_k, D_k, E_k of appropriate dimensions are defined on $(-\infty,+\infty)$.

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The state predictor we deal with is described by

$$\widehat{x}(k+1) = A_k\widehat{x}(k) + L_k[y(k) - C_k\widehat{x}(k)] \tag{1}$$

where

$$L_k = A_k S_k C_k' [C_k S_k C_k' + D_k R D_k']^{-1} \tag{2}$$

In the usual Kalman filter implementation, given an initial distribution $x(0) \sim N(\bar{x}_0, \Sigma_0)$, the covariance of the estimation error for each n in the interval $[0,k]$ is given by

$$\begin{aligned} P_n &= A_n[P_{n-1} - P_{n-1}C_n'(C_n P_{n-1} C_n' + D_n R D_n')^{-1} C_n P_{n-1}]A_n' \\ &\quad + E_n Q E_n' \qquad (3) \\ P_0 &= \Sigma_0 \end{aligned}$$

and we set $S_k = P_k$ in (2) at each time instant $k \geq 0$.

In the receding horizon strategy we consider, N is called horizon length or data window length. At each time k, the horizon length is $[k-N,k]$ and we define recursively the sequence $S_{n|k}$ for $k-N \leq n \leq k$, by means of the Riccati difference equation

$$\begin{aligned} S_{n|k} &= A_n[S_{n-1|k} - S_{n-1|k}C_n'(C_n S_{n-1|k} C_n' + D_n R D_n')^{-1} \\ &\quad \cdot C_n S_{n-1|k}]A_n' + E_n Q E_n' \qquad (4) \\ S_{k-N|k} &= H_{k-N} \end{aligned}$$

The initial condition H_{k-N} is picked from a given sequence $H_k, -\infty < k < \infty$ of symmetric positive semidefinite matrices. At each time k, we employ $S_{k|k}$ to evaluate the filter gain by setting $S_k = S_{k|k}$ in (2).

In order to compare the receding horizon Kalman filter (1), (2) and (4) with the standard Kalman filter (1), (2) and (3), suppose that we set at time step k the initial condition H_{k-N} for the former, to coincide with the error covariance matrix P_{k-N} of the latter. The solution of the Kalman filter will then coincide with the receding horizon Kalman filter on the time stage k only, since in this situation $P_k = S_{k|k}$, and generally, $P_n \neq S_{n|n}$ on the interval $k-N \leq n < k$.

The initial covariance matrix H_{k-N} in the receding horizon filter is associated to the accuracy of previous estimation and model confidence, and it can be employed for tuning purposes. In general, large values (in the positive definite sense) of H_{k-N} lead to high values of gain L_k, fast dynamics thus presenting a fast "forgetting rate" of old data. Assuming that the filter is stabilizable, setting the filter with large initial covariance matrix tends to yield stable closed-loop filters. In the other extreme, a small value of H_{k-N} creates a stronger dependence of the filter on the past data, and the stability of the filter is more difficult to ascertain.

Another possible choice of parameter that may contribute to the stability of the closed-loop receding filter, is to increase the horizon length N. One can infer from the infinite horizon filtering and control analysis, that under technical conditions (stabilizability and detectability of the filter in appropriate senses) the increase in the horizon length N provides asymptotically, stable closed-loop filters, cf. [1], [7]; however, nothing can be said beforehand from that analysis which could be relevant for finite values of N.

The main result of the present paper is to provide conditions for the stability of the closed-loop receding filter, thus, for finite values of horizon length N, no matter how the initial covariance matrix sequence $\{H_\ell\}$ is picked. The standard condition here are controllability and detectability.

More precisely, we say that the receding horizon Kalman filter (1), (2), (4) is exponentially stable when its closed-loop matrix $(A_k - L_k C_k), k \in (-\infty, +\infty)$ is exponentially stable, according with the following definition.

Definition 1 (Exponential Stability) $A_k, k \in (-\infty, +\infty)$ *is said to be exponentially stable if, for every integer k, there exist positive constants* α *and* β *such that*

$$||A_t \cdots A_k|| \leq \alpha e^{-\beta(t-k)}, \textit{for every } t \geq k.$$

The assumptions we consider throughout the paper are listed below.

A1. The sequence pair $[A_k, E_k Q^{1/2}]$ is uniformly controllable. D_k is full rank and $R > 0$.

A2. The sequence pair $[A_k, C_k]$ is uniformly detectable.

