An LMI Approach for Robust Stability of Linear Uncertain Systems with Time-Varying Multiple State Delays

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Abstract-This paper provides new stability criteria for a class of uncertain linear time-delay systems with time-varying delays. Based on Lyapunov-Krasovskii functionals combining with LMI techniques, improved delay-dependent robust stability criteria, which are given in terms of quadratic forms of state and LMI, are derived. Our results shown by an example are less conservative than the existing stability criteria.

IndexTerms- linear matrix inequality, Lyapunov-Krasovskii functional, uncertainty.

I. Introduction

A major subject in the analysis of linear dynamical systems with uncertain time-delay is related to the stability. The criteria for asymptotic stability of such systems can be classified as delay-independent, which do not include any information on the size of delay, for example, [1]-[2], these results obtained via Lyapunov and Riccati equation approach independent of the size of the delays, or delay-dependent, which include such information on the size of delay, for example [3]-[5]. The stability criteria have been proposed [6]-[7] via LMI approach. The delay-dependent robust stability criteria, which are included some suppleementary conditions, are developed in [6] by Lyapunov function method, Leibniz-Newton formula and matrix norm. The delay-dependent criteria have also been addressed in the time-varying delay case in [8].

This paper deals with robust stability criteria for a class of uncertain time-delay systems with time-varying multiple state delays. The system parameter uncertainties are unknown but bounded, and the delays are unknown time varying terms. Based on Lyapunov -Krasovskii functionals combined with LMI techniques, we obtain new and improved delay-dependent stability criteria.

II. System Description

Consider the following uncertain time-delay systems described by

$$\dot{x}(t) = (A_0 + \Delta A_0(x,t))x(t) + \sum_{i=1}^{k}(A_i + \Delta A_i(x,t))x(t - h_i(t)) \qquad (1)$$

$$x(t) = \phi(t), \quad \forall t \in [-h, 0] \qquad (2)$$

where $X(t) \in R^n$ is the state vector and $A_j, j = 0, 1, \ldots, k$ are known constant matrices with appropriate dimensions, $\Delta A_j, j = 0, 1, \ldots, k$ are matrix functions representing the uncertainties in the matrices $A_j, j = 0, 1, \ldots, k,$

$$\Delta A_j(x,t) = D_j F_j(x,t) E_j, \; j = o, 1, \ldots, k \qquad (3)$$

where $F_j(x,t) \in R^{k_j \times g_j}$ are unknown real time-varying matrices with Lebesgue measure elements bounded by

$$F_j^T(x,t) F_j(x,t) \le I, \forall t, \; j = 0, 1, \ldots, k \qquad (4)$$

$h_i, i = 1, \ldots, k$ are the unknown time varying delay terms, but bounded $0 \le h_i(t) \le h$, $\dot{h}_i(t) \le d < 1$, $\phi(t)$ is a smooth vector-valued initial function in $-h \le t \le 0$. The main aim of this paper is to develop delay-dependent conditions for robust stability of the uncertain time-delay system (1). More specifically, our objective is to determine bounds for the time-delay by using different arrangements of Lyapunov-Krasovskii functionals and LMI methods.

The following matrix inequality will be essential for the proofs.

Lemma [6]:

Let D, E and F be real matrices of appropriate dimensions with $\|F\| \le 1$, then we have the following:

$$DFE + E^T F^T D^T \le \varepsilon^{-1} D D^T + \varepsilon E^T E, \qquad (5)$$

for any scalar $\varepsilon > 0$.

III. Main Results

In this section, we describe our method for determining the robust stability of uncertian time-delay system (1)-(3). The main results are given in the following theorems.

