

Importance of Coefficient Diagram in Polynomial Method

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Abstract - Coefficient Diagram Method (CDM) is one of polynomial methods in control design. Its design effectiveness mainly stems from the usage of a diagram called “Coefficient Diagram”. Coefficient diagram shows the coefficients of characteristic polynomial and those of numerator polynomials corresponding to sensitivity and complementary sensitivity function in logarithmic scale, where the abscissa is the order for the coefficients. From the shape, designer can visualize the stability, response, and robustness. The frequency response of the sensitivity and complementary sensitivity function can be intuitively visualized, too. Thus H-inf problem can be solved without ordinary H-inf algorithm.

1. INTRODUCTION

With wide spread of control technology into various fields, simple and reliable control design approach is keenly needed. The classical control well answered to this need for the ordinary control design problems, but not for more complex plants. The modern control has been developed to answer to this need. However, in spite of the tremendous effort, it has not reached to the satisfactory state yet.

Polynomial method (or algebraic design approach) is the 3rd design approach, where the controller is algebraically obtained from the characteristic polynomial by solving Diophantine equation [1]. It is not difficult to define the characteristic polynomial from stability and response specification, but it is very difficult to choose it with guarantee of robustness.

The Coefficient Diagram Method (CDM) is one of the polynomial methods. It has been developed to solve this difficulty. The CDM is fairly new and not well-known, but its basic philosophy has been known in industry and in control community for more than 40 years with successful application in servo control [2], steel mill drive control [3], gas turbine control [4], and spacecraft attitude control [5]. The historical background is given in [6].

The effectiveness of CDM mainly stems from the usage of a diagram called “Coefficient Diagram”. Coefficient diagram shows the coefficients of characteristics polynomial and those of numerator polynomials corresponding to sensitivity and complementary sensitivity function in logarithmic scale, where the abscissa is the order for the coefficients in descending order. From the shape, designer can visualize the stability, response, and robustness. Also the frequency response of the sensitivity

and complementary sensitivity function can be intuitively visualized. Thus H-inf problem can be solved without ordinary H-inf algorithm.

In Section 2, a brief introduction of CDM is made. In Section 3, coefficient diagram is explained. In Section 4, interpretation of coefficient diagram is made. In Section 5, application to H-inf problem is briefly explained.

2. BASICS OF CDM

2.1 Basic Philosophy of CDM

The CDM is an algebraic control design approach with the following five features [6].

- (1) Polynomials and polynomial matrices are used for system representation.
- (2) Characteristic polynomial and controller are simultaneously designed.
- (3) Coefficient diagram is effectively utilized.
- (4) The sufficient condition for stability by Lipatov constitutes the theoretical basis of CDM [7][8].
- (5) Kessler standard form [3] is improved and used as the standard form of CDM.

CDM design is based on the stability index and equivalent time constant as defined later. Thus for the specified settling time, a controller of the lowest order with the narrowest bandwidth and of no-overshoot can be easily designed. CDM can be considered as “Generalized PID”, because the controller can be more complex than PID, and more reliable parameter selection rules are provided. Also CDM can be considered as “Improved LQG”, because the order of controller is smaller and weight selection rules are also given [9].

2.2 Mathematical Model

The standard block diagram of the CDM design for a single-input-single-output system is shown in Fig. 1. The extension to multi-input-multi-output can be made with proper interpretation, but it is not discussed here for simplicity. The plant equation is given as

$$A_p(s)x = u + d, \quad (1a)$$

$$y = B_p(s)x, \quad (1b)$$

where u , y , and d are input, output, and disturbance. The symbol x is called the basic state variable. $A_p(s)$ and $B_p(s)$ are the denominator and numerator polynomial of the plant transfer function $G_p(s)$. It will be easily seen that this expression has a direct correspondence with the control canonical form of the state-space expression, and x corresponds to the state variable of the lowest order. All the other states are expressed as the derivatives of x of high

order. Controller equation is given as

$$A_c(s)u = B_a(s)y_r - B_c(s)(y + n), \quad (2)$$

where y_r and n are reference input and noise on the output. $A_c(s)$ is the denominator of the controller transfer function. $B_a(s)$ and $B_c(s)$ are called the reference numerator and feedback numerator of the controller transfer function. Because the controller transfer function has two numerators, it is called two-degree-of-freedom system. This expression corresponds to the observer canonical form of the state-space expression. Elimination of y and u from (2) by (1a, b) gives

$$P(s)x = B_a(s)y_r + A_c(s)d - B_c(s)n, \quad (3a)$$

where $P(s)$ is the characteristic polynomial and given as

$$P(s) = A_c(s)A_p(s) + B_c(s)B_p(s). \quad (3b)$$

In a similar manner, equation for y and u can be obtained. Because this system has 3 inputs and 3 outputs, there are 9 transfer functions.

