Input-Output Passive Framework for Delay Systems

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Abstract

Multivariable linear systems with several delays are considered. Delays are allowed in the state as well as in the input or the output. Passivity and dissipativity of such systems are considered both in the input-output and state space framework in connection with hyperstability.
In the input-output framework necessary and sufficient conditions of passivity and hyperstability of delay systems are given in the language of positive real functions. Furthermore, *necessary and sufficient* conditions for positive realness are derived for a class of meromorphic functions describing delay systems.

1 Introduction

The effects of time delays in dynamical and control systems are contradictory. Usually, the time delay is associated with induced oscillations, instability or at least bad performance(see, e.g. Malek-Zavarei and Jamshidi [16] or Kolmanovskii and Myshkis [10]). But it is known since many years ago that delay may stabilize [5] and recent results confirm it [11]. As it is well known, time delay systems are infinite dimensional. For this reason many of the usual methods in the analysis and design of *finite-dimensional* control systems have to be modified appropriately.

Passivity theory is now an emerging design tool in control (see, e.g. [28], [29], [15]), and it has its roots in circuit theory(e.g.[1], [4]). Indeed, an energy-based analysis is easier to perform when dealing with electrical circuits or mechanical systems.

The passivity-based control is a direct consequence of the fact that the feedback connection of two passive (or dissipative) systems is also passive (dissipative). It is worth mentioning that the same property holds for two hyperstable systems [24]. In fact, as pointed out in the preface of [24], the entire hyperstability theory was constructed *"starting from the usual approach of the control engineer who would like to have at his disposal various dynamical blocks to combine in various connections without being bothered by possible stability loss"*. As a final step of this theory construction the cited reference [24] provides in its Appendix D "the list of main continuous time hyperstable blocks". Passivity-based control design has important robustness properties. Indeed, the plant parameters may change without affecting the closed-loop stability provided that the feedback interconnection scheme is composed of passive blocks.

The purpose of this paper is to extend such robustness properties for some classes of delay systems. Furthermore, we will revise the notions of passivity, dissipativity and hyperstability. It is worth mentioning that such attempts within passivity theory already exist, to cite but a few [13], [14], [30], [21]. In fact the present paper may be viewed as a continuation of [21]. It is organized as follows: some basic concepts are revisited and their interconnections presented. Then input-output results and some applications are discussed in Section 3.

2 Basic concepts

This paper is built around the following notions: dissipativity, passivity and hyperstability. They have to be inserted in the systems and control theory framework which has two quite independent aspects: the input-output (systemic, black-box) aspect and the state-space (Newtonian) aspect.

2.1 The input-output framework

We shall use here the basic framework in the form of [20] that may be found under various forms in more recent references (e.g. [31]). The *system* is considered to have m inputs and p outputs and *we identify the system with the set of signal pairs (u,y)* where $u \in \mathcal{U}_e^m, y \in \mathcal{U}_e^p$. Here $\mathcal{U}_e^m$ and $\mathcal{U}_e^p$ are extended spaces associated to the pre-Hilbertian spaces $\mathcal{U}^m$ and $\mathcal{U}^p$. The elements of these last spaces are vector signals(time functions) $u : \mathbf{R} \to \mathbf{R}^m$ and $y : \mathbf{R} \to \mathbf{R}^p$ respectively. In most cases $\mathcal{U}^m$ and $\mathcal{U}^p$ are $\mathcal{L}_m^2(\mathbf{R})$ and $\mathcal{L}_p^2(\mathbf{R})$ respectively.

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Let K, L, M be constant real matrices with K and M being symmetric. The dimensions of these matrices are such that the following integral has sense:

$$\eta(t_0, t_1) = \int_{t_0}^{t_1} (u^*(t)Ku(t) + 2u^*(t)L^*y(t) + y^*(t)My(t))dt, \quad (1)$$

the integration being performed along the input-output pairs of the system (this shows in fact that the input-output pairs have to be at least locally square integrable).

