

Input Shaping for a Simple Nonlinear System

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Abstract

Input Shaping is a command modification technique for reducing system vibration that has been implemented successfully on a number of mechanical systems. Input shaping was developed from linear system theory, but it has been applied successfully to many nonlinear systems. Of the approaches used to shape commands for nonlinear systems, very few address the specific nonlinearities of the systems directly. This paper addresses a special subset of configuration-dependent nonlinearities. The effect of input shaping on these types of systems is analyzed and key motion command parameters are extracted. This analysis is utilized to design effective zero-vibration input shapers for the class of nonlinear systems considered here.

1. Introduction

Input shaping is a command modification technique that has been used quite successfully to reduce residual vibration in a variety of mechanical systems. For example, residual vibrations of long-reach manipulators [1, 2], cranes [3-6], and silicon-handling robots [7] have all been significantly decreased with the use of input shaping. While many input shaping techniques have been derived primarily from linear system theory, there is much interest in the application of input shaping to nonlinear and time varying systems.

Input shaping is implemented by convolving a series of impulses with a reference command, as shown in Figure 1. If the impulse sequence causes no vibration, then the result of the convolution will also cause no vibration [8]. Such an impulse sequence is known as a zero-vibration (ZV) shaper. Techniques for applying input shapers to nonlinear systems can be roughly grouped into three categories: 1) built-in robustness approach, 2) adaptive input shaping approach, and 3) nonlinear input shaping approach.

1.1 Built-in Robustness Approach

Built-in robustness essentially imposes auxiliary constraints on the residual vibration to ensure low vibration when the frequencies and damping ratios of the system change due to nonlinearity. Singer and Seering were the first to include the concept of built-in robustness in their input shaper formulation [8]. Their robustness came from a requirement that the derivative of residual vibration with respect to frequency must be zero. The result is a zero vibration and derivative (ZVD) shaper that has been quite effective in dealing with moderate nonlinearity.

A number of other approaches for improving input shaper robustness have also been developed. In particular,

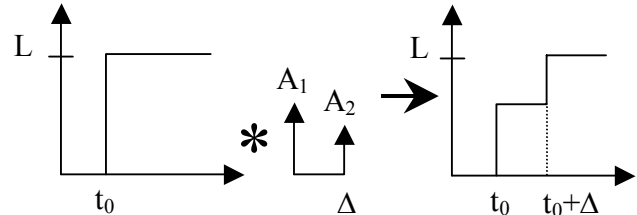


Figure 1: Convolution of Impulse Sequence with Reference Command.

the Extra Insensitive (EI) shaper exceeds the robustness of the ZVD formulation without increasing the shaper duration [9], and the Specified Insensitivity (SI) shaper allows the designer to specify any arbitrary range of system frequencies and damping ratios to be suppressed [10]. It should be noted that these approaches work well for moderate nonlinearity, but very robust shapers needed for highly nonlinear systems generally have long time durations and can make the system sluggish.

1.2 Adaptive Input Shaping Approach

Several researchers have used adaptive input shaper modification techniques to deal with system nonlinearity. These approaches use sensor feedback information to tune the input shaper such that the residual vibration is decreased. Some of these approaches include that of Tzes, Yurkovich, Khorrami, and Jain [11-12], who used on-line adaptive schemes to update the input shaper parameters. Bodson used a recursive least-squares technique to tune the input shaper parameters [13]. Magee and Book modified the input shaper as the system configuration changed [14].

More recently, Park and Chang used a learning scheme to update the input shaper parameters for repetitive motion tasks [15], while Rhim and Book developed an alternative adaptation scheme [16]. It should be noted that while these approaches can be successful in reducing residual vibration, sensor feedback is required, which may result in additional costs and difficulties when implementing.

1.3 Nonlinear Input Shaping Approach

Nonlinear input shaping is the use of the nonlinear system model to modify input shapers to reduce residual vibration. Much less work has been done in this area, but some approaches to date include [17] and [18].

