Modified Center Average Defuzzifier for improving the inverted pendulum dynamics.

Miguel A. Melgarejo
Department of Electronics Engineering.
Programmable logic applications and digital techniques group.
Universidad Distrital Francisco José de Caldas
Carrera 7ª numero 40-53 , Bogotá D.C. , A.A. 31156, Colombia.

Abstract-- **This paper presents an empirical modification of the Center Average Defuzzifier (CAD) which has been used to improve the inverted pendulum dynamics. The CAD is modeled as a two-stage process : a linear combination followed by a normalization. The second stage consists on a division operation in the traditional defuzzifier. In the proposed scheme, the division is replaced by others non linear functions. Some comparative experimental results between the traditional CAD and the proposed defuzzifier are presented. They show that the included modification can increase the balancing range of the inverted pendulum and makes it respond faster for small angles.**

Index Terms— **Defuzzification, Defuzzification schemes, Defuzzifiers, Fuzzy control, Fuzzy logic.**

I. INTRODUCTION

Center Average Defuzzifier has become in the most popular defuzzification scheme in practical fuzzy logic applications[1],[2].Even other schemes with less computational complexity have been reported, they do not offer some features like continuity and homogeneity[3].The basic idea of center average defuzzifier is to perform a linear combination over the computed weights by the fuzzy inference engine, and then modify this combination by means of a normalization. Several studies are focused to optimize the linear combination using different kind of methods like soft computing and learning algorithms [4],[5],[6], but the normalization stage is not considered in these studies.

Taking into account the network representation of a fuzzy inference system, it is clear that the normalization stage of the CAD consists on a division. This operation has a non linear nature and it can be modeled as a piece wise linear function. Each linear region is constructed taking into account particular points of the original non linear function. This approach was considered for optimizing the computational cost of the traditional CAD. Some experimental probes demonstrated that the proposed scheme can improve the balancing of the inverted pendulum [7].

This work presents a more formal conceptualization of the approach mentioned above, also validates some experimental results for the inverted pendulum system with a four rule-base Mamdani controller. It has allowed to propose other non linear functions to perform the required normalization. The balancing range of this non linear control system is increased by using those functions and its transient response for small angles is faster than the achieved by the traditional CAD.

II. MODIFYING THE CENTER AVERAGE DEFUZZIFIER

The commonly used center average defuzzifier is presented on (1) , where y is the center average , x the center of the l'th output fuzzy set and w its height. This scheme is computationally simple, intuitively plausible and homogenous [1].

$$y = \frac{\sum_{l=1}^{N} x_l w_l}{\sum_{l=1}^{N} w_l} \qquad (1)$$

Notice that y can be expressed as the product of two functions and one of them has a non linear behavior , as shown on (2).

$$\begin{pmatrix} y = \left(\sum_{l=1}^{M} x_l w_l\right) f(k) \\ f(k) = \dfrac{1}{k} \\ k = \sum_{l=1}^{M} w_l \end{pmatrix} \qquad (2)$$

It is possible to make an approximation *g(k)* of the non linear function *f(k)* expressing it as a set of linear combinations with defined ranges. These linear combinations are first order polynomials , and they are calculated according to (3).

$$\begin{pmatrix} g(k) = \begin{cases} m_1 k + b_1 & 0 < k \le k_1 \\ m_2 k + b_2 & k_1 < k \le k_2 \\ \vdots & \\ m_n k + b_n & k_{n-1} < k \le k_n \end{cases} \\ m_n = \dfrac{f(k_n) - f(k_{n-1})}{k_n - k_{n-1}} \\ b_n = f(k_{n-1}) - m_n k_n \end{pmatrix} \qquad (3)$$

As an example, an approximation function *g(k)* with five polynomials has been generated and it is compared with *f(k),* as shown on Fig. (1).Notice that a bigger set of polynomials will generate a better approximation for *f(k).*

