IMAGE PARTIONING BY LEVEL SET MULTIREGION COMPETITION

Abdol-Reza Mansouri, Amar Mitiche, Carlos Vázquez

INRS-EMT, Place Bonaventure, 800 de la Gauchetiere West, Suite 6900,
Montréal, Québec, Canada, H5A 1K6

ABSTRACT

The purpose of this study is to investigate a new representation of a partition of an image domain into a given number of regions, and its use in the context of region competition to provide an extensional level set *multiregion competition* algorithm. In contrast with the standard region competition formulation, this multiregion competition formulation leads to a system of coupled curve evolution equations which is easily amenable to a level set implementation. Minimization of the functional guarantees an unambiguous segmentation. We provide a common statement of the multiregion competition algorithm for intensity, motion, and disparity based segmentation. Experimental results are shown.

1. INTRODUCTION

Image segmentation is a fundamental problem in image processing and computer vision with numerous useful applications. Various algorithms have been developped following formulations of segmentation as a Bayesian estimation problem or, more generally, as a functional minimization problem, where the solution is provided by evolution equations of closed simple plane curves. Segmentation formulations are difficult to extend beyond the two-region case of foreground and background using the standard correspondence between the regions of segmentation and the regions bounded by the curves because this correspondence leads to ambiguities when the interiors of two or more curves overlap. How to guarantee that the functional minimization results in a *partition* of the image domain is a major question answered in various ways: (1) start with an initial partition into regions bounded by curves γ_i and evolve $\Gamma = \cup\gamma_i$ according to a functional that references Γ as a partition. This is the approach in [1] and called *region competition* for the case of two regions. However, its implementation requires a delicate initialisation and an explicit representation of Γ as a set of points on the image domain grid that does not accomodate changes in the topology of the curves during their evolution, (2) use a term in the functional that draws the solution towards a partition [2] [3]. This does not guarantee a partition. Curve evolution will likely give an ambiguous segmentation if the partition constraint is not sufficiently enforced, and if it is strongly enforced the curves will evolve more as a result of the partition constraint than of image statistics, (3) use a functional that results in curve evolution equations where the evolution of a curve involves a reference to the others [4]. In [4], however, a segmentation into N regions can be obtained only for vector images of dimension $N-1$ of higher. Also, the observation term in the functional measures an $N-1$-dimensional volume, resulting in excessive computational demand, (4) generalize directly the curve evolution equations of the two-region case, rather than the functional itself [5]. This allows the use of non-differentiable operators that cannot be introduced in the functional, such as $\min$ or $\max$, to express the competition for points between regions. Although this is a convenient generalization, the computed solution looses its tie to the original functional, (5) establish an explicit correspondence between the regions of segmentation and regions defined by the curves [6]. It guarantees a partition at all times during the curves evolution, and the computed solution is one that minimizes the original functional. The method in [6], seeks a segmentation into up to a power of 2 number of regions. There is no clear indication on the actual number of regions the method yields since this depends not just on the image but also on the weight of the regularization term. In some instances, unwanted division of regions can occur, as with image segments of planar intensity variation.

The purpose of this study is to investigate a new representation of a partition of an image domain into a fixed but arbitrary number of regions by explicit correspondence between the regions of segmentation and regions defined by simple closed planar curves, and the use of this representation in the context of region competition [1] to provide an extensional level set *multiregion competition* algorithm. The functional we use leads to a system of coupled curve evolution equations which is easily amenable to a level set implementation, in contrast with the functional in [1]. This formulation allows segmentation of images that are not necessarily vectorial into a fixed but arbitrary number of regions, in contrast with the formulations in [4] [6]. The functional minimization guarantees an unambiguous segmentation because at all time during curve evolution the evolving regions form a partition of the image domain. Finally, we provide a common statement of *multiregion competition* for intensity, motion, and disparity based segmentation.