For CDM design, the following four basic relations are selected as standard, namely

$$P(s)x = P(0)y_r, \quad (4a)$$

$$P(s)y = B_p(s)B_a(s)y_r, \quad (4b)$$

$$P(s)y = B_p(s)A_c(s)d, \quad (4c)$$

$$P(s)(-y) = B_p(s)B_c(s)n. \quad (4d)$$

Equation (4a) is the response of x to y_r when $B_a(s) = P(0)$, and it is the 0-th order canonical transfer function of $P(s)$, as will be explained in section 3.3. This equation specifies the characteristic polynomial, and it is a very good measure of stability and response speed. Equation (4b) is for the command following characteristics. Eq. (4c) is for the disturbance rejection characteristics. Equation (4d) corresponds to the complementary sensitivity function $T(s)$, and it is useful for checking the robustness. In the CDM design, these four basic relations are used as performance specification. The design of $P(s)$ is first made to satisfy specifications on (4a)(4c)(4d), and then $B_a(s)$ is adjusted to satisfy the specification on (4b).

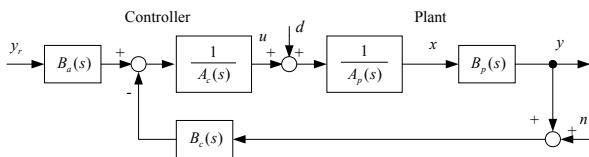


Fig. 1. Mathematical model

2.3 Mathematical Relations

Some mathematical relations extensively used in CDM will be introduced hereafter. The characteristic polynomial is given in the following form.

$$P(s) = a_n s^n + \dots + a_1 s + a_0 = \sum_{i=0}^n a_i s^i \quad (5)$$

The stability index γ_i , the equivalent time constant τ , and stability limit γ_i^* are defined as follows:

$$\gamma_i = a_1^2 / (a_{i+1} a_{i-1}), \quad i = 1 \sim n-1, \quad (6a)$$

$$\tau = a_1 / a_0, \quad (6b)$$

$$\gamma_i^* = 1/\gamma_{i+1} + 1/\gamma_{i-1}, \quad (6c)$$

$$i = 1 \sim n-1, \quad \gamma_n = \gamma_0 = \infty.$$

Also the equivalent time constant of the i -th order τ_i is defined as follows:

$$\tau_i = a_{i+1} / a_i, \quad i = 1 \sim n-1. \quad (7a)$$

Then the following relations are derived.

$$\tau_i = \tau_{i-1} / \gamma_i = \tau / (\gamma_i \dots \gamma_2 \gamma_1), \quad (7b)$$

$$a_i = \tau_{i-1} \dots \tau_2 \tau_1 \tau a_0, \quad (7c)$$

$$= a_0 \tau^i / (\gamma_{i-1} \gamma_{i-2}^2 \dots \gamma_2^{i-2} \gamma_1^{i-1}).$$

The characteristic polynomial will be expressed by a_0 , τ and γ_i as follows:

$$P(s) = a_0 \left[\sum_{i=2}^n \left(\prod_{j=1}^{i-1} 1/\gamma_{i-j}^j \right) (\tau s)^i \right] + \tau s + 1. \quad (8)$$

Notice here that $P(s)$ is expressed in $(\tau s)^i$, and its coefficients are sole functions of γ_i . Thus for given γ_i , the response shape of (4a) is similar irrespective of τ . For different τ , the response speed changes, while the response shape remains similar.