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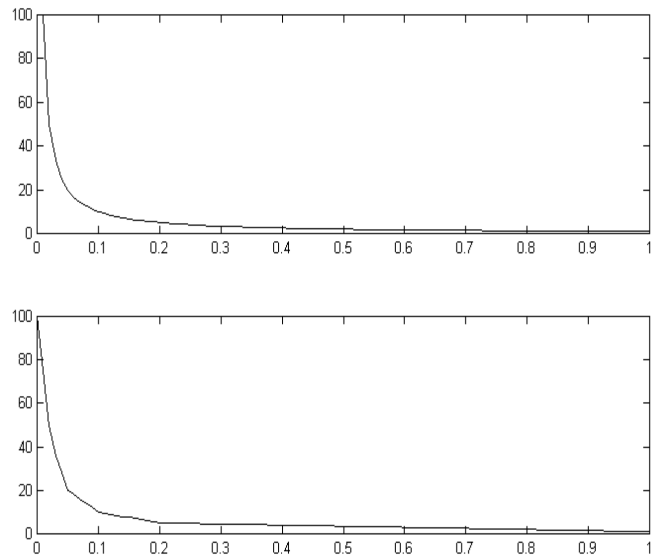

Fig.1. Comparison between *f(k)* and a *G(k)* with five polynomials.

III. EXTENDING THE CENTER AVERAGE DEFUZZIFIER

Taking into account the conceptualization presented in (2) and also the approximation of *f(k)* , other non linear functions have been considered in this work for replacing the division operation. This approach follows the axiomatic foundation for defuzzifiers [8] where they are considered as mappings with two independent variables : results of the fuzzy inference engine and a function which modifies those results.

The used non linear functions were proposed trying to preserve some important aspects of *f(k)*. This function has a decreasing behavior and it presents two interesting limits described by (4).

$$\lim_{k\to 0} f(k) = \infty \qquad \lim_{k\to\infty} f(k) = 0 \tag{4}$$

Two non linear functions have been chosen as candidates and they are presented on (5) and (6). Experimental results have demonstrated that the first limit is not strictly necessary for the considered application here.

$$f_1(k) = a - b\sqrt{k} \tag{5}$$

$$f_2(k) = am^{-bk} \tag{6}$$

IV. EXPERIMENTAL PROCEDURE AND RESULTS

A typical Mamdani fuzzy controller for balancing the inverted pendulum has been implemented in order to probe the proposed defuzzification scheme and compare it with the center average defuzzifier.

A typical approach has been followed to design the controller[9]. Two input variables were considered : *angular position* measured in radians and *angular velocity* measured in radians per second. Two exponential membership functions were used for setting each input variable, as shown on (7) and (8). The output universe has been described by means of three singletons : Negative big, zero and Positive big. The result is a fuzzy controller with a rule base of four rules, its network representation is shown on Fig 2.

$$negative(x) = \frac{1}{1+e^{30x}} \qquad positive(x) = \frac{1}{1+e^{-30x}} \qquad -0.34 < x < 0.34 \tag{7}$$

where x is angular position.

$$negative(y) = \frac{1}{1+e^{30y}} \qquad positive(y) = \frac{1}{1+e^{-30y}} \qquad -0.68 < x < 0.68 \tag{8}$$

where y is angular velocity.

Notice that the third layer of the designed fuzzy controller has been modified by including an operator named as *Z(k)*. This operator could be the traditional *f(k),* its approximation function *g(k)* or one of the considered non linear functions . In this case *g(k)* was configured with five polynomials according to (9).

$$G(k) = \begin{cases} -2500\,k + 100 & 0 < k \le 0.025 \\ -800\,k + 60 & 0.025 < k \le 0.05 \\ -200\,k + 30 & 0.05 < k \le 0.1 \\ -50\,k + 15 & 0.1 < k \le 0.2 \\ -0.5k + 5.1 & k > 0.2 \end{cases} \tag{9}$$

The controller was matched to the inverted pendulum through a mechanical actuator with a gain of 5N/100%. The pendulum parameters are shown on table 1 .