A3. The matrices A_k, C_k, D_k, E_k, H_k, $-\infty < k < \infty$, are uniformly bounded.

Assumptions A1–A3 are standard in the literature for linear time-varying systems; see [2], [1] and [8].

For simplicity we shall denote $S_{k|k}$ by S_k throughout the paper, since we will be only referring to the receding horizon filter, equations (1), (2) and (4).

3 Definitions and Basic Results

For a given sequence $g = \{G_t, \ldots, G_k\}$ where $G_n, k \leq n \leq t$, are matrices of appropriate dimensions, let $\Phi^g_{k,t}$ be defined by

$$\begin{cases} \Phi^g_{k,k} = I \\ \Phi^g_{k,t} = (A_{k-1} - G_{k-1}C_{k-1}) \cdots (A_t - G_t C_t), \quad k > t \end{cases} \tag{5}$$

We denote by Φ the state transition matrix of the system (1); we also drop the superscript g when the sequence is obvious from the context.

For $z \in R^n$ and $g = \{G_k, \ldots, G_{k-N+1}\}$ we define the trajectory

$$z(n+1) = \Phi'_{k,k-n} z, \quad n = 0, \ldots, N-1 \tag{6}$$

and the associated cost functional

$$J_g^{k,N}(z) := \sum_{n=0}^{N-1} \left[z(n)'(EQE' + GDRD'G')_{k-n}z(n)\right] + z(N)'H_{k-N}z(N) \quad (7)$$

where, for easy of notation, we write

$$(EQE' + GDRD'G')_{k-n} = (E_{k-n}QE'_{k-n} + G_{k-n}D_{k-n}RD'_{k-n}G'_{k-n})$$

Hereafter we seek the optimal cost, defined by $J^{k,N}(x) = \inf_g\{J_g^{k,N}(x)\}$.

The cost J can be regarded as the cost for which the sequence of gains (2) is optimal in a dual control problem. We formalize this in the next lemma; the proof is omitted.

Lemma 1 *Consider the receding horizon filter (1), (2) and (4), and the cost functional (7). Then,*

$$J^{k,N}(z) = z'S_kz$$

holds, and the optimal value of (7) is achieved with the sequence

$$G_t = A_tS_{t|k}C'_t[C_tS_{t|k}C'_t + D_tRD'_t]^{-1}$$

for $k-N+1 \le t \le k$; moreover, $L_k = G_k$ in (2).

There are two properties of the cost functional which will play a central role in our analysis: uniform boundness above and below. These properties are formalized in the next lemmas; the proofs are omitted.

Lemma 2 *Suppose that assumptions A2 and A3 hold. Then, there exists a scalar $J^+ > 0$ such that for every $N \ge 0$ and k, $\inf_g\{J_g^{k,N}(z)\} \le ||z||^2J^+$.*

Lemma 3 *Suppose that assumption A1 holds. Then, there exist $N_c > 0$ and $J^- > 0$ such that for every k, $\inf_g\{J_g^{k,N_c}(z)\} > ||z||^2J^-$, even if $H_{k-N_c} \equiv 0$.*

4 Preliminary Results

In this section we consider N_c, J^- and J^+ as in Lemma 3 and Lemma 2. Let us also denote

$$N_0 = \left\{(J^+)^2 \Big/ \delta J^- + 1\right\}N_c \quad (8)$$

$$\delta_0 = J^- \Big/ \left[\sum_{i=1}^{N_c}(J^+/J^-)^i\right] \quad (9)$$

Lemma 4 *For $0 < \delta \le \delta_0$, consider $g = \arg\inf_h\{J_h^{k,N}(z)\}$ for $N > N_0$, and the corresponding trajectory $z(t)$. Then there exists t, $0 \le t \le N_0$ such that*

$$J^{k-t,T}(z(t)) \le ||z||^2\delta, \quad \textit{for all } T \ge 0. \quad (10)$$