Several objections to input shaping that have been raised over the years apply to these types of approaches. In particular, an accurate model is needed for these nonlinear schemes to be effective. However, a better understanding of how nonlinearities affect the choice of input shaper

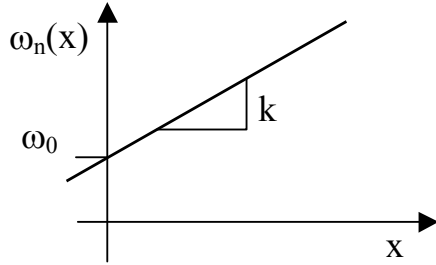


Figure 2: Linearized Natural Frequency Variation of Simple Nonlinear System.

parameters can only benefit the field. For instance, simple modifications of nonlinear shapers to improve robustness might be used to reduce the need for precise models.

2. Input Shaping on a Simple Nonlinear System

In this paper, we propose a simple nonlinear input shaping approach that generates a shaper by anticipating the nonlinear behavior of the system. In particular, we seek to apply input shaping schemes to a simple nonlinear oscillator – one with a natural frequency that varies linearly with position. The equation of motion of such a system is given by:

$$\ddot{x} + [\omega_n(x)]^2(x - x_{\text{des}}) = 0 \quad (1)$$

where $\omega_n(x) = \omega_0 + kx$, and x_{des} is the desired position. This system model is shown graphically in Figure 2.

Because even complicated functions can be locally linearized, such a model may be valid over some portion of the workspace of many machines. Additionally, the simple system is nonlinear in position. This nonlinearity in position is applicable to various mechanisms such as parallel manipulators under PD control [19]. For the baseline system to be studied, nominal values of $\omega_0 = 10$ and $k = 1$ are used.

The foremost goal of this research is to find a shaped command to move the system from an initial position, x_1 , to a final position, x_2 , with very little residual vibration. Figure 3 shows a sample response of a linear system to both shaped and unshaped reference commands. Residual vibration is eliminated through the use of a properly shaped command. This investigation attempts to find an input shaper that gives similar results for a system with nonlinear dynamics that can be approximated by (1).

2.1 Scope of Proposed Approach

To better understand how nonlinear systems behave when input shaping is applied, we analyze the dynamics of the simple nonlinear system described above. In particular, the factors that determine the percent residual vibration (*PRV*) are examined when a standard input shaper is applied to the system. The information obtained in the analysis is then used to more intelligently apply input shaping techniques to the system. Specifically, we seek to design better performing input shapers based on the following three approaches:

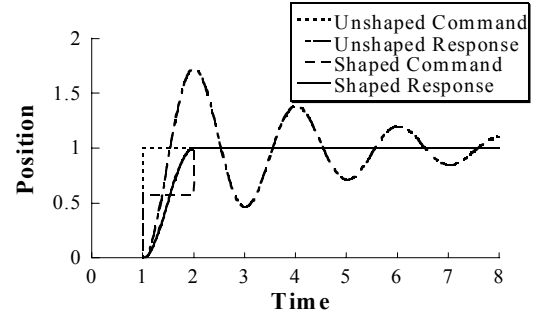


Figure 3: Linear System Response to Shaped and Unshaped Commands.

- 1) Built-in robustness (ZVD)
 - 2) Piecewise optimization
 - 3) Semi-analytical (using energy methods and simulation)
- The results of this investigation lead to the development of a novel input shaper that performs very well for the type of nonlinear system studied here.

2.2 Measuring Residual Vibration

The primary measure of input shaper performance used in this paper is the residual vibration resulting from a movement of the system. By measuring peak overshoot of the shaped response relative to that of the unshaped response, the effects of input shaping on the residual vibration resulting from movement of the system can be quantified. The percent residual vibration, *PRV*, as used in this paper is defined by:

$$PRV = \frac{\text{Peak Overshoot}_{\text{Shaped}}}{\text{Peak Overshoot}_{\text{Unshaped}}} (100 \%) \quad (2)$$

A *PRV* of 10 % would indicate that the response to the shaped command results in vibration that is 10 % of that resulting from an unshaped command.