2. BASIC MODELS

Let $\mathbf{I}_k : \Omega \to \mathbb{R}$, $k = 1, \dots, n$, be a sequence of image functions, with common domain $\Omega \subset \mathbb{R}^2$. Formulating segmentation as a Bayesian estimation problem, the segmentation estimate of maximum a posteriori probability $\{\widehat{\mathbf{R}}_i\}_{i=1}^N$ is given by:

$$\begin{aligned}\{\widehat{\mathbf{R}}_i\}_{i=1}^N &= \arg \max_{\{\mathbf{R}_i \subset \Omega\}} P((\mathbf{I}_k)_k | \{\mathbf{R}_i\}) P(\{\mathbf{R}_i\}) \\ &= \arg \max_{\{\mathbf{R}_i \subset \Omega\}} P(\mathbf{I}_l | (\mathbf{I}_k)_{k \neq l}, \{\mathbf{R}_i\}) P((\mathbf{I}_k)_{k\neq l} | \{\mathbf{R}_i\}) P(\{\mathbf{R}_i\})\end{aligned}$$

For image, motion, and stereo based segmentation, image formation can be modelled via a family of image-valued functions $(\phi_j)_{j=1}^N$ defined on the space of images, such that

$$\mathbf{I}_l = \sum_{j=1}^N \phi_j((\mathbf{I}_k)_{k\neq l})\chi_{\mathbf{R}_j} + \mu$$

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where $\chi_{\mathbf{R}_j}$ is the indicator function of the set $\mathbf{R}_j$, defined by $\chi_{\mathbf{R}}(\mathbf{x}) = 1$ (resp. 0) if $\mathbf{x} \in \mathbf{R}$ (resp. $\mathbf{x} \notin \mathbf{R}$), and μ is a stationnary white Gaussian noise process with zero mean and covariance matrix Σ. This model is general enough to approximate the image formation process for intensity, motion, and stereo based segmentation. For intensity image segmentation, for instance, the family (ϕ_j) could be defined by

$$\phi_j((\mathbf{I}_k)_{k\neq l}) = \alpha_j$$

where $\alpha_j : \Omega \to \mathbb{R}$ is the restriction to region $\mathbf{R}_j$ of the noiseless image underlying the observed image. For motion and stereo based segmentation, the family (ϕ_j) could be defined by

$$\phi_j((\mathbf{I}_k)_{k\neq l}) = \mathbf{I}_{l+1} \circ T_j$$

where $T_j : \Omega \to \Omega$ is the motion (resp. stereo) transformation corresponding to region $\mathbf{R}_j$. More generality can be attained by assuming dependence of the ϕ_j on the segmentation $\{\mathbf{R}_i\}$. However, this level of generality is in no way essential to the presentation of our multiregion competition formulation. Therefore, we will assume that the ϕ_j are independent of the segmentation $\{\mathbf{R}_i\}$. We will also assume that the ϕ_j are either computed prior to the segmentation, as in [3] [5], for instance, or are computed iteratively as suggested in [1] in parallel with the segmentation. Assuming $(\mathbf{I}_k)_{k\neq l}$ is independent of the segmentation $\{\mathbf{R}_i\}$ and using independence of the noise process μ, we have:

$$\begin{aligned}
&\{P(\mathbf{I}_l|(\mathbf{I}_k)_{k\neq l}, \{\mathbf{R}_i\}) \\
&\quad = P(\sum_{j=1}^{N} \phi_j((\mathbf{I}_k)_{k\neq l})\chi_{\mathbf{R}_j} + \mu|(\mathbf{I}_k)_{k\neq l}, \{\mathbf{R}_i\}) \\
&\quad = \prod_{j=1}^{N} \prod_{\mathbf{x}\in\mathbf{R}_j} \mathcal{N}(\mathbf{I}_l(\mathbf{x}) - \phi_j((\mathbf{I}_k)_{k\neq l})(\mathbf{x}), \Sigma)
\end{aligned}$$

where $\mathcal{N}(\nu, \Sigma)$ denotes the Gaussian with mean ν and covariance matrix Σ. Finally, taking the logarithm of the right hand side, the Bayesian estimation problem is converted to the following energy minimization problem:

$$\{\widehat{\mathbf{R}}_i\}_{i=1}^{N} = \arg \min_{\{\mathbf{R}_i \subset \Omega\}} \sum_{j=1}^{N} \int_{\mathbf{R}_j} \xi_j(\mathbf{x}))d\mathbf{x} - \log P(\{\mathbf{R}_i\}) \quad (1)$$

$$\xi_j(\mathbf{x}) \equiv \frac{1}{2}(\mathbf{I}_l(\mathbf{x}) - \phi_j((\mathbf{I}_k)_{k\neq l})(\mathbf{x}))^t \Sigma^{-1}(\mathbf{I}_l(\mathbf{x}) - \phi_j((\mathbf{I}_k)_{k\neq l})(\mathbf{x}))$$

3. MULTIREGION COMPETITION ALGORITHM

3.1. Representation of a partition

Ambiguities in segmentation can be eliminated altogether without the use of any penalty terms in the energy functional as in [3] [2], or a requirement on the number N of regions such as N being a power of 2 as in [6], by establishing a suitable correspondence between regions enclosed by closed simple plane curves and regions in the segmentation which guarantees that at all time the partition constraint is maintained. Consider a family $\vec{\gamma}_i : [0, 1] \to \Omega, i = 1, \ldots, N-1$, of plane curves parametrized by the arc parameter $s \in [0, 1]$. We propose the following correspondence between the family $\{\mathbf{R}_{\vec{\gamma}_i}\}$ of regions enclosed by the curves $\{\vec{\gamma}_i\}$ and the segmentation $\{\mathbf{R}_i\}$ of the image domain Ω: We associate to region $\mathbf{R}_1$ of the segmentation the region $\mathbf{R}_{\vec{\gamma}_1}$ inside the curve $\vec{\gamma}_1$; to region $\mathbf{R}_2$ of the segmentation, however, we associate the region $\mathbf{R}^c_{\vec{\gamma}_1} \cap \mathbf{R}_{\vec{\gamma}_2}$; to region $\mathbf{R}_3$ of the segmentation, we similarly associate region $\mathbf{R}^c_{\vec{\gamma}_1} \cap \mathbf{R}^c_{\vec{\gamma}_2} \cap \mathbf{R}_{\vec{\gamma}_3}$, and continuing this construction, we associate to region $\mathbf{R}_k$ of the segmentation (for $k \leq N-1$) the region $\mathbf{R}^c_{\vec{\gamma}_1} \cap \mathbf{R}^c_{\vec{\gamma}_2} \cap \cdots \cap \mathbf{R}^c_{\vec{\gamma}_{k-1}} \cap \mathbf{R}_{\vec{\gamma}_k}$ defined by the plane curves $\{\vec{\gamma}_i\}$, while region $\mathbf{R}_N$ of the segmentation is finally associated to the region $\mathbf{R}^c_{\vec{\gamma}_1} \cap \mathbf{R}^c_{\vec{\gamma}_2} \cap \cdots \cap \mathbf{R}^c_{\vec{\gamma}_{k-1}} \cap \mathbf{R}^c_{\vec{\gamma}_{N-1}} = (\cup_{j=1}^{N-1} \mathbf{R}_j)^c$. The family $\{\mathbf{R}_{\vec{\gamma}_1}, \mathbf{R}^c_{\vec{\gamma}_1} \cap \mathbf{R}_{\vec{\gamma}_2}, \mathbf{R}^c_{\vec{\gamma}_1} \cap \mathbf{R}^c_{\vec{\gamma}_2} \cap \mathbf{R}_{\vec{\gamma}_3}, \ldots\}$ thus obtained is clearly a partition of the image domain, for any family of plane curves $(\vec{\gamma}_i)_{i=1}^{N-1}$. The partition representation is shown in Figure 1 for four regions.