Proof: Let $r := ||z||^2J^+/\{J^-((N_0/N_c)-1)\} > 0$, and we first show that there exist $0 \le t \le N_0$ such that $||z(t)||^2 \le r$. Let us deny this, and assume that $||z(t)||^2 > r$, for all $0 \le t \le N_0$. Recall that $g = \{G_k, \ldots, G_{k-N+1}\}$. By optimality of g and Lemma 2 we have that

$$\begin{aligned} ||z||^2J^+ &\ge J_g^{k,N}(z) \\ &= \sum_{n=0}^{N-1} z(n)'(EQE' + GDRD'G')_{k-n}z(n) + z(N)'H_{k-N}z(N) \\ &\ge \sum_{n=0}^{N_0} z(n)'(EQE' + GDRD'G')_{k-n}z(n) \\ &\ge \sum_{q=1}^{\bar{q}}\left(\sum_{n=(q-1)N_c}^{qN_c} z(n)'(EQE' + GDRD'G')_{k-n}z(n)\right) \quad (11) \end{aligned}$$

where $\bar{q}$ is the smallest integer such that $\bar{q} \ge N_0/N_c$. Recall that we assumed above that $||z((q-1)N_0)||^2 > r$, and from Lemma 3,

$$\sum_{n=(q-1)N_c}^{qN_c} z(n)'(EQE' + GDRD'G')_{k-n}z(n) \ge rJ^-$$

The choice of δ implies that $N_0/N_c > 1$, and (11) yields that

$$\begin{aligned} ||z||^2J^+ &\ge \sum_{q=1}^{\bar{q}} rJ^- = \bar{q}rJ^- = \frac{\bar{q}\,||z||^2J^+}{(N_0/N_c - 1)} \\ &\ge \frac{N_0/N_c}{(N_0/N_c - 1)}||z||^2J^+ > ||z||^2J^+ \end{aligned}$$

which is an absurd, and hence a t with $0 \le t \le N_0$ should exist in such a way that

$$||z(t)||^2 \le \frac{||z||^2J^+}{J^-(N_0/N_c - 1)} = r \quad (12)$$

Once again, the optimality of g and Lemma 2 provide that

$$J^{k-t,T}(z(t)) \le ||z(t)||^2J^+ \le \frac{||z||^2(J^+)^2}{J^-(N_0/N_c - 1)} \le ||z||^2\delta \quad (13)$$

where the last inequality in (13) holds with the choice of number N indicated in the lemma. ■

Lemma 5 *For $0 < \delta \le \delta_0$ let $N, M > N_0$. Then,*

$$|J^{k,M}(z) - J^{k,N}(z)| \le ||z||^2\delta, \quad \forall z \in R^n \quad (14)$$

Proof: Let us denote $g = \arg\inf_h\{J_h^{k,N}(z)\} = \{G_k, \ldots, G_{k-N+1}\}$ and consider the corresponding trajectory $z(t)$. According to Lemma 4, there exists t, $0 \le t \le N_0$, such that $J^{k-t,T}(z(t)) \le ||z||^2\delta$, for all $T \ge 0$; in particular, choose $T = M - t$ (notice that $T \ge M - N_0 > 0$) to get that

$$J^{k-t,M-t}(z(t)) \le ||z||^2\delta \quad (15)$$

Apart from this, we have for $t \leq N_0 < N$ that

$$J^{k,N}(z) = J_g^{k,N}(z) \geq \sum_{n=0}^{t-1} z(n)'(EQE' + GDRD'G')_{k-n} z(n) \tag{16}$$

Let us denote $f = \arg\inf_h \{J_h^{k-t,M-t}(z(t))\} = \{F_{k-t}, \ldots, F_{k-M+1}\}$. From (15) and (16), we obtain

$$\begin{aligned}
&J^{k,N}(z) + \|z\|^2 \delta \\
&\geq \sum_{n=0}^{t-1} z(n)'(EQE' + GDRD'G')_{k-n} z(n) + J^{k-t,M-t}(z(t)) \\
&= \sum_{n=0}^{t-1} z(n)'(EQE' + GDRD'G')_{k-n} z(n) \\
&+ \sum_{n=0}^{M-t-1} z(n+t)'(EQE' + FDRD'F')_{k-n-t} z(n+t) \\
&+ z(M)' H_{k-M} z(M) = J_h^{k,M}(z) \geq J^{k,M}(z)
\end{aligned} \tag{17}$$

where h denotes the sequence concatenation $h = \{K_k, \ldots, K_{k-t+1}, F_{k-t}, \ldots, F_{k-M+1}\}$. Following the same procedure with N replaced by M and vice-versa, we obtain that $J^{k,N}(z) - J^{k,M}(z) \leq \|z\|^2 \delta$ and the proof is completed. ■