Definition 1 *It is said that the system is (K, L, M)-dissipative or dissipative with respect to the* supply rate*:*

$$w(u, y) = u^*Ku + 2u^* * L^*y + y^*My,$$

if $\eta(t_0, t_1) \geq 0$ for all real $t_0, t_1, t_1 > t_0$ and all input-output pairs of the system.

As pointed out in [20] if we choose $K = \gamma^2 I, L = 0, M = -I$ with the identity matrices of appropriate dimensions and $\gamma > 0$ some fixed real the above Definition 1 reduces to

$$\int_{t_0}^{t_1} |y(t)|^2 dt \quad \leq \quad \gamma^2 \int_{t_0}^{t_1} |u(t)|^2 dt \qquad (2)$$

nothing more but finite-gain input-output stability. If $m = p$ i.e.*the system is "square"* and the choice for the supply rate is $K = 0, M = 0, L = \frac{1}{2}I$ the above definition will give

$$\int_{t_0}^{t_1} u^*(t)y(t)dt \quad \geq 0 \qquad (3)$$

nothing more but passivity of the considered system.

On the other hand inequality (3) is the so-called *hyperstability at large* [24] provided $t_0 = 0$ (in fact this property was called hyperstability at large only in the Romanian edition of the book in 1966; afterwards it is simply called property (h_{p0}) but we think that for our purposes it is more useful and more suggestive the first name).

As it will be seen in the sequel the property of passivity(or hyperstability at large) is equivalent, even in the input-output framework to *stronger* qualitative properties as *hyperstability in the strict sense* that allow deduction of stability properties for feedback connections of systems.

2.2 The state space framework

It is interesting to point out that the input-output framework for dissipativity originates from circuit theory. However, if we want to get closer to energy concepts even in the abstract(non-physical) sense we need a state space framework since the *energy* is a *state function*.

The state space approach is (generally, even philosophically speaking) quite independent from the input-output one. The state realization theory for linear time invariant systems with lumped parameters establishes the rigorous equivalence of the two approaches in that case. In the case of distributed parameters however the input-output approach seems more intuitive: in fact, as pointed out by Willems [33] *"the system is always described by the relations between signals at a finite number of terminals regardless how complicated the internal description may be"*.

If this philosophy is taken into account, then it appears as obvious that the state space framework should be constructed as close as possible to the input-output one. This line has been taken in several papers and we cite a few [32] [3] [34] and **Assumption 1** of [20] well summarizes the state of the art of this approach. We shall not reproduce here this **Assumption 1** but just point out that for dissipativity studies(as well as for hyperstability and stability ones) the choice of the state space $\mathcal{X}$ within the Assumption is not an easy task since all results will be obtained in the sense of that state space topology. Not only should it be a *metric space* as pointed out in [20] but *it is useful to be a Hilbert space in order to be able to apply - in the linear case - the complex domain methods and to use quadratic storage(i.e.Liapunov) functions.*

In the state space framework the definition of dissipativity(and, therefore, of passivity) is chosen the same as in the input-output case, starting from the same supply rate and with the same inequality, but with the additional condition that it holds for the relaxed(i.e. zero initial state) system.

The **Lemma 1** of [20] is crucial for the state space framework theory and we reproduce it here.

Lemma 1 *If a system satisfying Assumption 1 of [20] is dissipative with respect to the supply rate $w(u, y)$ then there exists a storage function $v : \mathcal{X} \to \mathbf{R}_+$ (in fact a functional defined on the state space $\mathcal{X}$) such that the following* **dissipativity inequality** *holds*

$$v(x(t_1)) \leq v(x(t_0)) + \int_{t_0}^{t_1} w(u(t), y(t))dt \qquad (4)$$

For zero input systems(provided $u(t) \equiv 0$ is an admissible input) the above inequality is a Liapunov like inequality that gives stability of the *equilibrium at* 0 (whose existence is also a consequence of dissipativity - or hyperstability - see [24], Proposition 8 of Section

13). Moreover, if we have the storage function satisfying

$$\alpha(\|x\|) \le v(x) \le \beta(\|x\|)$$

where $\|\cdot\|$ is the norm on $\mathcal{X}$ and α, β are *Massera functions* (the class $\mathcal{K}$-functions of stability theory - see the book of Hahn [7]) then (4) implies

$$\alpha(\|x(t_1)\|) \le \beta(\|x(t_0)\|) + \eta(t_0, t_1) \tag{5}$$

This is exactly the early form of the strict hyperstability property and looks like the sufficient conditions for hyperstability as expressed in ([24], Proposition 4 of Section 13).

