A Stability Analysis Based on Economic Principles for the Control of the Cotton Aphid

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Abstract

The cotton aphid is an important pest affecting the profitability of cotton production. We study the problem of the optimal timing of pesticide application to control the aphid. The problem is complicated by the presence of a significant predator insect. The predator serves as a natural control of the aphid and is adversely affected by application of pesticide. We determine optimal state dependent rules for application of pesticide. We show that first application of pesticide is a switching time between two dynamic systems.

1 Introduction

The optimal application of pesticide is a problem made difficult by the short half-lives of pesticides. All approved pesticides deteriorate very quickly in the natural environment. With pesticides which deteriorate quickly in the natural environment, the optimization becomes dynamic and the timing of the initial application becomes important.

We study the problem of the optimal timing of pesticide application. The problem is complicated by the presence of a predator insect. The predator is a control of the aphid and is adversely affected by application of pesticide. The short half-life of the pesticide implies that the aphids and the predator insects are killed in great numbers for a short period of time after which the kill is negligible. The recovery rate of the predator insect is slower than the aphid.

In order to emphasize the problem of application of pesticide, we use a very simple model of cotton plant, aphid and predator interaction. We allow all other factors to be viewed as random perturbations to the model. The farmer's problem is to maximize profits through selective application of pesticide. Pesticides are costly to apply and first application of pesticide radically affects the model dynamics.

In the absence of the predator insect, the problem is simple. We find an evenly spaced pattern of application, such that, the marginal cost of spraying is equal to the expected increase in revenue. This solution has been found be many authors (Hall and Norgaard (1973), etc.). The idea that the impact on the predator insect must be explicitly recognized is given by Longworth and Rudd (1975).

The first application of pesticide alters the dynamics of the model by effectually eliminating the predator. Hence, the relevant marginal cost now includes both the direct cost of spraying today and the increase in future cost associated with more frequent spraying in the future. The increase in future cost of spraying is directly related to the proportion of predators in existence. If the proportion of predators is very small, we are very close to the dynamics system after spraying and the future additional cost is small.

The decision problem of our farmer will include a state dependent decision rule for first spraying and a decision rule for determining the frequency of spraying after the first application. In this paper, we determine when first spraying should be postponed and under what conditions the first application of pesticide should be undertaken. If applied too early insecticide will have to be applied more times during the growing season increasing the cost. If applied too late, the cotton plant itself may be so damaged that the profitability of the crop will be compromised.

2 Model

The goal of the mathematical model is to develop a hybrid control model for the interaction. We include in the model pesticide application. The pesticide has the potential to radically affect the dynamics of the basic model and actually introduces a new model that is effective for a few days. Thus the application time of the pesticide is a switching time between two dynamical systems. Stability is not asked in this situation.

We describe a simple predator prey model of the aphid and predator insect interaction. For this to be meaningful we need to, at the same, time develop a model of the cotton plant and its interaction with the cotton aphid. We will include in the model a representation of the effect of the application of insecticide. The insecticide acts on both the aphid and the predator insect. The model will be set up to provide input for a cost and production model and the model will allow for quantitative predictions. The primary assumptions are: 1) the aphids are harmful to the cotton plant and have an effect on the total amount of cotton that can be harvested from a given field. 2) There are predator insects that feed primarily on the cotton aphid. 3) Under some conditions the cotton plant, the aphid population and the predator insect population with reach a mutually stable equilibrium. 4) Insecticide application is costly

2.1 The Predator-Prey-Plant Model

The model presented here is based on the masters thesis of David Hogan of the Department of Mathematics and Statistics at Texas Tech University. The model is based on simple concepts of birth and death models. We assume that the number of aphids present in a cotton field at time t+h is dependent on the number at time t, the number hatched in the time interval h, the number that have died in the time interval h. We will assume that the increase due to emigration is negligible and we will assume that the number that have immigrated is likewise negligible. We lump all predator insects into one group. The model for the cotton plant is somewhat more problematic. The rate of growth of the cotton plant is not a function time but a complicated function of the environment. Our approach to the problem allows us to incorporate all factors which are outside the farmer's control into the underlying probability structure. In reality, the farmer can control some of the determinates we take as exogenous. As long as we consider the decision process for these actions as independent of both the evolution of aphids and their interaction with the cotton crop, the affect of these actions on production is incorporated into the law of motion for cotton. . We are now able to completely describe the evolution of our system in the following equations:

$$\begin{aligned}
A_{t+h} &= A_t + \alpha_{1t} A_t - \delta_{1t} A_t \\
P_{t+h} &= P_t + \alpha_{2t} P_t - \delta_{2t} P_t \\
C_{t+h} &= C_t + \gamma_t C_t - \eta_t C_t
\end{aligned}$$

where A_t is number of aphids at time t, P_t is the number of predators at time t and C_t represents the amount t of foliage on the cotton plant at time t. The functions α_{it} are birth rates as are the functions δ_{it} death rates. The function γ_t is the growth rate of the cotton and η_t is rate at which the plant is loosing foliage.

We can likewise examine the death coefficient. It is a function of the natural death rate of the aphids, the number consumed by the predator insects and most importantly of the application of the pesticide. We will assume that the pesticide kills on the order of 99% plus of the aphids. The problem arises in that it kills the same percentage of predators. Since the doubling time of the predators is measured in days instead of hours we see that an application of pesticide has the effect of removing the natural controls from the picture.

3 Agent's Problem

We assume separability in the farmer's production and consumption decisions. To elaborate further, the farmer in this model cares about cotton and pesticide only to the extent to which they affect the outcome in terms of end of period net profit. We then obtain risk neutrality by thinking of the farmer as having access to a complete set of contingent claims over the relevant states.

We assume that the environmental effects of the pesticide are appropriately reflected in the price of the pesticide which the farmer faces. This causes the farmer to act as if we explicitly enter care for the environment into his utility function. We also assume the intensity of spraying is fixed.

Our final assumption is that decisions this season affect only this seasons crop. In particular, this implies that aphids do not develop resistance to pesticide over time. Again we assume any tie ins between periods is accounted for in the price of pesticide.

The cost of these actions is accounted for by an adjustment in the price of cotton - i.e. we consider the only the net price of cotton.

We are now prepared to write the farmer's decision problem:

$$\max_{\{S_t\}} E\left\{\sum_{t=o}^{T-1} \beta^t \left(-P_t^s S_t\right) + \beta^T P_T^C C_T\right\}$$
$$s.t. \text{ The above difference equations}$$

T is the harvest date. We assume a fixed growing season for cotton. $\{\mathrm{P}^c, P^s\}$ are the price of spraying and cotton. For the computations, we choose a decision period of one week. To make the model realistic we update the evolution equations much more frequently. We analyze this problem as a T period Bellman Equation.

Which dynamic system we are in depends on the level of P. If P is large, there exists a natural control of the aphid population and we consider ourselves to be in the prespraying dynamic system. If P is small, there is no natural control of the aphid population and we are in the post-spraying dynamics system. The doubling time of the predator population is such that we never move from a post to a pre spraying system.

We can see immediately, the solution to the problem. In the post-spraying dynamic system, the farmer will spray at regular intervals with the time between spraying determined solely by the cost of spraying and the growth rate of the aphids. In the pre-spraying system, the farmer will postpone spraying so long as the predators are acting as a significant control for the aphids. In the next section, we give the results of our model.

4 Results

To emphasize our main result, we have chosen not to display results for alternative specifications of the underlying difference equations. Instead, we take the difference equations as a given and explore the implications of different initial conditions - e.g. when will a farmer optimally spray if he starts with twice as many predators as aphids. We also explore the implications for different relative price of spraying.

A large aphid population early in the season has large effects on harvested cotton. The aphid population grows so quickly the cotton foliage never recovers. Hence, there exist may configurations of initial conditions for which the farmer will always spray.

Figure 1 gives the solution of the stopping problem for different levels of predator insects, different levels of spray price and at different dates. The decision to spray is increasing in the number of aphids, increasing in the amount of cotton foliage, and decreasing in the number of predators. Harvest occurs at the end of week 16.

We can see the existence of the two dynamic systems to which we have been referring. Notice in the early weeks, the rule for spraying when the minimum number of predators exist is to spray any time the aphids grow appreciably. The rule takes this form because the aphids are effectively an uncontrolled population in this state. The aphids grow so quickly that the probability for cotton damage is very high. We see less spraying toward the end of the season is that the aphids do to have as much time to damage the crop as they do earlier.