Lemma 6 *Let $N > N_c$. Consider the receding horizon gain sequence $g = \{L_k, \ldots, L_{k-N+1}\}$ and the corresponding trajectory $z(t)$. Then,*

$$\|z(t)\|^2 \leq \|z\|^2 (J^+/J^-)^t, \quad 1 \leq t \leq N \tag{18}$$

Proof: We start by showing that

$$\|z(t)\|^2 \leq (J^+/J^-) \|z(t-1)\|^2, \quad 1 \leq t \leq N \tag{19}$$

holds. For $\|z(t-1)\|_2 = 0$ the assertion is trivial since $z(t) = 0$. Now consider $\|z(t-1)\|_2 > 0$. From the optimality of g (Lemma 1), we can write

$$\begin{aligned}
J_g^{k-t,N}(z(t-1)) &= z(t-1)'(EQE' + LDRD'L')_{k-t} z(t-1) \\
&+ J_g^{k-t-1,N-1}(z(t))
\end{aligned}$$

and hence,

$$\begin{aligned}
&\|z(t-1)\|^2 J^+ \geq J_g^{k-t,N}(z(t-1)) \\
&= z(t-1)'(EQE' + LDRD'L')_{k-t} z(t-1) + J_g^{k-t-1,N-1}(z(t)) \\
&\geq J_g^{k-t-1,N-1}(z(t)) > \|z(t)\|^2 J^-
\end{aligned} \tag{20}$$

where Lemma 3 and Lemma 2 lead respectively to the last and the first inequality. Finally we apply (19) successively to obtain

$$\begin{aligned}
\|z(t)\|^2 &\leq \|z(t-1)\|^2 (J^+/J^-) \leq \cdots \\
&\leq \|z(0)\|^2 (J^+/J^-)^t = \|z\|^2 (J^+/J^-)^t
\end{aligned}$$

■

Lemma 7 *Let $0 < \delta \leq \delta_0$ and $N > N_0 + 1$. Consider the receding horizon gain sequence $g = \{L_k, \ldots, L_{k-N+1}\}$ and the corresponding trajectory $z(t)$. Then, there exists $0 < \gamma < 1$ such that*

$$J^{k-N_c,N}(z(N_c)) \leq \gamma J^{k,N}(z), \quad \forall z \in R^n \tag{21}$$

Proof: We set in this proof

$$\kappa := J^- - \delta \sum_{i=1}^{N_c} (J^+/J^-)^i$$

$$\bar{N} := N - 1 > N_0$$

For $z = 0$, (21) holds trivially. Now, consider $z \neq 0$ and any t, $0 \leq t \leq k$. One gets from Lemma 1 that $G_{k-t} = L'_{k-t}$ and thus,

$$\begin{aligned}
J^{k-t,N}(z(t)) &= z(t)'(EQE' + LDRD'L')_{k-t} z(t) \\
&+ J^{k-t-1,N-1}(z(t+1))
\end{aligned}$$

and,

$$\begin{aligned}
&J^{k-t,N}(z(t)) - J^{k-t-1,N}(z(t+1)) \\
&= z(t)'(EQE' + LDRD'L')_{k-t} z(t) \\
&+ J^{k-t-1,N-1}(z(t+1)) - J^{k-t-1,N}(z(t+1))
\end{aligned} \tag{22}$$

Apart from this, we apply Lemma 5 to conclude that $J^{k,\bar{N}+1}(z) - J^{k,\bar{N}}(z) \leq \|z\|^2 \delta$, $\forall z \in R^n$ and thus, we can write

$$\begin{aligned}
&J^{k-t-1,N}(z(t+1)) - J^{k-t-1,N-1}(z(t+1)) \\
&\leq \|z(t+1)\|^2 \delta \leq \|z\|^2 (J^+/J^-)^{t+1} \delta
\end{aligned} \tag{23}$$

for all $0 \leq t \leq k$; where the last inequality is justified by Lemma 6. Now, we substitute (23) in (22) to obtain

$$\begin{aligned}
&J^{k-t,N}(z(t)) - J^{k-t-1,N}(z(t+1)) \\
&\geq z(t)'(EQE' + LDRD'L')_{k-t} z(t) - \delta \|z\|^2 (J^+/J^-)^{t+1}
\end{aligned}$$

Summing for $t = 0, \cdots, N_c - 1$ we get that

$$\begin{aligned}
&J^{k,N}(z(0)) - J^{k-N_c,N}(z(N_c)) \\
&\geq \sum_{n=0}^{N_c-1} z(n)'(EQE' + LDRD'L')_{k-n} z(n) \\
&- \delta \|z\|^2 \left[\sum_{n=1}^{N_c} (J^+/J^-)^n\right]
\end{aligned} \tag{24}$$

