LMI-BASED STATE ESTIMATION OF A CLASS OF UNCERTAIN NONLINEAR STOCHASTIC SYSTEMS

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Abstract: A general class of discrete-time uncertain nonlinear stochastic systems corrupted by finite energy disturbances is considered. Linear state estimators are designed according to a variety of performance criteria which include guaranteed-cost suboptimal versions of estimation objectives like H_2, H_∞, stochastic passivity, etc. A common matrix inequality formulation is used in characterization of estimator design equations.

1. Introduction

In the present work, the problem of state estimator design is formulated using linear matrix inequalities (LMI) for a general class of uncertain nonlinear stochastic systems. The purpose behind this approach is the possible utilization of efficient numerical schemes for solving LMI [1]. In the signal model used, the system and measurement vectors are assumed to be corrupted by additive noises with finite energy and the same vectors are also assumed to be affected by white noises whose powers are determined by unknown nonlinear functions of the state. Such nonlinear models are introduced in [2], and their system theoretic properties are investigated using LMI approach in [3]. Mean-square optimal control and state estimator designs can be found in [4] and [5], respectively. In the present work, various estimation problems including guaranteed cost suboptimal versions of H_2, H_∞, stochastic passivity, etc. are tackled within a common framework. In that sense, the present work can be viewed as an extension of minimum-variance results of [5] to the case of generalized performance criteria.

2. Signal Model and Estimator Design

We assume that the signal is generated by the following system and the measurement equations

$$x_{k+1} = Ax_k + Bw_k + f_k \tag{1}$$

$$y_k = C_y x_k + D_y w_k + g_k \tag{2}$$

where the state $x_k \in \mathbf{R}^n$ and the measurement $y_k \in \mathbf{R}^p$. $w_k \in \mathbf{R}^m$ is a white (timewise uncorrelated) stochastic l_2-type (finite energy) signal with

$$\sum_{k=0}^{\infty} E\{\|w_k\|^2\} \leq 1 \tag{3}$$

The initial state x_0 is assumed to have the known mean $E\{x_0\} = \bar{x}_0$, covariance $E\{x_0 x_0{}^T\} = X_0$, and to be uncorrelated with other noise sources. The nonlinear functions $f_k = f(x_k, v_k)$ and $g_k = g(x_k, v_k)$, where v_k is a zero-mean white noise, are defined by their statistical properties as follows:

$$E\{\begin{pmatrix} f_k \\ g_k \end{pmatrix} (f_j{}^T \quad g_j{}^T)\} = 0, \tag{4}$$

for all $k \neq j$ and

$$E_{x_k}\{\begin{pmatrix} f_k \\ g_k \end{pmatrix}\} = 0 \tag{5}$$

$$E_{x_k}\{\begin{pmatrix} f_k \\ g_k \end{pmatrix} (f_k{}^T \quad g_k{}^T)\} \leq \sum_{i=1}^{r} \begin{pmatrix} T_{11}{}^i & T_{12}{}^i \\ * & T_{22}{}^i \end{pmatrix} x_k{}^T M^i x_k \tag{6}$$

This description is quite general as shown in [2], [3], and [6].

We assume that the measurement sequence given by (2) is available and we design a full-order linear state estimator:

$$\hat{x}_{k+1} = A\hat{x}_k + K(y_k - C_y\hat{x}_k), \quad \hat{x}_0 = \bar{x}_0 \tag{7}$$

Due to the zero-mean nature of f_k and g_k, this estimator is an unbiased one. The use of this estimator leads to the estimation error $e_k = x_k - \hat{x}_k$ dynamics

$$e_{k+1} = (A - KC_y)e_k + (B - KD_y)w_k + f_k - Kg_k \tag{8}$$

The following main result summarizes our estimator design procedure:

Theorem 1. Consider the model (1) - (6), the performance output

$$z_k = C_z e_k + D_z w_k \tag{9}$$

the linear unbiased state estimator given by (7). If the following LMI holds

$$\begin{pmatrix} X_{11} & X_{12} \\ * & X_{22} \end{pmatrix} \geq 0 \tag{10}$$

where $X_{11} = X - A^T X A - I - \sum_{i=1}^{r} \mathrm{tr}[X T_{11}{}^i] M^i$, $X_{12} = -A^T X B$, $X_{22} = \gamma I - B^T X B$ for some $X > 0$ and $\gamma > 0$ and if it is true that

$$\begin{pmatrix} P_{11} & P_{12} & P_{13} \\ * & P_{22} & P_{23} \\ * & * & P_{33} \end{pmatrix} \geq 0 \tag{11}$$

where $P_{11} = P - C_z{}^T Q C_z$, $P_{12} = -C_z{}^T(QD_z + S)$, $P_{13} = A^T P - C_y{}^T J^T$, $P_{22} = -R - D_z{}^T Q D_z - D_z{}^T S - S^T D_z$, $P_{23} = B^T P - D_y{}^T J^T$, $P_{33} = P$, then, the estimation error dynamics given by (8) satisfy:

$$E\{e_N{}^T P e_N\} \leq E\{e_0{}^T P e_0\} - \sum_{k=0}^{N-1} E\{(z_k{}^T \quad w_k{}^T) \times \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} \begin{pmatrix} z_k \\ w_k \end{pmatrix}\} + \epsilon \frac{(1-\delta^N)}{(1-\delta)} \tag{12}$$

for any integer $N \geq 1$, where $K = P^{-1}J$ and the constants $\delta \in (0,1)$ and $\epsilon > 0$.

Theorem 1 given above allows one to design different estimators for a variety of performance criteria for this class of systems. For example, taking $Q = 0$, $S = 0$, and $R = -\mu I$, $\mu > 0$ yields $E\{e_N{}^T P e_N\} \leq E\{e_0{}^T P e_0\} + \mu + \epsilon(1-\delta^N)/(1-\delta)$. This means that by maximizing $\lambda_{\min}(P)$ and minimizing $\lambda_{\max}(P)$ and μ, we can obtain a tight bound on the mean-square estimation error.

By taking $Q = I$, $S = 0$, $R = 0$, $B = 0$, $D_y = 0$, and $D_z = 0$, we obtain $\sum_{k=0}^{N-1} E\{z_k{}^T z_k\} \leq E\{e_0{}^T P e_0\} + \epsilon(1-\delta^N)/(1-\delta)$ which yields a bound on the energy of the performance output in terms of the initial estimation error. This bound can be tightened by minimizing $\lambda_{\max}(P)$ which produces a suboptimal H_2 result in estimation.

If we set $Q = I$, $S = 0$, and $R = -\mu I$, $\mu > 0$, this produces $\sum_{k=0}^{N-1} E\{z_k{}^T z_k\} \leq \mu + \epsilon(1-\delta^N)/(1-\delta)$ which is a bound on the stochastic (mean-square) l_2 to l_2 gain of the estimator. By minimizing μ, we can get a tighter bound on the stochastic H_∞ norm of the estimator.

Several dissipative estimator designs are also possible using this formulation. For example, taking $e_0 = 0$, $Q = 0$, $S = -0.5I$, and $R = \mu I$, $\mu > 0$ will yield the stochastic (mean-square) version of the input strict passivity result $\sum_{k=0}^{N-1} E\{z_k{}^T w_k\} \geq \mu + \epsilon(1-\delta^N)/(1-\delta)$ Other similar dissipativity results are also possible. For example, setting $Q = 0$, $S = -0.5I$, and $R = 0$ will give stochastic passivity. Setting $Q = \nu I$, $\nu > 0$, $S = -0.5I$, and $R = 0$ will yield output strict passivity. Also, setting $Q = \nu I$, $S = -0.5I$, $R = \mu I$, ν, $\mu > 0$ will give strict passivity both in terms of the input and the output (very strict passivity in the mean-square sense). So, one can see that this LMI formulation allows one to consider a variety of performance criteria in a common framework.

3. References

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