2.3 Assumptions

Several assumptions are utilized in this investigation: 1) the dynamic system is defined or approximated by Equation 1, 2) the baseline reference commands are step functions, and 3) for simplicity, only forward moves are considered (i.e. moves in the positive direction along the x -axis).

3. End-Point-Frequency ZV Shaper

Generally, an input shaper is designed based on the system frequency and damping ratio. For a nonlinear system with the dynamics described by (1), the frequency and damping ratio are poorly defined. For the analysis performed in this section, the frequency used to compute the ZV shaper is the frequency of the system at the end of the maneuver (end-point frequency). The damping ratio will be set to zero because the system is conservative. In order to determine the effectiveness of designing shapers in this manner, the dynamic time response of the system must be examined over a wide range of conditions.

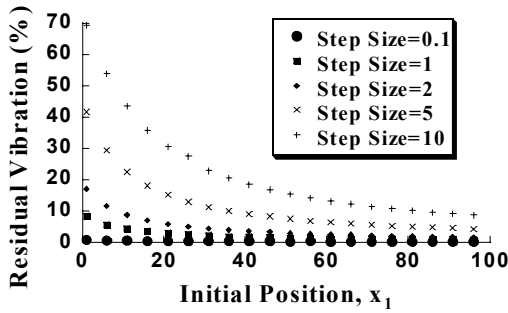


Figure 4: Residual Vibration for Constant Step Sizes.

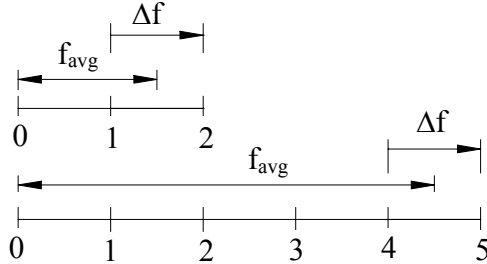


Figure 5: Example of Relative Frequency Moved (f_{rel}).

For the first phase of this investigation, the step size, $(x_2 - x_1)$ where x_1 is the initial position and x_2 is the final position, was held constant. The percent residual vibration (PRV) was measured for a range of starting positions. Results for several different step sizes are shown in Figure 4. The residual vibration is high for low values of the initial position (x_1) and steadily decreases as x_1 gets larger. This result suggests that step size does not uniquely determine PRV . One explanation for this behavior is that, at low values of x_1 , the natural frequency of the system is quite low. As a result, the change in natural frequency during the moves is very high. This result reinforces the notion that for a system to be approximately linear, any changes in system frequency (in time and/or position) must be small relative to a nominal frequency of the system.

These results suggest a new approach to display the performance – hold the relative percentage change in natural frequency constant. The relative frequency change is defined as:

$$f_{rel} = \frac{|f(x_2) - f(x_1)|}{0.5[f(x_2) + f(x_1)]} = \frac{\Delta f}{f_{average}} \quad (3)$$

where $f(x)$ represents the linearized natural frequency around the operating point x . As an illustration of f_{rel} , consider two different moves where the change in natural frequency is 1 Hz. Figure 5 shows this situation, where the top diagram is a move from 1 Hz to 2 Hz and the bottom is from 4 Hz to 5 Hz. Even though the absolute frequency change is 1 Hz in both cases, the frequency change relative to the average frequency is much larger in the low frequency move.