In view of Lemma 2 and the first term on the right-hand side of (24), we can write that

$$\sum_{n=0}^{N_c-1} z(n)'(EQE' + LDRD'L')_{k-n} z(n) > \|z\|^2 J^- \tag{25}$$

and, returning to (24), we get that

$$\begin{aligned}
&J^{k,N}(z(0)) - J^{k-N_c,N}(z(N_c)) \\
&> \|z\|^2 \left[J^- - \delta \sum_{n=1}^{N_c} (J^+/J^-)^n\right] = \|z\|^2 \kappa
\end{aligned} \tag{26}$$

Applying again Lemma 2, we introduce the inequality $||z||^2 \geq J^{k,N}(z(0))/J^+$ in the inequality in (26) to obtain

$$J^{k,N}(z(0)) - J^{k-N_c,N}(z(N_c)) > (\kappa/J^+)J^{k,N}(z(0))$$

or, equivalently,

$$\gamma J^{k,N}(z) > J^{k-N_c,N}(z(N_c)), \quad z \neq 0 \tag{27}$$

where $\gamma := (1 - \kappa/J^+)$. Finally, notice that with the choice of κ and δ one has that $0 < \gamma < 1$. ■

5 Stability of the Closed-Loop Filter

Theorem 1 *Consider J^- and J^+ as in Lemma 3 and Lemma 2 respectively, and let $N > N_0 + 1$ with N_0 as in (8). Then the receding horizon Kalman filter (1), (2) and (4) with horizon N is exponentially stable.*

Proof: We show that $(A_k + A_k L_k C_k)$ is exponentially stable. Consider the receding horizon gain sequence $g = \{L'_k, L'_{k-1}, \dots\}$ and the corresponding trajectory $z(t)$. From Lemma 7, there exists $0 < \gamma < 1$ such that

$$J^{k-N_c,N}(z(N_c)) \leq \gamma J^{k,N}(z), \quad \forall z \in R^n$$

From Lemma 1,

$$J^{k-N_c,N}(z(N_c)) = z(N_c)' S_{k-N_c} z(N_c) \text{ and } J^{k,N}(z) = z' S_k z$$

and, since $z(N_c) = \Phi'_{k,k-N_c+1} z$,

$$\Phi_{k,k-N_c+1} S_{k-N_c} \Phi'_{k,k-N_c+1} \leq \gamma S_k \tag{28}$$

Since (28) holds for every k, we obtain

$$\begin{aligned} &\Phi_{k,k-2N_c+1} S_{k-2N_c} \Phi'_{k,k-2N_c+1} \\ &\qquad \leq \gamma \Phi_{k,k-N_c+1} S_{k-N_c} \Phi'_{k,k-N_c+1} \leq \gamma^2 S_k \end{aligned} \tag{29}$$

and repeating this procedure n times, we can evaluate

$$\Phi_{k,k-nN_c+1} S_{k-nN_c} \Phi'_{k,k-nN_c+1} \leq \gamma^n S_k \tag{30}$$

From Lemma 3 and Lemma 2, $J^- \leq \text{eig}(S_k) \leq J^+, \forall k$, and (30) leads to

$$\Phi_{k,k-nN_c+1} \Phi'_{k,k-nN_c+1} \leq (J^+/J^-)\gamma^n I, \quad \forall n, \forall k \tag{31}$$

Now, for $-\infty < k \leq t < \infty$, let n be the largest integer such that $n \leq (t-k)/N_c$ and we write

$$\begin{aligned} \Phi_{t,k}\Phi'_{t,k} &= \Phi_{t,t-nN_c+1}\Phi_{t-nN_c,k}\Phi'_{t-nN_c,k}\Phi'_{t,t-nN_c+1} \\ &\leq (J^+/J^-)\gamma^n \Phi_{t-nN_c,k}\Phi'_{t-nN_c,k} \\ &\leq (J^+/J^-)\chi_k e^{\ln(\gamma)\, n} I \leq (J^+/J^-)\chi_k e^{\frac{\ln(\gamma)}{N_c}(t-k)} I \end{aligned}$$

where $\chi_k = \max_{0 \leq \eta \leq N_c - 1} \lambda_{\max}(\Phi_{k+\eta,k}\Phi'_{k+\eta,k})$. Thus, from (5) we obtain

$$||(A_t - L_t C_t) \cdots (A_k - L_k C_k)|| \leq (\chi_k J^+/J^-)^{1/2} e^{\frac{\ln(\gamma)}{2N_c}(t-k)}$$

implying the exponential stability of the receding control gain sequence, according to Definition 1. ■