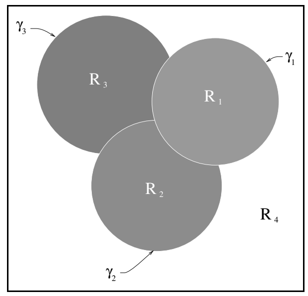

Fig. 1. Representation of a partition.

3.2. Multiregion Competition Functional

With our choice of representation of a partition of the image domain into N regions, the energy functional (1) becomes:

$$\begin{aligned}
E[\{\vec{\gamma}_i\}_{i=1}^{N-1}] &= \int_{\mathbf{R}_{\vec{\gamma}_1}} \xi_1(\mathbf{x})d\mathbf{x} + \int_{\mathbf{R}^c_{\vec{\gamma}_1} \cap \mathbf{R}_{\vec{\gamma}_2}} \xi_2(\mathbf{x})d\mathbf{x} \\
&+ \int_{\mathbf{R}^c_{\vec{\gamma}_1} \cap \mathbf{R}^c_{\vec{\gamma}_2} \cap \mathbf{R}_{\vec{\gamma}_3}} \xi_3(\mathbf{x})d\mathbf{x} + \ldots \\
&+ \int_{\mathbf{R}^c_{\vec{\gamma}_1} \cap \mathbf{R}^c_{\vec{\gamma}_2} \cap \mathbf{R}^c_{\vec{\gamma}_3} \cdots \cap \mathbf{R}_{\vec{\gamma}_{N-1}}} \xi_{N-1}(\mathbf{x})d\mathbf{x} \\
&+ \int_{\mathbf{R}^c_{\vec{\gamma}_1} \cap \mathbf{R}^c_{\vec{\gamma}_2} \cap \mathbf{R}^c_{\vec{\gamma}_3} \cdots \cap \mathbf{R}^c_{\vec{\gamma}_{N-1}}} \xi_N(\mathbf{x})d\mathbf{x} \\
&+ \lambda \sum_{j=1}^{N-1} \oint_{\vec{\gamma}_j} ds \qquad (2)
\end{aligned}$$

3.3. Curve Evolution Equations

The minimization of the functional E in equation (2) with respect to the curves $(\vec{\gamma}_j)_j$ is again performed by embedding the family $\vec{\gamma}_i : [0, 1] \to \Omega, i = 1, \ldots, N-1$ of plane curves into a one-parameter family $\vec{\gamma}_i : [0, 1] \times \mathbb{R}^+ \to \Omega, \quad i = 1, \ldots, N-1$ of plane curves constructed by solving the following system of evolution equations:

$$\frac{d\vec{\gamma}_j}{dt} = -\frac{\delta E}{\delta \vec{\gamma}_j}, \quad j = 1, \ldots, N-1.$$

The functional derivatives $\frac{\delta E}{\delta \vec{\gamma}_j}$ can be easily computed by suitably rewriting the area integrals appearing in the energy functional. Starting with $\vec{\gamma}_1$, we can rewrite the energy functional (2) as follows:

$$\begin{aligned} E[\{\vec{\gamma}_i\}_{i=1}^{N-1}] &= \int_{\mathbf{R}_{\vec{\gamma}_1}} \xi_1(\mathbf{x})d\mathbf{x} + \int_{\mathbf{R}^c_{\vec{\gamma}_1}} \Phi_1(\mathbf{x})d\mathbf{x} \\ &+ \lambda \oint_{\vec{\gamma}_1} ds + \lambda \sum_{j=2}^{N-1} \oint_{\vec{\gamma}_j} ds \end{aligned}$$