2.4 Controller Design

The stability index has been introduced already under different names by several authors. Kessler called it "damping factor" [3]. Naslin called it "characteristic ratio" [10]. Brandenburg called it "double ratio" [11][12]. The new name is given here, because it is a measure of stability in conjunction with the newly introduced stability limit. If all stability indices and equivalent time constant are given, all the coefficients a_i of the characteristic polynomial $P(s)$ can be defined by (7c). Then controller is obtained by solving the Diophantine equation, (3b). This design process is nothing but the pole assignment design approach.

There are two shortcomings in pole assignment design. The first and the severest shortcoming is that robustness is not guaranteed at all, although stability and response are guaranteed. In order to guarantee robustness, the designer is not allowed to specify all poles freely. He must choose all poles carefully such that robustness is guaranteed. The second shortcoming is that controller order is high, because it is automatically specified as the order of the plant minus one.

The most important feature of CDM is the simultaneous design of characteristic polynomial and controller. The controller structure has a great effect on robustness. The traditional design principle of sticking to the minimum-phase controller wherever possible, with the lowest-possible order, and with the narrowest possible bandwidth is actually found to be a strong guarantee of robustness. If such consideration is to be given, the stability index and equivalent time constant can not be chosen freely, but should be chosen with such care that the specific Diophantine equation is satisfied. The problem is then as

follows:

“Given some parts of controller parameter and some parts of characteristic polynomial, Find the rest of controller parameters and characteristic polynomial.”

In solving such vaguely defined problem, human intuition on the basis of graphical representation is found to be most powerful. The coefficient diagram is a very powerful means for this purpose. By intuition, the designer can finish a rough design. For the rest of the refinement, he can rely on computer power.

Usually stability index and equivalent time constant are chosen in the following range:

$$\begin{aligned} \gamma_1 &= 2.5, \gamma_2 = \gamma_{n-1} = 2, \\ \gamma_i &> 1.5\gamma_i^*, \quad i = n-1 \sim 3, \\ \gamma_i &= 1.5 \sim 4, \quad i = n-1 \sim 1, \\ \tau &\approx (1/3) \text{ of settling time.} \end{aligned} \quad (9)$$

Contrary to pole assignment, the equivalent time constant becomes a trade-off issue as well as stability index and controller parameters. Kessler proposed 2 as the standard stability index for all order [3]. In CDM standard form, $\gamma_1 = 2.5$ and rest of stability indices are 2. For simple system, the CDM standard form suffices. But for more complex system, some adjustment of γ_i is usually required.

3. COEFFICIENT DIAGRAM

3.1 Coefficient Diagram

When the plant and controller polynomials are given as

$$A_p(s) = 0.25s^4 + s^3 + 2s^2 + 0.5s, \quad B_p(s) = 1,$$

$$A_c(s) = l_1s, \quad B_c(s) = k_2s^2 + k_1s + k_0,$$

$$l_1 = 1, \quad k_2 = 1.5, \quad k_1 = 1, \quad k_0 = 0.2, \quad (10a)$$

the characteristic polynomial is expressed as

$$P(s) = 0.25s^5 + s^4 + 2s^3 + 2s^2 + s + 0.2. \quad (10b)$$

Then

$$a_i = [a_5 \cdots a_2 \ a_1] = [0.25 \ 1 \ 2 \ 2 \ 1 \ 0.2], \quad (11a)$$

$$\gamma_i = [\gamma_4 \cdots \gamma_2 \ \gamma_1] = [2 \ 2 \ 2 \ 2.5], \quad (11b)$$

$$\tau = 5, \quad (11c)$$

$$\gamma_i^* = [\gamma_4^* \cdots \gamma_2^* \ \gamma_1^*] = [0.5 \ 1 \ 0.9 \ 0.5]. \quad (11d)$$

The coefficient diagram is shown as in Fig. 2, where coefficient a_i is read by the left side scale, and stability index γ_i , equivalent time constant τ , and stability limit γ_i^* are read by the right side scale. The τ is expressed by a line connecting 1 to τ . The stability index γ_i can be graphically obtained (Fig. 3a). If the curvature of the a_i becomes larger (Fig. 3a), the system becomes more stable, corresponding to larger stability index γ_i . If the a_i curve is left-end down (Fig. 3b), the equivalent time constant τ is small and response is fast. The equivalent time constant τ specifies the response speed.