Example. Consider system Φ with

$$C_k = [1\ 0];\ E_k = I_2;\ D_k = 1; R = 0.2$$

and let matrices A_k be given by

$$A_k = \exp\left\{ \frac{1}{4} \begin{bmatrix} 0 & 1 \\ -f(k)^2 & -2\xi f(k) \end{bmatrix} \right\}$$

where $\xi = 0.2$ and f is a parameter described by

$$f(k) = -0.9 + 0.5 \sin((2\pi/T)k)$$

where $T = 4$ is the oscillation period. We assume $H_k = 0$, $-\infty < k < \infty$. Regarding the covariance matrix of w, we consider the next two situations.

(i) $Q = I_2$. In this case, $N_c = 1$. We adopt J^+ and J^- as follows. The solution P_n as in (3) for the usual Kalman filter converges to a periodic solution as n increases; then, we adopt J^+ as the maximum eigenvalue of P_n during one period,

$$J^+ = 8.321$$

We also set $J^- = \lambda_{\min}(Q) = 1$. It can be checked that J^+, J^- and N_c satisfy Lemmas 3 and 2 and thus, Theorem 1 guarantees that the filter is stable under the condition

$$N \geq N_0 = 578.1$$

where N_0 is evaluated as in (8). Figure 1 (a) depicts the spectral radius of the state transition matrix:

$$\sigma = \max_{0 \leq n \leq T} (\lambda_{\max}(\Phi'_{n+T,n}\Phi_{n+T,n})^{1/2})$$

for increasing values of N. As one can see, σ decreases as N increases, and the filter is stable for $N \geq 3$.

(ii) $Q = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}$. In a similar manner to case (i) we adopt $J^+ = 14.20$. Bearing in mind that $S_{n+1|k} \geq S_{n|k}$, we can write for each $N \geq 2$ and $k \geq 0$,

$$\begin{aligned} J^{k,N}(z) &= z' S_{k|k} z \geq z' S_{k-N+2|k} z \geq \lambda_{min}(S_{k-N+2|k})||z||^2 \\ &\geq \min_{\ell} \lambda_{min}(S_{\ell-N+2|k})||z||^2 \end{aligned}$$

and, since the system is periodic with oscillation period T, one can check that $\min_\ell(\lambda_{min}(S_{\ell-N+2|k})) = \min_{0 \leq \ell \leq T}(\lambda_{min}(S_{\ell-N+2|k})) = 0.1539$. Hence, we have that $J^{k,2}(z) \geq 0.1539||z||^2$ in such a manner that we can adopt $N_c = 2$ and $J^- = 0.1539$ and Theorem 1 guarantees that the filter is stable for a sufficiently large horizon. The stability behavior of the filter is illustrated in Figure 1 (b) for increasing values of N; as one can see, the filter is stable for $N \geq 2$.

