# VECTOR ZONE PLATES AS TEST PATTERNS FOR LINEAR VECTOR FILTERS

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**ABSTRACT**

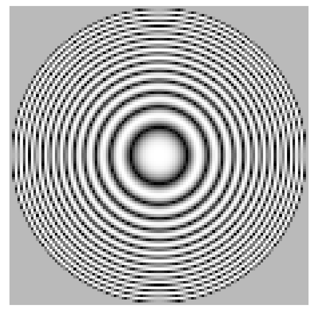
Zone plate test images have been used in image processing and television for some time. In this paper the concept is generalized to a vector zone plate, which proves to be a useful test pattern for linear vector filters, in this case colour image filters. The vector zone plate introduced in this paper consists of a radial frequency sweep oscillating between opponent colours. The direction of the opponent colour vectors varies around the image, so that all colours within a plane in colour space are contained within the pattern. Using one of the few examples of a linear vector filter published so far, the paper shows how different vector zone plate patterns may be used to reveal the frequency response of a filter in different directions in colour space.

## 1. INTRODUCTION

Linear vector filtering of color images is an emerging field and only a few examples of linear filters with color-dependent responses have been published to date [1, 2, 3], but the authors are confident that others will be discovered. The theory of color-dependent frequency responses is as yet undeveloped, although the authors believe they are beginning to make some progress towards an understanding. In this paper we present a new type of color test image derived from an existing idea in grayscale test images (the zone plate pattern), and we show how it reveals color-dependent structure in the frequency response of a color-selective linear image filter.

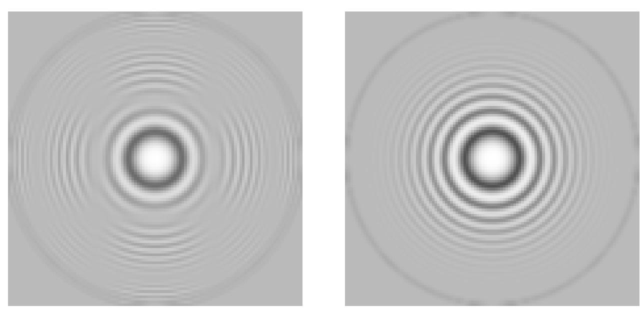
The zone plate pattern is essentially a frequency sweep pattern, radially symmetric about the centre of the image[1]. They are used in test patterns for television and video [4]. In its grayscale form luminance varies sinusoidally from black to white, increasing in frequency from the centre of the pattern outwards (Figure 1).

The zone plate pattern may be convolved with a linear image filter and the resulting filtered image reveals the frequency response of the filter. This is easiest to understand from an example.



**Note:** The zone plate images in this paper contain frequency content up to the Nyquist frequency. They must be viewed on screen with the image pixels in 1:1 correspondence with the display pixels to avoid aliasing. They are unlikely to appear correctly when printed.

**Fig. 1**. Grayscale zone plate pattern.



(a) averager (b) binomial smoothing

**Fig. 2**. Filtered grayscale zone plate patterns.

Figure 2(a) shows the zone plate image of Figure 1 after filtering with a $5 \times 5$ averager. It is evident that the filtered zone plate pattern reveals any directional sensitivity of the filter, and in this case it shows the characteristic rectangular pattern of 'notches' where the black/white cosinusoid has been averaged to mid-gray. A spatially isotropic low-pass

[1] Physicists use the term *zone plate* to mean a pattern of annular apertures which focus radiation by diffraction.

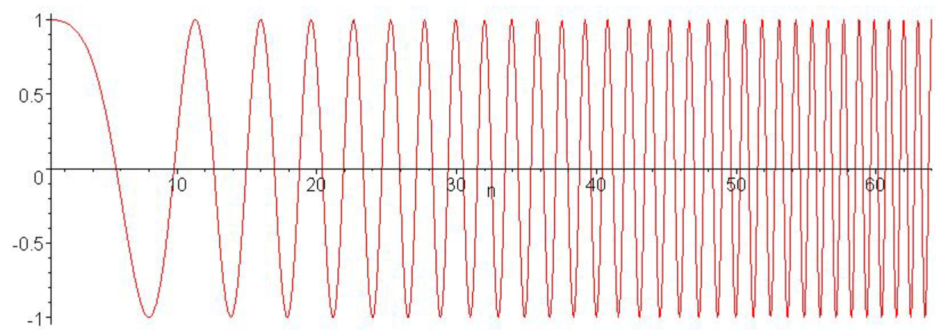



**Fig. 3**. Swept frequency cosine function, 64 samples.

filter would be revealed by uniform attenuation at all angles from the pattern centre (we qualify the word *isotropic* here because in a later section we refer to vectorially isotropic filters). Figure 2(b) shows the test pattern after filtering with an approximately isotropic filter (a 5×5 binomial smoothing filter [5, page 152]).