It will be seen in the sequel that for linear systems some sufficient conditions may become necessary and sufficient and weak and strong properties may be equivalent.

3 Passivity in the input-output framework

We shall start here with systems described by an input-output equation of the form

$$y(t) = r(t) + \int_0^t K(t-\tau)u_+(\tau)d\tau \tag{6}$$

where $u_+ : \mathbf{R}_+ \to \mathbf{C}^m, K : \mathbf{R}_+ \to \mathbf{C}^{m \times m}$ and $r : \mathbf{R}_+ \to \mathbf{C}^m$ are piecewise continuous. This is a rather general input-output framework for linear systems: for instance, usual lumped parameter linear systems described by

$$\dot{x} = Ax + Bu(t) \quad , \quad y = Cx \tag{7}$$

may be given form (6) *via* the Cauchy formula of the variations of constants which gives

$$K(t) = Ce^{At}B \quad , \quad r(t) = Ce^{At}x_0 \tag{8}$$

thus showing the connection of $r(t)$ with the initial state of the system.

If we consider time delay systems of the form

$$\begin{cases} \dot{x} = \sum_{i=0}^{r}[A_i x(t-\tau_i) + B_i u(t-\tau_i)] \\ y(t) = \sum_{i=0}^{r} C_i x(t-\tau_i), \end{cases} \tag{9}$$

with the convention that $0 = \tau_0 < \tau_1 < ... < \tau_r$. Note that this representation is quite general since the matrix coefficients (A_i, B_i, C_i) may be zero for some delay values. Furthermore, there are no other constraints on the delays. The discrete-delays systems considered in [21] are a special case of (9), etc.

Then (9) can be rewritten in the form (6) with

$$\begin{aligned} r(t) &= (\sum_0^r C_i X(t-\tau_i))x_0 \\ &+ \sum_{i=0}^{r}\sum_{j=1}^{r}\int_{-\tau_j}^{0} C_i X(t-\tau_i-\tau_j-\theta)B_j\psi_0(\theta)d\theta \\ &+ \sum_{j=1}^{r}\int_{-\tau_j}^{0}(\sum_{i=0}^{r} C_i X(t-\tau_i-\tau_j-\theta)B_j)u_0(\theta)d\theta, \end{aligned} \tag{10}$$

and

$$K(t) = \sum_{i=0}^{r} K_i(t) \tag{11}$$

where K_i are defined by:

$$K_i(t) = \begin{cases} \sum_{j=0}^{r} C_j X(t-\tau_i-\tau_j)B_i \quad , \quad t \ge \tau_i \\ 0 \quad , \quad 0 < t < \tau_i \end{cases}$$

Here $X(t)$ is the state transition matrix of (9) defined by

$$\begin{aligned} \dot{X} &= \sum_0^r A_i X(t-\tau_i) \quad , \quad t > 0 \\ X(t) &\equiv 0 \quad , \quad t < 0 \quad , \quad X(0) = I \end{aligned} \tag{12}$$

and may be constructed by steps. The couple $(x_0; \psi_0(\theta), -\tau_r \le \theta < 0)$ defines the initial state while $u_0(\theta), -\tau_r \le \theta < 0$ defines the initial control function.

In the sequel we shall assume that $r(t)$ in (6) is such that there exists $t_0 < 0$ and $u_- : (t_0, 0) \to \mathbf{C}^m$ piecewise continuous in order that

$$r(t) = \int_{t_0}^{0} K(t-\tau)u_-(\tau)d\tau \quad , \quad \forall t > 0 \tag{13}$$

Remark 1 *Notice that this is a* Fredholm-type integral equation *of 1st type which has a (non-unique) solution only under special assumptions. If $r(t)$ and $K(t)$ are given by (8) i.e. in the lumped-parameter case then (13) has a solution provided (A, B) is a* controllable pair *[24] (Appendix A).*

For systems with time lags this problem has not been fully investigated: some early results were obtained in [25]: further investigation is performed in [18] [22] [17] [19].