TABLE 1. INVERTED PENDULUM PARAMETERS

Parameter	Value
Pole Mass	0.1Kg
Pole Length	1 m
Car Mass	1Kg

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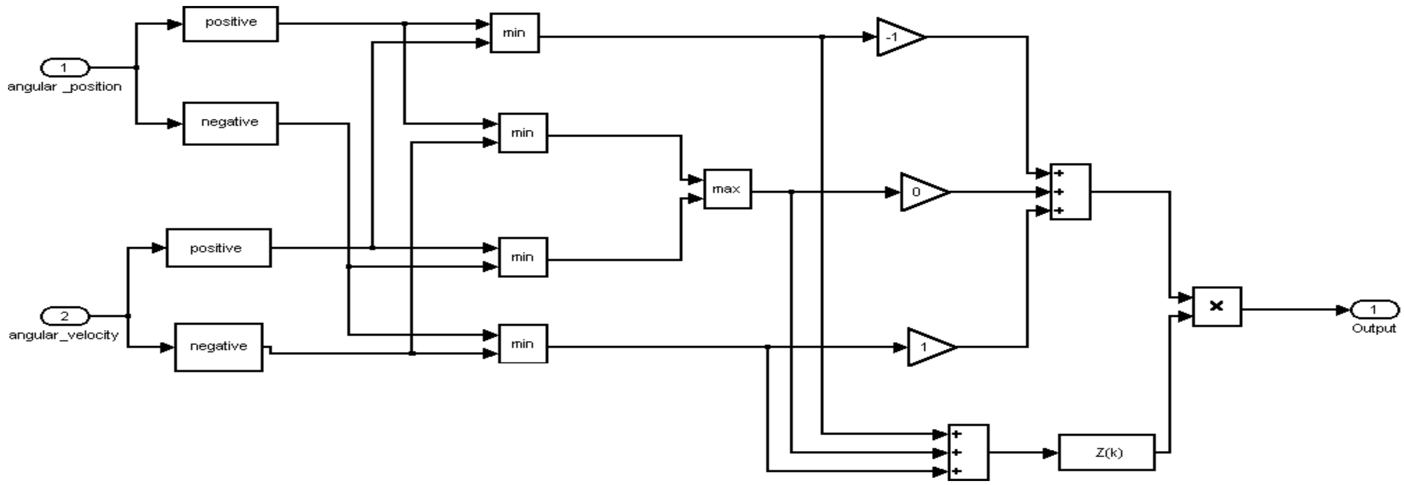

Fig. 2. Network representation of the designed fuzzy controller.

A set of experimental probes have been performed in order to compare the inverted pendulum transient response using the traditional CAD and the proposed scheme based on *g(k)*. The results of these probes are shown on Fig 3 and Fig 4. They demonstrate that the Mamdani controller with the modified CAD has a larger balancing range than the same controller using the traditional CAD. Also the proposed scheme allows to obtain smaller setting times for angles between zero and thirteen degrees. The outputs of both controllers are shown on Fig. 5, they have been captured for a initial condition of ten degrees. The modified CAD allows the fuzzy controller to Supply a bigger initial force than the allowed by the traditional CAD, representing a faster transient response.

Two non linear functions haven been proposed in previous section , they were selected due to their monotonic decreasing behavior and their right limit. Another consideration taken into account was their easily implementation in practical applications.

For instance square root can be approximated by using an arrangement of min and max operators[10] and exponentiation with finite length Taylor series. These functions also increased the balancing range of the inverted pendulum by setting their parameters adequately , in this case they were established empirically by a trial and error process.