In the next section we present a mathematical derivation of the grayscale zone plate pattern in discrete form. In section 3 we introduce the idea of a *vector* zone plate test pattern, in which it is not luminance that varies sinusoidally, but the amplitude of a vector (pixel direction in color-space), such that all pixel values in the vector zone plate pattern are co-planar in color-space. We show some filtered vector zone plates in section 4 using already published linear vector filters. In section 5 we discuss the concept of frequency response as it applies to linear vector filters. Finally we draw some conclusions in section 6.

## 2. DISCRETE ZONE PLATES

In this section we present a simple mathematical derivation of the discrete zone plate test pattern. Derivations of the zone plate pattern we have seen have been based on continuous mathematics, but a much simpler derivation of the pattern is possible in direct discrete (sampled) form. Consider an image which is $N$ pixels square ($N$ may be odd or even, but we do not consider the difference between the two cases here). Since an image is sampled, the frequency sweep should extend from zero frequency (at the center of the image) to the Nyquist frequency at the center of the edges. Since the sweep pattern is radially symmetric, we can consider only a half row (or half column) of pixels starting at the center of the image and extending to the center of an edge. The number of pixels in this half row or column will be $N/2$. If the pattern was of constant frequency, the following function would define the pixel values for a single cycle extending from the center to the edge of the image:

$$f(k, n) = \cos\left(\frac{2\pi kn}{N}\right) \quad (1)$$

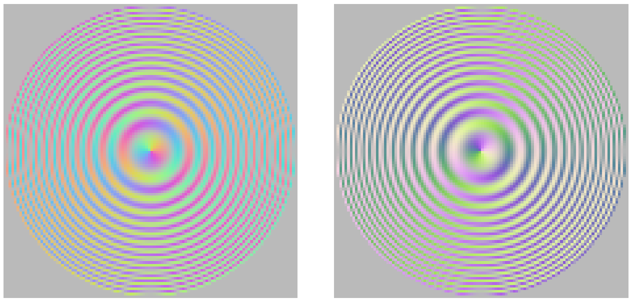
where $n$ is sample position, and $k$ determines the spatial frequency. To obtain a sweep function, we need only make $k$ proportional to $n$. $k$ must be zero at the center of the image, and at the edge of the image $k$ must correspond to the Nyquist frequency. The value of $k$ required for this latter case is $N/2$. Hence, the function we require is:

$$f(n) = \cos\left(\frac{2\pi n^2}{N}\right) \quad (2)$$

and this is illustrated in Figure 3 for $N = 64$. The choice of the cosine function ensures that the sweep starts with a non-zero DC value at the center.

## 3. VECTOR ZONE PLATES

The concept of the zone plate test pattern may be extended to vector images (for example, color images). However, there are many ways in which the cosinusoidal sweep introduced in section 2 might be extended. We could define a single vector value (direction in color space) and simply scale the amplitude of this vector by the function $f(n)$ as defined in equation 2. This would not be a very useful test pattern, because the effect of a given filter when applied to the pattern would only be revealed for one direction in color space. It happens that we can do better than this by including in the test pattern all directions within a single *plane* in color space, because this reveals the response of the filter to all of these co-planar vectors at once. A caveat here is that if the response of the filter is not spatially isotropic we may need to rotate the test pattern to different angles to expose the full response of the filter even to the set of co-planar vectors. We return to this issue in section 5. Figure 4 shows a vector zone plate of this type with pixel values derived as follows. Let $P$ denote a plane in color space, defined by its normal vector, with the center of the plane coinciding with the origin of color space. (We assume from here on that, in the case of RGB space, the origin is located at the center of the color cube, corresponding to a mid-gray color, with normalized coordinates $(0.5, 0.5, 0.5)$ in conventional RGB



(a)
Normal = (1,1,1)
Reference = (1,-1,-1)

(b)
Normal = (1,-1,-1)
Reference = (1,½,½)

**Fig. 4**. Vector zone plates.

coordinates as used in standard image formats.). Let $R$ be a unit vector in the plane $P$ representing an arbitrary reference direction. We define a reference direction in the image to be horizontally to the right of the image center and define all pixels along this half row to have angle zero. All

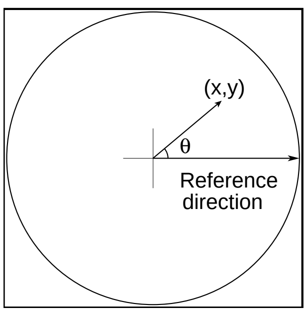
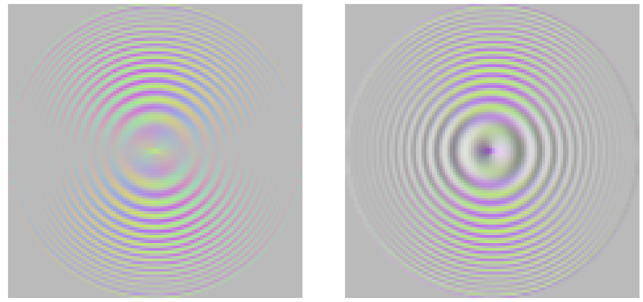