Theorem 1: Consider the uncertain delay system (1), for all delays $h_i \in [0, h]$, this system is robustly stable if there exist symmetric and positive-definite matrices $P > 0$, $R_i > 0, i = 1, \ldots, k$ and scalar, $\varepsilon_j > 0, j = 0, 1, \ldots, k$, such that the following LMI holds:

$$\begin{bmatrix} M & G \\ G^T & L \end{bmatrix} < 0, \qquad (6)$$

$$M = (A_0 + \alpha I)^T P + P(A_0 + \alpha I) + \sum_{i=1}^{k} R_i + X D_0 D_0^T P + J_0 + \sum_{i=0}^{k} X_i e^{2\alpha h} D_i D_i^T P$$

$$G = \begin{bmatrix} e^{\alpha h} P A_1 & \cdots & e^{\alpha h} P A_k \end{bmatrix}$$

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$$L=-diag\left[(1-d_1)R_1-J_1,\ .\ .\ .,(1-d_k)R_k-J_k\right]$$

$$X=\varepsilon_0^{-1}P,\quad X_i=\varepsilon_i^{-1}P,\ J_i=\varepsilon_i E_i^T E_i,\ i=0,\ldots,k$$

Remark 1:

If we let k=1, then the system (1) is a single state delay, the (6) can be transformed into (7) as LMI problem on a single state delay

$$\begin{bmatrix} M_0 & e^{\alpha h} P A_l \\ e^{\alpha h} A_1^T P & -((1-d)R-J_1) \end{bmatrix} < 0, \qquad (7)$$

$$\begin{aligned} M_0 = {} & (A_0+\alpha I)^T P + P(A_0+\alpha I) + R \\ & + X D_0 D_0^T P + J_0 \\ & + e^{\alpha h} X_1 D_1 D_1^T P e^{\alpha h} \end{aligned}$$

$$\varepsilon_0>0, \varepsilon_1>0, P, R>0 \ ,\ J_0=\varepsilon_0 E_0^T E_0 \ ,\ J_1=\varepsilon_1 E_1^T E_1 \ ,$$
$$X=\varepsilon_0^{-1}P, X_1=\varepsilon_1^{-1}P$$

Theorem 2: Consider the uncertain delay system (1), for all delays $h_i \in [0,h]$, this system is robustly stable if there exist symmetric and positive-definite matrices $P>0, R_i>0, Q_{i0}>0,\ i=1,\ldots,k$ and scalar $\varepsilon_j>0,\ j=0,\ldots,k$, such that the following LMI holds:

$$\begin{bmatrix} M_1 & G \\ G^T & L \end{bmatrix} < 0, \qquad (8)$$

$$\begin{aligned} M_1 = {} & (A_0+\alpha I)^T P + P(A_0+\alpha I) + \\ & \sum_{i=1}^{k} R_i + X D_0 D_0^T P + J_0 + \\ & \sum_{i=0}^{k} e^{2\alpha h} X_i D_i D_i^T P + \sum_{i=1}^{k} h\ Q_{i0}. \end{aligned}$$

$$G=\left[e^{\alpha h}\ P A_1\ \ .\ .\ .\ \ e^{\alpha h}\ P A_k\right]$$

$$L=-diag\left[(1-d_1)R_1-J_1,\ .\ .\ .,(1-d_k)R_k-J_k\right]$$

$$X=\varepsilon_0^{-1}P, X_i=\varepsilon_i^{-1}P,\ \ J_k=\varepsilon_i E_i^T E_i, i=0,\ldots,k$$

Remark 2:

If we let k=1, then the system (1) is a single state delay, the (8) can be transformed into (9) as LMI problem on a single state delay

$$\begin{bmatrix} M_2 & e^{\alpha h} P A_l \\ e^{\alpha h} A_1^T P & -((1-d)R-J_1) \end{bmatrix} < 0, \qquad (9)$$

$$\begin{aligned} M_2 = {} & (A_0+\alpha I)^T P + P(A_0+\alpha I) + R + \\ & X D_0 D_0^T P + J_0 + \\ & e^{2\alpha h} D_1 D_1^T P + hQ \end{aligned}$$

$$P>0,\ R>0,\ Q>0,\ \alpha>0,\ \varepsilon_0>0,\ \varepsilon_1>0, X=\varepsilon_0^{-1}P,$$
$$X_1=\varepsilon_1^{-1}P, J_0=\varepsilon_0 E_0^T E_0, J_1=\varepsilon_1 E_1^T E_1$$

Remark 3:

The delay term appears in (8), if $\alpha=0$ then the stability condition is still dependent on delay. Theorem2 improves theorem1 with this $\alpha=0$ problem for not dependent on delay.