F(k) and $F_2(k)$ have been probed with an initial angular position of 25 degrees. System setting time can be controlled only by modifying the constant *a* for F_1 and F_2, as shown on Fig. 6 and Fig 7. respectively. $F_2(k)$ is of special interest due to its parameter *m* can be established in any base. For example m=2 is convenient for accelerating the defuzzification stage of digital-binary fuzzy inference systems. Computation of the respective exponentiation is efficiently done by several well knowledge algorithms. Also Gaussian functions could be considered as *Z(k)* but experiences demonstrated they are computationally inefficient.

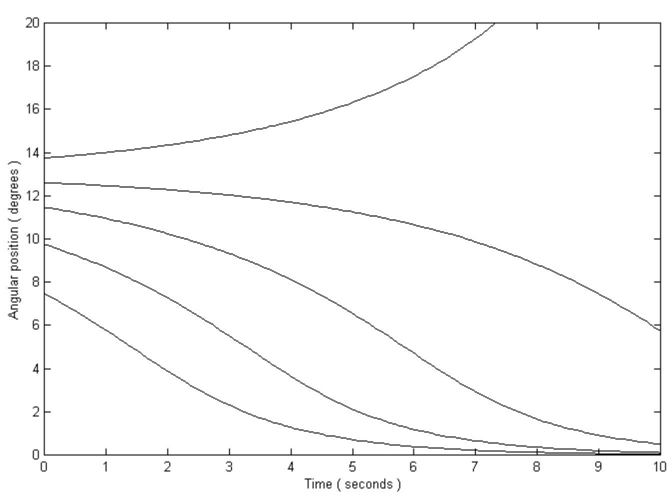

Fig 3. Angular position dynamics for six initial conditions using the CAD.

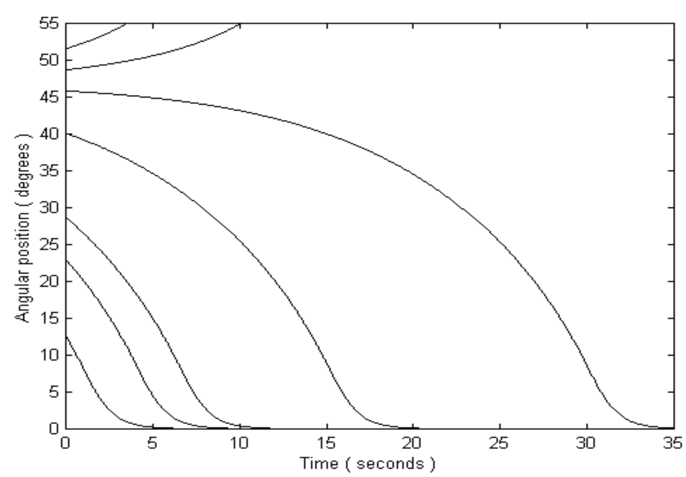

Fig 4. Angular position dynamics for seven initial conditions using the modified CAD.

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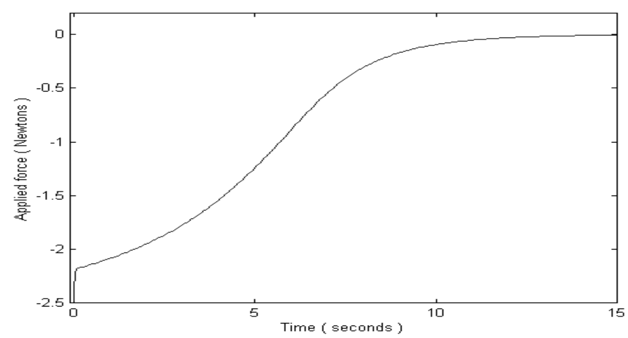
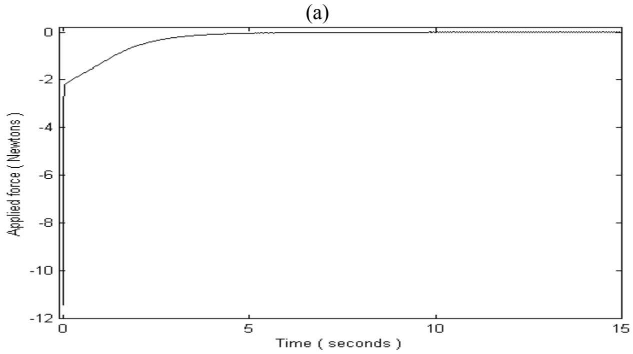


(a)



(b)

Fig. 5.Controllers'outputs: (a) Controller with CAD.
(b) Controller with modified CAD based on *g(k)*.