5 Conclusion

In this paper, we have demonstrated the importance of using state dependent decision rules in order to determine the frequency of pesticide application for the control of the cotton aphid. The key result is that the farmer should consider not only the aphid population but should also consider the predator population. The existence of a significant predator can decrease the cost of spraying significantly. We have also shown that for many cases - those where the predator population is small relative to the aphid population - the solution is equivalent to the case with no predators in existence. In these cases, our model yields equivalent solutions to the older pesticide application papers which ignore the predator population.

References

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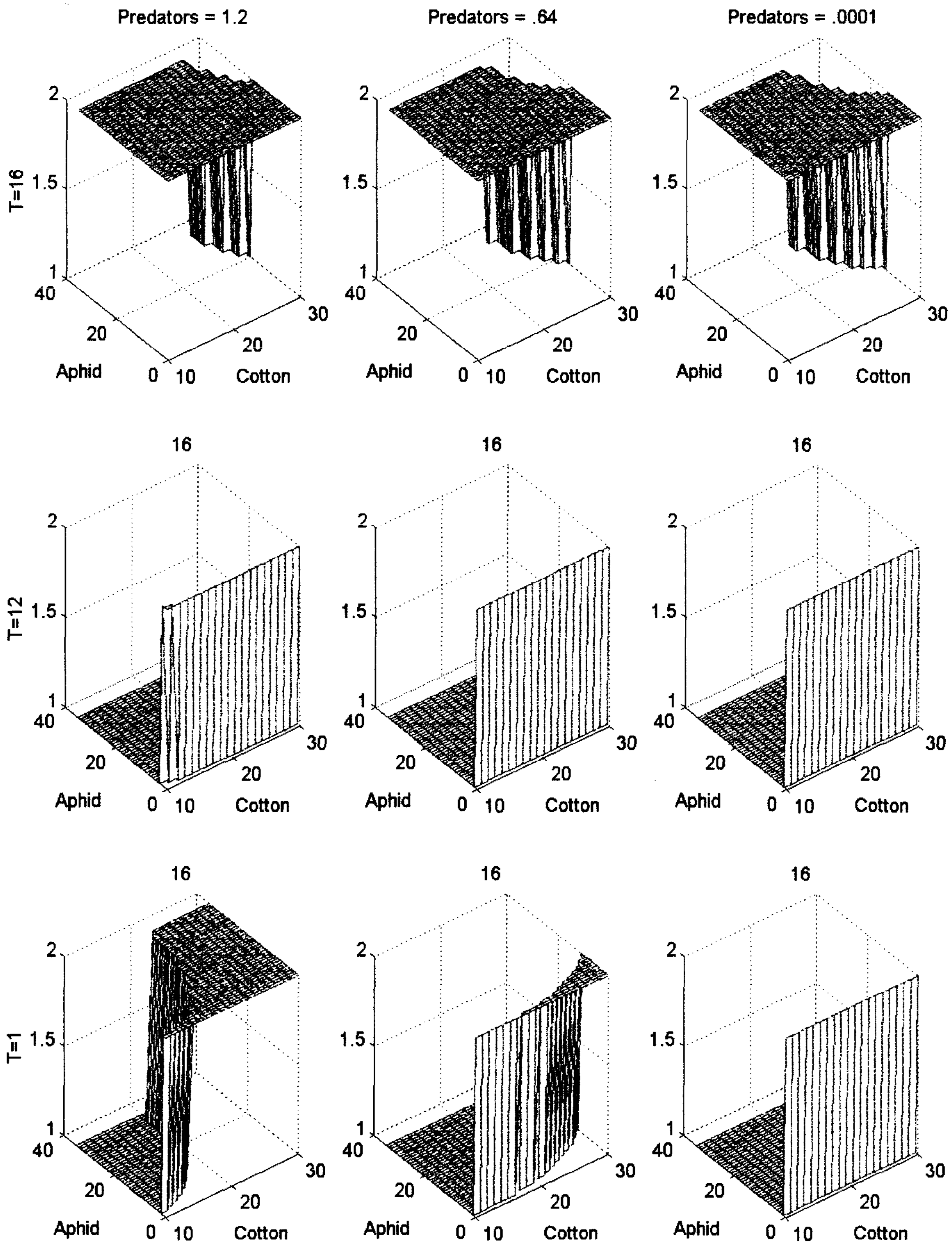

Spray Price = 3; Policies for Small Number of Predators

Figure 1