To test this new approach to performance evaluation, end-point-frequency ZV shapers were used while holding the relative frequency constant. Results for several

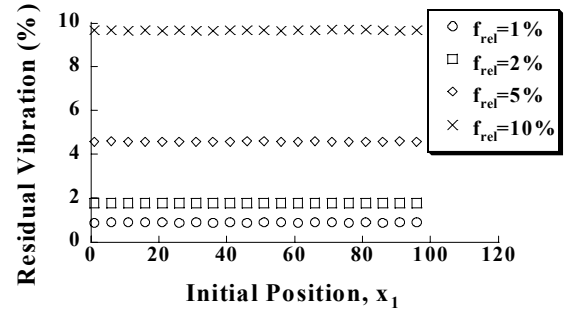


Figure 6: Residual Vibration for Constant f_{rel} .

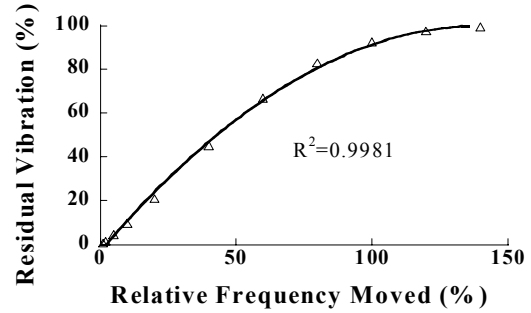


Figure 7: Residual Vibration as a Function of f_{rel} for ZV Shaper Designed for End-Point Frequency.

different relative frequency moves are shown in Figure 6. The percent residual vibration is essentially constant for a given relative frequency move. The standard deviation for each of the relative frequency moves was computed to be less than 0.5 %, which is within the bounds of measurement and simulation error. These results suggest that, for a given system and desired move, the relative change in natural frequency that occurs during the move uniquely determines the residual vibration. Thus, PRV is purely a function of the relative frequency move (of course, PRV is also implicitly a function of the shaper used).

Because the residual vibration is a constant for a given relative frequency change, a plot can be generated showing residual vibration as a function of relative frequency change. This plot is shown in Figure 7 with a second-order polynomial fit to the data.

To ensure that the relative frequency moved is the only independent datum that must be considered when evaluating the performance of end-point-frequency shapers, another variation was tested. The relative frequency change was held constant and the system parameters, k and ω_0 , were varied. For this trial, the relative frequency moved was fixed at 10 %. Figure 8 shows that the different values of the system parameters all resulted in virtually the same percentage residual vibration. From these results, the conclusion can be drawn that the system parameters do not affect the percent residual vibration resulting from a relative frequency change when a ZV shaper is used. If an initial position and a 10 % relative frequency change were desired, changing the system parameters would change the final position that corresponds to the given relative frequency

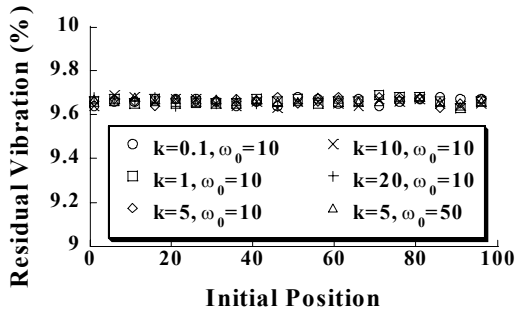


Figure 8: Residual Vibration from $f_{rel}=10\%$ for Various Values of the System Parameters Using an End-Point-Frequency ZV Shaper.

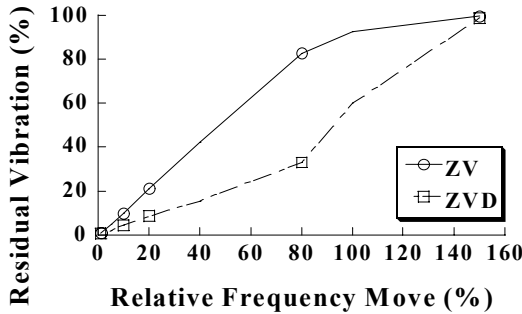


Figure 9: Residual Vibration Comparison of End-Point-Frequency ZV and ZVD Shapers.

change, but the percentage residual vibration would remain the same. Again, this implies $PRV = PRV(f_{rel})$.