V. CONCLUSIONS

A modification of the center average defuzzifier has been proposed. Two approaches were considered , first the required division in the CAD was approximated by using a set of first order polynomials. Second, two non linear functions were chosen to replace the original normalization function due to some similarities with it. Experimental results have demonstrated that the two approaches can provide a larger balancing range for the inverted pendulum than the offered by the original defuzzification scheme.

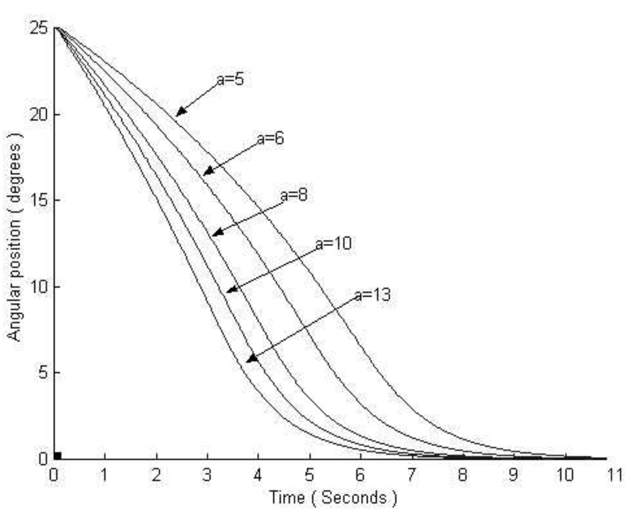


Fig 6. Inverted pendulum balancing using $F_1(k)$, b=1.

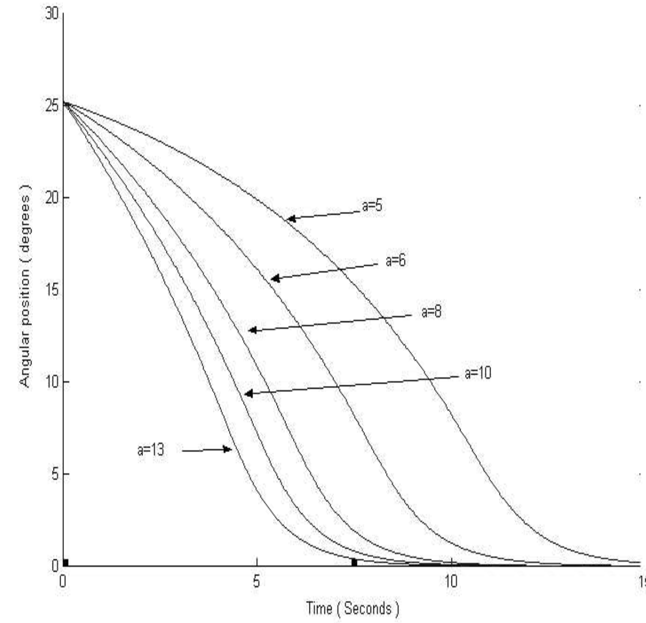


Fig 7. Inverted pendulum balancing using $F_2(k)$. (b=1, m=2).

The used non linear functions present some interesting properties : they make the fuzzy controller to provide larger initial forces avoiding instability of the pendulum, Also these functions are computationally simple and they can be implemented easily.

The proposed defuzzification scheme could be considered as an extension of the traditional center average defuzzifier. So its necessary to apply it in other practical applications in order to probe its functionality , also a formal study of the proposed scheme is required taking into account the axiomatic foundation of defuzzifiers.

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