IV.Example

Consider the following uncertain time delay system provided in [6], [9]:

$$\dot{x}(t)=[A_0+\Delta A_0(t)]x(t)+[A_1+\Delta A_1(t)]x(t-h(t)) \qquad (10)$$

$$A_0=\begin{bmatrix} -2 & 0 \\ 1 & -3 \end{bmatrix}, A_1=\begin{bmatrix} -1 & 0 \\ -0.8 & -1 \end{bmatrix}$$

and $\Delta A_0(t)$ and $\Delta A_1(t)$ are uncertain matrices satisfying

$$\|\Delta A_0(t)\| \le 0.2,\ \ \|\Delta A_1(t)\| \le 0.2, \qquad \forall t. \qquad (11)$$

$$D_0=D_1=\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, E_0=E_1=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \qquad (12)$$

Applying Theorem 1 to this uncertain time-delay system, it is found, using the software package LMI Lab, that this system is robustly stable for any time varying time-delay satisfying $0 \le h(t) \le 3.0073$, Theorem 2 then satisfying $0 \le h(t) \le 3.0369$. We note that the result of [6] guarantees the robust stability of (41) when $0 \le h(t) \le 0.0433$, whereas by the method of [9] the time-delay is $0 \le h(t) \le 0.3188$. This example shows that the method of this paper is an improvement of the previous results of [6] and [9].

V.Conclusion

This paper deals with the problem of robust stability criteria for a class of uncertain linear time-delay systems. Based on Lyapunov-Krasovskii functionals combining the LMI techniques, new and improved delay-dependent stability criteria are derived. An example is illustrated to show that the criteria perform much better than existing stability criteria.

References

[1] H. Trinh and M. Aldeen, "On the stability of linear systems with delayed perturbations,"IEEE Trans. Auto. Control, vol. 39, pp. 1948-1951, 1994.

[2] J.H. Kim, "Robust stability of linear systems with delayed perturbations,"IEEE Trans. Auto. Control., vol.41(12), pp. 1820-1822, 1996.

[3] S.I. Niculescu," H_∞ memoryless control with and α -stability constraint for time-delay systems: an LMI approach,"IEEE Trans. Auto. Control. Vol.43, no.5, pp.739-743, 1998.

[4] T. J. Su and C.G. Huang, "Robust stability of delay-dependence for linear uncertain systems,"IEEE. Trans. Auto. Control., vol.37, no.10, pp.1656-1659, 1992.

[5] Y.Y. Cao, Y.X. Sun and C. Chang, "Delay-dependent robust stabilization of uncertain systems with multiple state delays,"IEEE Trans. Auto. Control.,vol.43, no.11, pp. 1608-1612, 1998.

[6] X. Li and C.E. de Souza, "Criteria for robust stability and stabilization of uncertain linear systems with state delay,"Automatica., vol.33, pp.1657-1662, 1997.

[7] S. Boyd, L. Ei Ghaoui, E. Feron and V. Balakrishnan, Linear matrix inequalities in system and control theory, PA:SIAM, philadelphia, 1994.

[8] S.I. Niculescu, C.E. de Souza, L. Dugard, J.M. Dion, "Robust exponential stability of uncertain systems with time-varying delays,"IEEE Trans. Automat. Control, vol. 43, pp. 743-748, 1998.

[9] J.H. Su,"Further results on the robust stability of linear systems with a single time delay,"Syst. Control.Lett., vol. 23, pp.375-379, 1994.