The coefficient diagram is also used for parameter sensitivity analysis and robustness analysis. In this example, the characteristic polynomial $P(s)$ is composed of

two component polynomials: denominator polynomial $P_{11}(s)$ and numerator polynomial $P_k(s)$.

$$P(s) = P_{11}(s) + P_k(s), \quad (12a)$$

$$P_{11}(s) = l_1(0.25s^5 + s^4 + 2s^3 + 0.5s^2), \quad (12b)$$

$$P_k(s) = k_2s^2 + k_1s + k_0. \quad (12c)$$

The sensitivity and complementary sensitivity functions, $S(s)$ and $T(s)$ are expressed as

$$S(s) = P_{11}(s) / P(s), \quad (12d)$$

$$T(s) = P_k(s) / P(s). \quad (12e)$$

Equation (12b) is shown in Fig. 2 with small circles and dash-dot lines. Equation (12c) is shown with small squares and broken lines. Designer can visually assess the deformation of the coefficient diagram due to the parameter variation of k_2 , k_1 , and k_0 . Then he can visualize the variation of stability and response. Also from (12e), it is clear that robustness can be analyzed by comparison of coefficients a_i and k_i at the coefficient diagram.

Thus the coefficient diagram indicates stability, response, and robustness (three major properties in control design) in a single diagram, enabling the designer to grasp the total picture of control system. At present, Bode diagram is used for this purpose. However coefficient diagram is more accurate and easy to use in actual design.

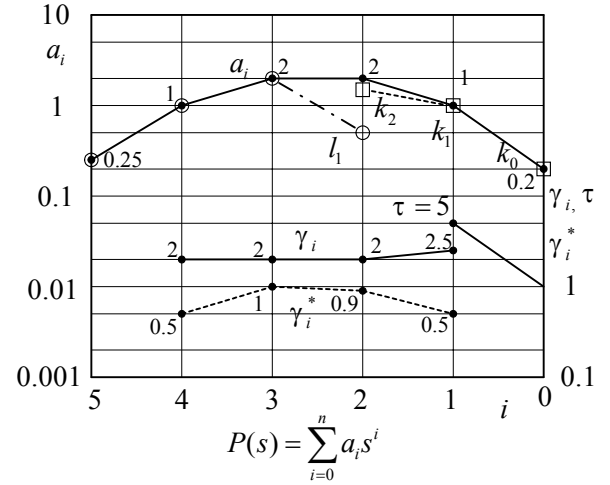


Fig. 2. Coefficient diagram

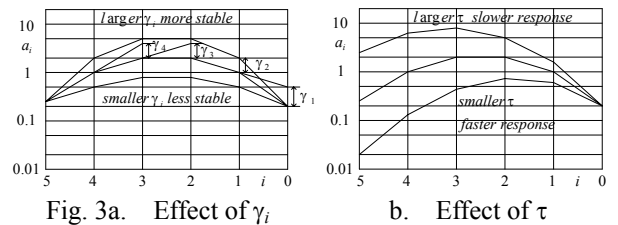


Fig. 3a. Effect of γ_i b. Effect of τ

3.2 Stability Condition

From the Routh-Hurwitz stability criterion, the stability condition for the 3rd order is given as

$$a_2 a_1 > a_3 a_0. \quad (13a)$$

If it is expressed by stability index,

$$\gamma_2 \gamma_1 > 1. \quad (13b)$$

The stability condition for the fourth order is given as

$$a_2 > (a_1 / a_3)a_4 + (a_3 / a_1)a_0, \quad (14a)$$

$$\gamma_2 > \gamma_2^*. \quad (14b)$$

For the system higher than or including 5th degree, Lipatov gave the sufficient condition for stability and instability in several different forms [7][8]. The conditions most suitable to CDM can be stated as follows:

"The system is stable, if all the partial 4th order polynomials are stable with the margin of 1.12. The system is unstable if some partial 3rd order polynomial is unstable."