4. Application of System Analysis to the Design of Nonlinear Input Shapers

4.1 End-Point-Frequency ZVD Shaper

The results obtained from the preceding analysis show two important results of the application of input shaping to the simple nonlinear system: 1) the system vibration never exceeds 100 % using the measure in (2), which is a consequence of using a positive input shaper. 2) the end-point-frequency ZV shaper is only effective when the relative frequency change during a move is small. In an effort to reduce vibration further, a more robust shaper was tested on the system given by (1). A ZVD shaper was used for a series of constant relative frequency moves, and the results were compared with those of a ZV shaper. As with the ZV, the residual vibration was a function only of the relative frequency change during the move. Lower residual vibration resulted from using the ZVD shaper, but at the expense of a longer rise time. Twenty data points were taken for each relative frequency move by using different starting positions. Figure 9 shows the residual vibration resulting from ZV and ZVD shapers as a function of relative frequency change.

The ZVD shaper is clearly more effective than the ZV shaper at reducing residual vibration, but as the relative frequency moved becomes very large, the performance of the two shapers approach one another. Compared to the ZV

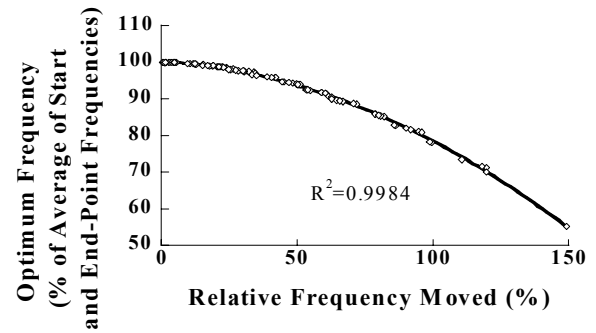


Figure 10: Shaper Frequency Minimizing Residual Vibration (with $\zeta=0$) as a Function of f_{rel} .

shaper, the more significant reduction of residual vibration of the ZVD shaper is an indication that the robustness of a shaper in the linear domain transfers into robustness for nonlinear systems. Likewise, these results support the notion that built-in robustness is appropriate for dealing with moderate nonlinearity.

4.2 Piecewise-Minimal-Vibration ZV Shaper

The simulations thus far have utilized shapers designed for the frequency at the end point of the move and for zero damping ratio. The next logical step is to try to find the best ZV shaper that can be used for a given move. Intuition suggests that perhaps some type of average frequency (rather than end-point frequency) during the move should be used to create the shaper [20]. To test this hypothesis, a piecewise optimization was performed. The first step obtained the shaper frequency that minimizes residual vibration while holding the shaper damping ratio constant at $\zeta=0$. The second step found the shaper damping ratio that further minimizes residual vibration while holding the shaper frequency constant (at the optimal frequency computed in the first step).

Figure 10 shows the optimal design frequency obtained by performing step 1 of the piecewise optimization. A best-fit parabola is also shown. In this figure, the value 100 on the vertical axis represents the average of the start and end-point frequencies, i.e. $0.5[f(x_1)+f(x_2)]$. For small relative frequency moves, shaping for the average frequency is very close to the optimum shaper. As the relative frequency change associated with the move increases, a frequency that is lower than the average value minimizes the residual vibration.

Figure 11 shows the optimal damping ratio values obtained by performing step 2 of the piecewise optimization. The portion of this graph between 0 and about 40 % is highly linear, but the trend breaks down at higher relative move frequencies. The introduction of a non-zero damping ratio into the shaper design may seem somewhat contrary to intuition due to the fact that the system is conservative, but by doing so PRV is reduced more than by optimizing frequency alone.