**Fig. 5**. Image reference direction and pixel angle.

vectors in the test pattern located horizontally to the right of the image center have direction $R$. Their amplitudes are scaled by $f(n)$ in equation 2. Since $f(n)$ takes negative as well as positive values, the vectors will reverse in direction at the zero crossings of $f(n)$ to give vectors opponent to $R$. If $R$ corresponds to red, for example, the opponent vectors will correspond to cyan. At other pixel positions in the test pattern, each pixel subtends an angle $\theta$ to the horizontal at the center of the image, as defined in Figure 5. We define the direction of the vector at each pixel such that the vector subtends the same angle $\theta$ to $R$ in the plane $P$ as the pixel subtends to the reference direction in the image.

## 4. RESULTS: VECTOR SOBEL FILTER

Figure 6 shows the two vector zone plates of Figure 4 after convolution with the vector Sobel filter presented in [2]. Each vector zone plate has its vectors in a different plane,



(a) (b)

**Fig. 6**. Vector zone plates of Figure 4 after filtering with vector Sobel filter.

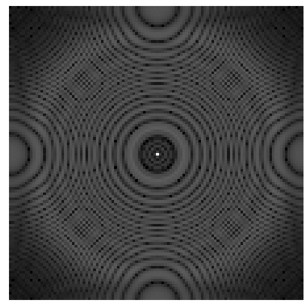
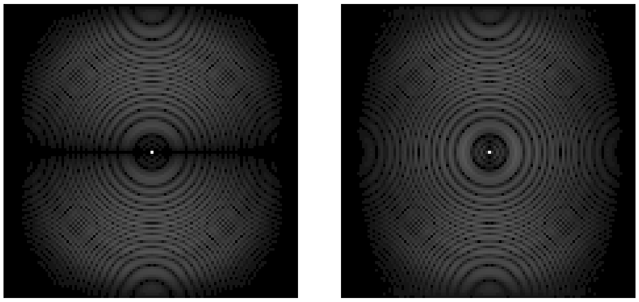


**Fig. 7**. Fourier transform (modulus) of the vector zone plates in Figure 4. (The modulus is independent of color, so both vector zone plates have the same transform modulus.)

and the response of the filter differs in each case. These filtered zone plate images reveal the structure of the frequency response directly, but this is clearer still when examining the Fourier transforms of the filtered zone plates. Figure 7 shows the modulus of the hypercomplex Fourier transform of the vector zone plate of Figure 4. Hypercomplex Fourier transforms represent the frequency content of a vector image in a holistic manner. Linear vector filters can be implemented using hypercomplex Fourier transforms as an alternative to direct convolution in the spatial (image) domain [6]. The hypercomplex Fourier transform which we use here is defined in [7] and some further discussion is given in [8]. Figure 8 shows the Fourier transforms of the two filtered images in Figure 6. In 8(a) the attenuation of horizontal frequencies is clear, but in 8(b) this attenuation is not present, but a general attenuation of frequencies close to the horizontal Nyquist frequency is seen.

## 5. FREQUENCY RESPONSE OF A LINEAR VECTOR FILTER

A linear grayscale image filter has a 2-dimensional frequency response which expresses the gain and phase shift



(a) (b)

**Fig. 8**. Filtered vector zone plate Fourier transforms (modulus).

of the filter at each combination of horizontal and vertical frequencies.

It is clear from the examples presented in this paper, that a linear vector filter can have a frequency response that is also dependent on the direction in colour space of the sinusoids which stimulate the filter. Therefore, at each pair of horizontal and vertical frequencies, the frequency response must have a 3-dimensional character expressing the gain and phase shift of the filter at the given frequency. The vector zone plate in which all vectors are co-planar in colour space allows us to visualize a plane section of this 3-dimensional frequency response, and by obtaining the response of the filter to several orthogonal vector zone plates we can glimpse the shape of the 3-dimensional frequency response.

An obvious possibility which we cannot realise in a conference paper, is to create a movie (e.g. in MPEG format) showing the filtered zone plate pattern (or its Fourier transform) for a smoothly varying range of vector zone plate directions, ideally covering all possible directions in a systematic sequence.

## 6. CONCLUSIONS

In this paper we have introduced a new concept in color image test patterns by generalizing an existing test pattern (the zone plate) to vector images, and we have shown how particular choices of vector pixels within a vector zone plate (co-planar vectors) allow the vector directional response of a vector filter to be explored. The theoretical understanding of the frequency response of linear vector filters is clearly an important topic for further research. An equally promising avenue for future work is a taxonomy of linear vector filters, classifying them according to the characteristics of their frequency responses. For example, a linear vector filter may be isotropic within a color space plane, but have a spatially varying response, or it may be spatially isotropic with a response varying within a color space plane.

## 7. ACKNOWLEDGEMENT

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## 8. REFERENCES

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