Thus the sufficient condition for stability is given as

$$a_i > 1.12 \left[\frac{a_{i-1}}{a_{i+1}} a_{i+2} + \frac{a_{i+1}}{a_{i-1}} a_{i-2} \right], \quad (15a)$$

$$\gamma_i > 1.12 \gamma_i^*, \quad \text{for all } i = 2 \sim n - 2. \quad (15b)$$

The sufficient condition for instability is given as

$$a_{i+1} a_i \leq a_{i+2} a_{i-1}, \quad (16a)$$

$$\gamma_{i+1} \gamma_i \leq 1, \quad \text{for some } i = 1 \sim n - 2. \quad (16b)$$

These conditions can be graphically expressed in the coefficient diagram. Fig. 4a is a 3rd-order example. Point A is $(a_2 a_1)^{0.5}$ and point B is $(a_3 a_0)^{0.5}$. Thus if A is above B, the system is stable. Point C is $(\gamma_2 \gamma_1)^{0.5}$. If it is above 1, the system is stable. Fig. 4b is a 4th-order example. Point A is obtained by drawing a line from a_4 in parallel with line $a_3 a_1$. Similarly point B is obtained by drawing a line from a_0 in parallel with line $a_3 a_1$. The stability condition is $a_2 > (A + B)$. The other condition is $\gamma_2 > \gamma_2^*$.

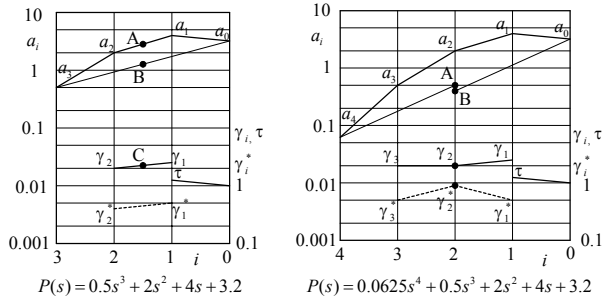


Fig. 4a. 3rd order

b. 4th order

3.3 Transfer Function

In order to express the transfer function, the numerator polynomial has to be specified as well as the denominator polynomial. In any transfer function of closed-loop system, the denominator is always the characteristic polynomial but the numerator polynomial varies depending on the input/output relations. Such transfer function is given generally in the following form.

$$T(s) = (b_m s^m + \dots + b_1 s + b_0) / (a_n s^n + \dots + a_1 s + a_0) \quad (17)$$

Now a special transfer function, called " i -th order canonical transfer function", derived from the characteristic polynomial, is defined as follows;

$$T_i(s) = a_i s^i / (a_n s^n + \dots + a_i s^i + \dots + a_1 s + a_0) \quad (18)$$

Then any transfer function can be expressed by the weighted sum of the canonical transfer functions.

$$T(s) = \sum_{i=0}^m (b_i / a_i) T_i(s). \quad (19)$$

Also a short hand notation as follows is used to denote the sum of canonical transfer functions.

$$T_{i,j}(s) = T_i(s) + T_j(s). \quad (20)$$

This sum is also called $[i,j]$ -th order canonical transfer function. When unity feedback is assumed, the open-loop transfer function is uniquely determined from the closed-loop transfer function. If the closed-loop transfer function is a canonical transfer function as in the form of (20), the corresponding canonical open-loop transfer function is given as follows;

$$G_{i,j}(s) = T_{i,j}(s) / [1 - T_{i,j}(s)]. \quad (21)$$

The following two canonical open-loop transfer functions are helpful to visualize the characteristics of $P(s)$.

$$G_0(s) = a_0 / (a_n s^n + \dots + a_1 s), \quad (22)$$

$$G_{1,0}(s) = (a_1 s + a_0) / (a_n s^n + \dots + a_2 s^2). \quad (23)$$

It is clear that (22) corresponds to the so-called "system type 1" and (23) to "system type 2".

Also break point ω_i is defined as

$$\omega_i = a_i / a_{i+1} = 1 / \tau_i. \quad (24)$$

The ω_i is the reciprocal of the equivalent time constant of high order τ_i . The ratio of adjacent break points is equal to the stability index γ_i . This is the reason, why stability index is called as "double ratio" [11][12].

$$\gamma_i = \omega_i / \omega_{i-1} \quad (25)$$

Fig. 5 shows an example of Bode diagram for the 0-th order canonical open-loop transfer function (system type 1) and for $[1,0]$ -th order (system type 2). The straight-line approximation of Bode diagram or asymptotic Bode diagram (A-Bode) used here is somewhat different from the ordinary way. The break points are chosen from the ratio of the coefficients and not from the poles and zeros of the transfer function as in the usual case. However this way is more accurate and the relation with the coefficient diagram is closer.