Figure 12 compares the performance of the piecewise minimal vibration shaper to the end-point-frequency shaper

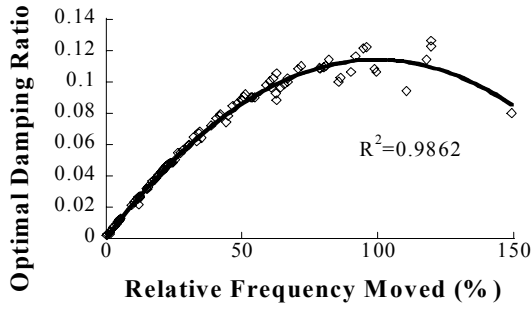


Figure 11: Damping Ratio Used in Shaper Minimizing Residual Vibration When Optimal Frequency from Figure 10 is Used.

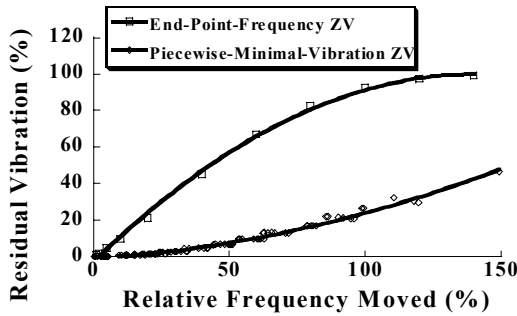


Figure 12: Residual Vibration for Piecewise-Minimal-Vibration ZV Shaper and End-Point-Frequency ZV Shaper.

from Figure 7. The piecewise-minimal-vibration shaper produces significantly lower residual vibration than the end-point shaper for all relative frequency moves tested.

4.3 Concurrently-Optimized-Minimal-Vibration ZV Shaper

In order to find the optimal minimal vibration shaper, a semi-analytical approach was developed. The analytical portion of the technique first computes the impulse amplitudes of the input shaper. To do so, a conservation of energy argument is used. The potential energy of the system stored initially in a step move to x from some intermediate point x_{int} is given by the expression:

$$V(x) = \int_{x_{\text{int}}}^x (\omega_0 + kx)^2 (\xi - x_{\text{int}}) d\xi \quad (4)$$

This equation can be integrated in closed form. Let the indefinite integral be equal to $I(x)$. The definite integral is then $V(x) = I(x) - I(x_{\text{int}})$. For the step input from x_1 to x_2 , the problem is to find the intermediate point x_{int} such that the step command from x_1 to x_{int} will carry the mass to x_2 with zero kinetic energy at that point. Because the system is conservative, the energy stored in the step input from x_1 to x_{int} would be the same as the energy stored in the overshoot from x_{int} to x_2 . Therefore, x_{int} can be found by solving the following expression:

$$V_1 = V_2 = I(x_1) - I(x_{\text{int}}) = I(x_2) - I(x_{\text{int}}) \quad (5)$$

Or equivalently, by solving $I(x_1) - I(x_2) = 0$ for x_{int} .

The impulse amplitudes can then be computed directly.

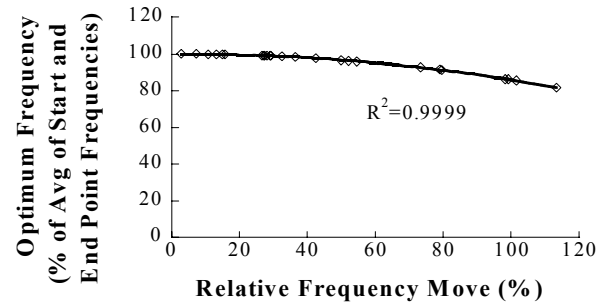


Figure 13: Frequency to Use in ZV Shaper for Zero Vibration as a Function of Relative Frequency Moved.

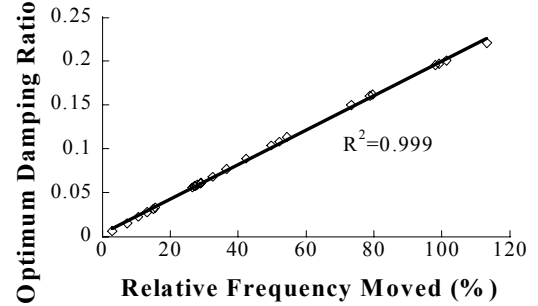


Figure 14: Damping Ratio to Use in ZV Shaper for Zero Vibration as a Function of Relative Frequency Moved.