Due to (13) we may write (6) as

$$y(t) = \int_{t_0}^{t} K(t-\tau)u(\tau)d\tau \tag{14}$$

with

$$u(t) = \begin{cases} u_+(t) & , \quad t > 0 \\ u_-(t) & , \quad t_0 < t < 0 \end{cases} \tag{15}$$

The kernel $K(t)$ will be extended to the entire real axis as follows

$$K(t) = K^*(-t) \quad , \quad detK(0) \neq 0 \tag{16}$$

where $*$ denotes transpose and complex conjugate.

As in *passivity theory* we introduce

$$\begin{aligned} \eta(t_0, t) &= Re \int_{t_0}^{t} u^*(\tau) y(\tau) d\tau \\ &= \frac{1}{2} \int_{t_0}^{t} [u^*(\tau) y(\tau) + y^*(\tau) u(\tau)] d\tau \end{aligned} \tag{17}$$

the only difference with respect to Section 1 being that we allow *complex signals*. We may state now

Theorem 1 *Consider the system (14) with its matrix kernel $K(t)$ satisfying (16). Then the following properties are equivalent*

1^o *System (14) is* **hyperstable** *in the sense that there exists some real $\delta_0 > 0$ such that for any input-output pair $(u(t) \quad , \quad y(t))$ of (14) the inequality*

$$\eta(t_0, t) \geq \delta_0 |y(t)|^2 \tag{18}$$

holds for all $t \geq t_0$.

2^o *There exists some real $\delta > 0$ such that for any input-output pair $(u(t) \quad , \quad y(t))$ of (14) and any $\gamma \geq 0$ satisfying*

$$\eta(t_0, t) \leq \gamma^2 + \gamma \cdot \sup_{0 \leq \tau \leq t} |y(\tau)| \quad , \quad \forall t \geq 0 \tag{19}$$

the following inequality

$$|y(t)| \leq \delta(\gamma + \sqrt{|\eta(t_0, 0)|}) \tag{20}$$

holds for any $t \geq 0$.

3^o *The matrix transfer funcion $T(s)$ of (14) defined as the Laplace transform of the kernel $K(t)$ is* **positive real** *i.e.$T(s) + T*(s) \geq 0$ (a nonnegative definite matrix) for any s such that $Re(s) >$* 0.

4^o *The kernel matrix $K(t)$ is* **of positive type in the sense of Bochner** *i.e. the matrix*

$$S(N) = \sum_1^N \sum_1^N \alpha_m K(t_m - t_n) \overline{\alpha_n} \tag{21}$$

is nonnegative definite for any positive integer N, any reals $t_1, ..., t_N$ and any complex numbers $\alpha_1, ..., \alpha_N$.

5^o *The system is* **passive** *i.e. the inequality*

$$\eta(t_0, t_1) \geq 0 \tag{22}$$

holds for all $t_1 \geq t_0$ and all input-output pairs of (14).

We shall not insist on the proof of this result that was obtained for $m = 1$ in [23] [24] and in the multivariable case $m > 1$ in [8]. Instead we shall point out some significant aspects of its statement.

The *passivity property* 5^o has the last position in the assertion sequence because it seems to be the *"weakest"* property. Nevertheless, it has very broad applications in stability and stabilization. This is due to its equivalence with the *"much stronger"* property 1^o which is *hyperstability in the strict sense.* As mentioned in the Introduction, passivity stands in the framework of [24] as *hyperstability in the broad sense.* This *equivalence of "weak" and "strong" properties* is an interesting feature of this theorem. To strengthen this remark we just mention that the proof scheme follows the line

$$1^o \to 2^o \to 3^o \to 4^o \to 1^o$$
$$1^o \to 5^o \to 3^o$$

The implication $1^o \to 5^o$ is obvious and quite normal (a stronger property implies a weaker one). Then $5^o \to 3^o$ i.e. *passivity implies real positivity* of the transfer function is also expectable. But the cycle of equivalences closes with two "instrumental properties", property 2^o that is useful in proving boundedness and stability and property 4^o which is in fact a very deep mathematical result. In the engineering language it means that *a positive real transfer function is associated to a weighting pattern of positive type in the sense of Bochner*; this connection seems to be not very exploited up to now even in the lumped parameter case.