Thus it becomes clear that the coefficient diagram has a one-to-one correspondence with A- Bode of the canonical open-loop transfer function.

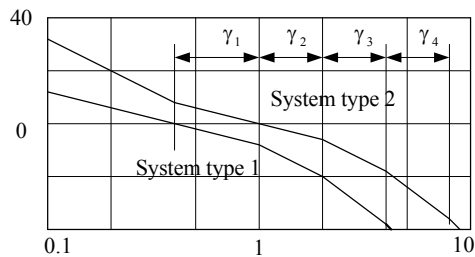


Fig. 5. Canonical open-loop transfer function

4. INTERPRETATION OF COEFFICIENT DIAGRAM

4.1 Pole Estimation

By proper interpretation of coefficient diagram, much of

useful information can be derived. For small variation of coefficients, the step response and frequency response are almost the same, while the pole changes conspicuously. This makes the quick estimation of the poles difficult. However there are some cases where the poles can be estimated by the coefficient diagram. This technique is shown by the following example.

The characteristic polynomial to be considered is as follows;

$$P(s) = 0.2s^4 + s^3 + 1.4s^2 + 4s + 0.8 = \sum_{i=0}^4 a_i s^i. \quad (26)$$

The exact poles are as follows;

$$p_i = -4.3955, -0.19541 \pm j2.0546, -0.21364. \quad (27)$$

When the coefficient diagram (Fig. 6) is drawn, it is immediately seen that stability index γ_3 and γ_1 are very large. When stability index is large, the characteristic polynomial can be separated into smaller polynomials. This leads to

$$P(s) \approx P_1(s)P_2(s)P_3(s), \quad P_1(s) = 0.2s + 1, \quad (28a)$$

$$P_2(s) = s^2 + a_2^*s + 4, \quad P_3(s) = s + 0.2, \quad (28b)$$

$$a_2^* = a_2 - (a_4 a_1 / a_3 + a_0 a_3 / a_1) = 0.4. \quad (28c)$$

Thus the poles estimated by coefficient diagram are

$$p_i = -5, -0.2 \pm j2, -0.2. \quad (29)$$

The results compare favorably with the exact values. The Equation (21c) is intuitively deduced from (14a).

4.2 Transient Response

For CDM standard form, the step response for 0-th order canonical transfer function, $T_0(s)$, and that for [1,0]-th order, $T_{1,0}(s)$, are reported previously [13]. For non-standard case, the overshoot can be estimated as,

$$M_p \approx 0.8(1/\gamma_1 - 0.4), \quad \text{in per unit}, \quad (30)$$

after some simulation. In this example, the rest of stability indices are all 2. The step responses of canonical transfer function $T_i(s)$ is shown in Fig. 6, where $\tau = 2.5$ and the order of characteristic polynomial $n = 8$. Maximum values are almost 1, and the time at peak is

$$t_p \approx 1.5(0.5)^i \tau. \quad (31)$$

With the aid of these curves, the designer can visualize the step response of transfer functions given in the coefficient diagram.

4.3 Frequency Response

The A-Bode can be obtained graphically from the coefficient diagram. The mathematical expression of A-Bode for (17) is expressed as

$$T(j\omega)_{A-Bode} = \max[abs(b_i \omega^i)] / \max[abs(a_i \omega^i)]. \quad (32)$$

This can be obtained by drawing two parallel lines tangent to the numerator and denominator polynomials at coefficient diagram. The difference of two parallel lines corresponds to $T(j\omega)_{A-Bode}$. The slope of the parallel lines corresponds to ω . By this way, the frequency responses of the sensitivity and complementary sensitivity functions