The first impulse amplitude, a_1 , is simply equal to x_{int} normalized by the total move distance, or $a_1 = x_{\text{int}}/(x_2 - x_1)$. The impulse amplitudes in the input shaper must sum to one, so $a_2 = 1 - a_1$.

The second step in computing the shaper finds the time location of the second impulse. This step is done by simulating the system using the desired step input convolved with the first impulse a_1 and measuring the time when the system reaches maximum displacement. This will be the time location of the second impulse, t_2 .

To test the effectiveness of this approach, simulations were run for both 10 % and 80 % relative frequency moves over a wide range of starting positions. The residual vibrations were all less than 1 %. The very small amount of vibration occurs because the time of the second impulse cannot be computed exactly from the simulation.

The exact same value of ζ was found for each data point for a given relative frequency move, and the natural frequency of the shaper (relative to the average endpoint frequency) had a standard deviation of less than 0.1 %. The results indicate these values remain constant for a given relative frequency move.

Given these results, data was collected for many different relative frequency moves and trends were documented. Simulations were run for random forward moves, and values for the natural frequency and damping ratio to use in the ZV shaper design were calculated. Figures 13 and 14 show the resulting curves that are used to approximate these values from the relative frequency change of the desired move.

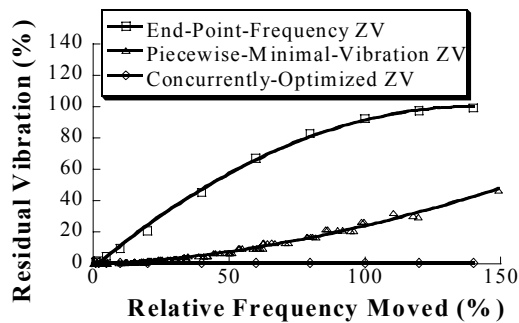


Figure 15: Residual Vibration Resulting When Each of the Three Shapers Examined is Used.

Using curve fits to the data in Figures 13 and 14, the system parameters, and the start and end points of the move, a ZV shaper resulting in essentially zero residual vibration can be calculated directly. The frequency and damping values to use in the ZV design equations are:

$$\omega_{\text{shap}} = \frac{6(10^{-7})}{2\omega_0 + kx_1 + kx_2} \left(\begin{array}{l} 343059k^2x_2^2 + 2685808k^2x_1x_2 + \\ 302749k^2x_1^2 + 3371926kx_2\omega_0 \\ + 3291306kx_1\omega_0 + 3331616\omega_0^2 \end{array} \right)$$

$$\zeta_{\text{shap}} = \frac{2(10^{-6})}{2\omega_0 + kx_1 + kx_2} (198123kx_2 - 194477kx_1 + 3646\omega_0)$$

Figure 15 shows the resulting residual vibration using this shaper compared to the other shapers examined here. Although the method for finding this zero-vibration shaper would require running a simulation for many data points, empirical results showed a strong trend that allows for the direct computation of shaper parameters that will eliminate virtually all residual vibration.

5. Conclusions

An investigation of the response of a simple nonlinear system to shaped inputs was conducted. It was shown that the only factor that affects the residual vibration is the relative frequency change between the starting and ending points of a motion. Piecewise optimization to determine the natural frequency and damping ratio to use in designing an input shaper for the system yielded vibrations significantly less than what resulted from shaping for the end-point frequency, but it did not eliminate vibration altogether. A more complete solution calculated the shaper that results in zero vibration from energy-based arguments, and then backed out the shaper design parameters from those results. A closed-form description of a zero vibration input shaper was then developed. While somewhat limited in scope by the assumptions and conditions placed on the system and the commands, the results provide basic insights into the application of input shaping to nonlinear systems.

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