3.1 On positive realness of the transfer function

Positive real functions were considered in Circuit Theory [4] [6] in the rational case i.e. when they were ratios of polynomials; this is also the framework of Appendix C of [24]. Nevertheless, as mentioned in a footnote of page 410 in [6] "it is not necessary that the function be rational although... ". If we search practical criteria for positive realness in the distributed parameter case we need knowledge about other than rational functions of complex variables.

The class of non-rational functions is quite rich and the cornerstone in its analysis is the problem of the singularities. In order to simplify our analysis we shall restrict ourselves to *meromorphic functions* which are somehow *the infinite dimension analogue of the rational functions.*

Moreover we shall consider only those meromorphic functions that correspond to the transfer functions of delay systems (9). These functions are ratios of exponential polynomials (also called quasi-polynomials) and it is known that in this case the principal term of the denominator has constant coefficient. Consequently (see, for instance, [2], Chapter 12) the number of the roots at the right of any vertical line is at most finite. We may state now

Theorem 2 *Let $H(s) = n(s)/d(s)$ be a meromorphic function with $n(s)$ and $d(s)$ quasi-polynomials whose coefficients may be complex and whose principal term(the highest power of s) has constant coefficient. Then the following properties are equivalent*

(A) The function $H(s)$ is positive real in the sense that $ReH(s) \geq 0$ for all s such that $Re(s) > 0$ and $H(s)$ is well defined.

(B) There is no s with $Re(s) > 0$ such that $sH(s)$ and $H(s)/s$ should be finite, real and strictly negative.

(C) The frequency domain inequality $ReH(j\omega) \geq 0$ holds for any real ω such that $H(j\omega)$ is well defined. Besides this the following conditions hold

(a) The function $H(s)$ has no poles with $Re(s) > 0$.

(b) On the imaginary axis $j\mathbf{R}$ the poles of $H(s)$ may be at most simple and with real positive residues.

The proof of this theorem is quite involved, even in the case of rational functions [24] (Appendix C).

Remark 2 *The proof can be found in the full version of the paper [27], in which there are pointed out the differences with respect to [24] and especially, the role of the specific assumptions on the meromorphic function.*

We shall end this section with a few comments about the class of linear systems that may be analyzed via the above theorems.

Remark 3 *Since no condition is mentioned concerning the weighting pattern kernel $K(t)$ - usually it is required that at least $K \in \mathcal{L}^1(0,\infty)$ - we deduce that not only input-output stable blocks are allowed but also - remember that K was assumed bounded - critical blocks i.e. with nonvoid intersection of the spectrum with the imaginary axis $j\mathbf{R}$.*

For instance, we can consider a time delay system with a finite dimensional "critical part" [9]; its transfer function may have the form

$$H(s) = \frac{\gamma}{s} + \sum_{1}^{p} \frac{\alpha_k s}{s^2 + \omega_k^2} + \frac{r(s)}{p(s)}$$

where $r(s)$ and $p(s)$ are quasi-polynomials satisfying the assumption about the principal term and with the zeros of $p(s)$ in the left half-plane. It is easy to see that (C) holds provided $\gamma > 0$, $\alpha_k > 0$ and $Re[r(j\omega)/p(j\omega)] \geq 0 \forall \omega \in \mathbf{R}$.

Remark 4 *Even if we cannot separate the critical part from the stable one, we still may check real positiveness according to Theorem 2. Moreover, Theorem 1 offers an equivalence between real positiveness in some critical cases and passivity and other properties in the time domain.*

Remark 5 *Other examples can be found in [27]. Note that connections with the time-domain extending [21] are also given in the full version of the paper [27].*

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