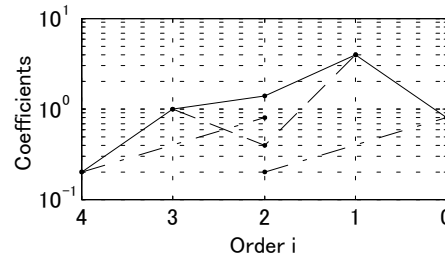


Fig. 6. Coefficient diagram for polynomial separation

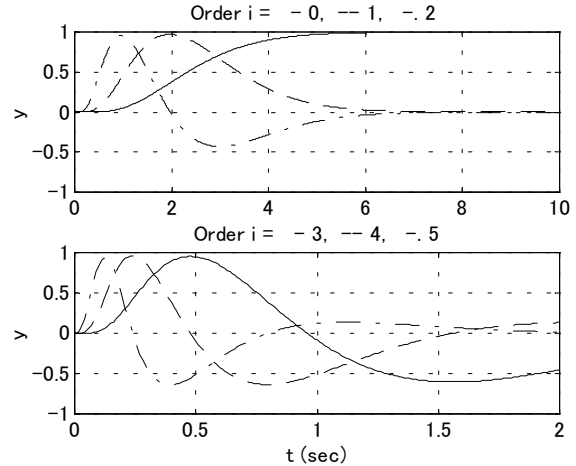


Fig. 7. Step response of canonical transfer function

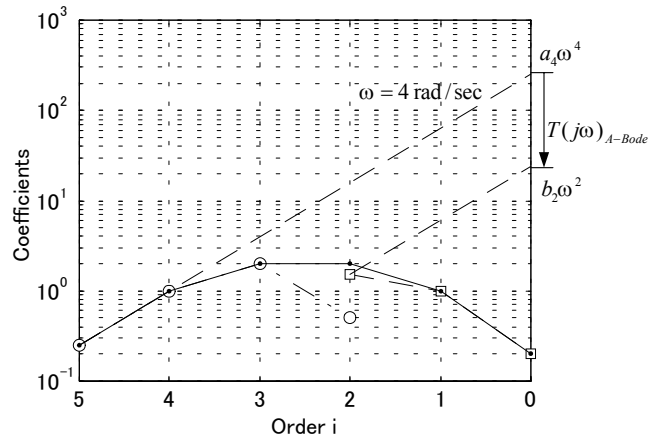


Fig. 8. Frequency response estimation

are intuitively read from the coefficient diagram. In Fig. 8, complementary sensitivity function at 4 rad/sec is estimated for the system of Fig. 2.

5. SOLUTION OF H-INF PROBLEM BY CDM

The most popular H-inf design is mixed-sensitivity analysis. In SISO case, this problem is equivalent to the problem setting that the amplitude response of complementary sensitivity function, $|T(j\omega)|$, is to be kept smaller than some specified function $|W_T(j\omega)|$, while that of sensitivity function, $|S(j\omega)|$, is made as small as possible in

comparison with the specified function $|W_s(j\omega)|$.

The CDM design procedure equivalent to this problem is as follows:

- (1) On the basis of $W_T(s)$ and $W_S(s)$, the controller structure will be determined. The order of controller is determined. Some parameters of denominator of controller, $A_c(s)$, will be specified.
- (2) Make CDM design for some assumed equivalent time constant τ .
- (3) Decrease τ until $|T(j\omega)|$ touches $|W_T(j\omega)|$.

This approach has advantages over the standard H-inf approach as listed below:

- (1) Poles are guaranteed to stay in favorable region, because CDM is used.
- (2) No pole-zero cancel problem arises.
- (3) The order of controller is small.

For MIMO case, the similar approach can be used. In MIMO, singular values are used instead of the absolute value. Optimization over singular value tends to make the characteristics of each channel equal. If some channel is inferior to other channel, there is always a way to improve the inferior one until all channels show the same characteristics. For MIMO, each channel is first designed by standard CDM. Then some de-coupling controller, which does not affect characteristic polynomial of the total system, is added. A trial design was made for the longitudinal control of modern fighter in MATLAB robust control toolbox [14][15].

6. CONCLUSION

The important points of this paper are as follows:

- (1) Coefficient diagram plays a very important role in CDM. The trade-off between performance specification (Stability, response, and robustness) and controller limitation (Order/complexity of controller, response, and power requirement) can be made intuitively.
- (2) The convexity of coefficient diagram represents stability. This is the most important feature.
- (4) Coefficient diagram has the same amount of information as Bode diagram. Coefficient diagram is more convenient than Bode diagram in design stage.
- (5) H-inf problem can be solved without H-inf algorithm.

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