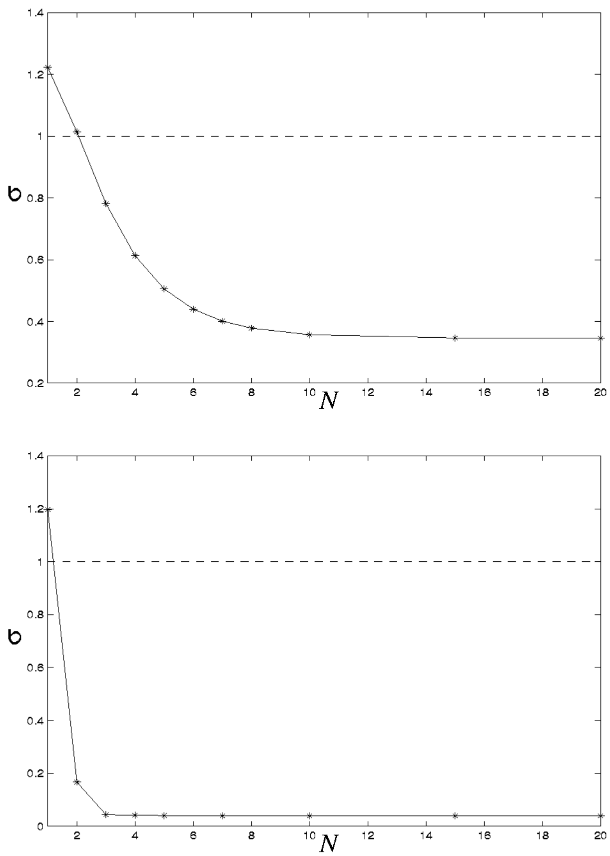

Figure 1: Maximal spectral radius of the closed loop filter, σ, versus the horizon length N. (a) Case (i); (b) Case (ii).

6 Conclusions and Extensions

The paper presents a state predictor for linear time-varying systems using a Kalman filter, combined with receding horizon strategy. The stability of the closed-loop filter is addressed and we have shown that the filter is exponentially stable under uniform detectability of the pair $[A_k, C_k]$ and uniform controllability of the pair $[A_k, E_k Q^{1/2}]$, provided the horizon length N is large enough. This number is subject to the evaluation in Theorem 1.

One of the most important features of the analysis developed here is that the associated Riccati equation is nowhere employed in deriving the stability condition. The focus is on the cost function. This feature allows us to extend the results to other classes of problems involving time varying linear systems with extra elements that precludes the Riccati formulation. This is the subject of ongoing investigation.

References

[1] B. D. O. Anderson and J. B. Moore. Detectability and stabilizability of time-varying discrete-time linear systems. *SIAM Journal on Control and Optimization*, 19(1):20–32, 1981.

[2] R. R. Bitmead and M. Genvers. Riccati difference and differential equations: Convergence, monotonicity and stability. *S. Bittani, A. J. Laub and J. C. Willems (eds) The Riccati Equation (Springer Verlag, Berlim)*, pages 263–291, 1991.

[3] G. De Nicolao. On the time-varying Riccati difference equation of optimal filtering. *SIAM Journal on Control and Optimization*, 30(6):1251–1269, 1992.

[4] G. De Nicolao and S. Strada. On the stability of receding-horizon LQ Control with zero-state terminal contraint. *IEEE Transactions on Automatic Control*, 42(2):257–260, 1997.

[5] J.B.R. do Val and T. Başar. Receding horizon control of jump linear systems and a macroeconomic policy problem. *Journal of Economic Dynamics & Control*, 23:1099–1131, 1999.

[6] C. E. Garcia, D. M. Prett, and M. Morari. Model predictive control: Theory and practice - a survey. *Automatica*, 25(3):335–348, 1989.

[7] W. W. Hager and L. L. Horowitz. Convergence and stability properties of the discrete Riccati operator equation and the associated optimal control and filtering problems. *SIAM Journal on Control and Optimization*, 14(2):295–312, 1976.

[8] S. S. Keerthi and E. G. Gilbert. Optimal infinite-horizon feedback laws for a general class of constrained discrete-time systems: Stability and moving-horizon approximations. *Journal of Optimization Theory and Applications*, 57(2):265–293, 1988.

[9] W. H. Kwon and D. G. Byun. Receding horizon tracking control as a predictive control and its stability properties. *Int. J. Control*, 50(5):1807–1824, 1989.

[10] W. H. Kwon, H. Choi, D. G. Byun, and S. Noh. Recursive solution of generalized predictive control and its equivalence to receding horizon tracking control. *Automatica*, 28(6):1235–1238, 1992.

[11] W. H. Kwon, P. S. Kim, and P. Park. A receding horizon Kalman FIR filter for discrete time-invariant systems. *IEEE Transactions on Automatic Control*, 44(9):1787–1791, 1999.

[12] J. W. Lee, W. H. Kwon, and J. Choi. On stability of constrained receding horizon control with finite terminal weighting matrix. *Automatica*, 34(12):1607–1612, 1998.

[13] K. V. Ling and K. W. Lim. Receding horizon recursive state estimation. *IEEE Transactions on Automatic Control*, 44(9):1750–1753, 1999.

[14] H. Michalska and D. Q. Mayne. Moving horizon observers and observer-based control. *IEEE Transactions on Automatic Control*, 40(6):995–1006, 1995.

[15] Z. Wang and J. Zhang. A Kalman filter algorithm using a moving window with applications. *International Journal on Systems Science*, 26(8):1465–1478, 1995.