where $\Phi_1(\mathbf{x})$ is defined as

$$\begin{aligned} \Phi_1(\mathbf{x}) &= \xi_2(\mathbf{x})\chi_{\mathbf{R}_{\vec{\gamma}_2}}(\mathbf{x}) \\ &+ \xi_3(\mathbf{x})\chi_{\mathbf{R}^c_{\vec{\gamma}_2}}(\mathbf{x})\chi_{\mathbf{R}_{\vec{\gamma}_3}}(\mathbf{x}) + \ldots \\ &+ \xi_{N-1}(\mathbf{x})\chi_{\mathbf{R}^c_{\vec{\gamma}_2}}(\mathbf{x})\chi_{\mathbf{R}^c_{\vec{\gamma}_3}}(\mathbf{x}) \ldots \chi_{\mathbf{R}^c_{\vec{\gamma}_{N-2}}}(\mathbf{x})\chi_{\mathbf{R}_{\vec{\gamma}_{N-1}}}(\mathbf{x}) \\ &+ \xi_N(\mathbf{x})\chi_{\mathbf{R}^c_{\vec{\gamma}_2}}(\mathbf{x})\chi_{\mathbf{R}^c_{\vec{\gamma}_3}}(\mathbf{x}) \ldots \chi_{\mathbf{R}^c_{\vec{\gamma}_{N-2}}}(\mathbf{x})\chi_{\mathbf{R}^c_{\vec{\gamma}_{N-1}}}(\mathbf{x}) \end{aligned}$$

Since $\Phi_1(\mathbf{x})$ and $\sum_{j=2}^{N-1} \oint_{\vec{\gamma}_j} ds$ have no dependence on $\vec{\gamma}_1$, the functional derivative $\frac{\delta E}{\delta \vec{\gamma}_1}$ is computed as for the standard region competition functional described in [1], yielding:

$$\frac{\delta E}{\delta \vec{\gamma}_1}(\vec{\gamma}_1(s,t)) = [\xi_1(\vec{\gamma}_1(s,t)) - \Phi_1(\vec{\gamma}_1(s,t)) + \lambda\kappa_1(s,t)]\vec{n}_1(s,t)$$

where $\vec{n}_1$ is the outward unit normal to $\vec{\gamma}_1$, and κ_1 the curvature function of $\vec{\gamma}_1$.

To compute the functional derivative $\frac{\delta E}{\delta \vec{\gamma}_2}$ yielding the evolution equation of $\vec{\gamma}_2$, we rewrite the energy functional (2) as follows:

$$\begin{aligned} E[\{\vec{\gamma}_i\}_{i=1}^{N-1}] &= \int_{\mathbf{R}_{\vec{\gamma}_2}} \chi_{\mathbf{R}^c_1}(\mathbf{x})\xi_2(\mathbf{x})d\mathbf{x} \\ &+ \int_{\mathbf{R}^c_{\vec{\gamma}_2}} \chi_{\mathbf{R}^c_1}(\mathbf{x})\Phi_2(\mathbf{x})d\mathbf{x} + \lambda \oint_{\vec{\gamma}_2} ds + \lambda \sum_{j\neq 2} \oint_{\vec{\gamma}_j} ds \end{aligned}$$

where $\Phi_2(\mathbf{x})$ is defined as

$$\begin{aligned} \Phi_2(\mathbf{x}) &= \xi_3(\mathbf{x})\chi_{\mathbf{R}_{\vec{\gamma}_3}}(\mathbf{x}) \\ &+ \xi_4(\mathbf{x})\chi_{\mathbf{R}^c_{\vec{\gamma}_3}}(\mathbf{x})\chi_{\mathbf{R}_{\vec{\gamma}_4}}(\mathbf{x}) + \ldots \\ &+ \xi_{N-1}(\mathbf{x})\chi_{\mathbf{R}^c_{\vec{\gamma}_3}}(\mathbf{x})\chi_{\mathbf{R}^c_{\vec{\gamma}_4}}(\mathbf{x}) \ldots \chi_{\mathbf{R}^c_{\vec{\gamma}_{N-2}}}(\mathbf{x})\chi_{\mathbf{R}_{\vec{\gamma}_{N-1}}}(\mathbf{x}) \\ &+ \xi_N(\mathbf{x})\chi_{\mathbf{R}^c_{\vec{\gamma}_3}}(\mathbf{x})\chi_{\mathbf{R}^c_{\vec{\gamma}_4}}(\mathbf{x}) \ldots \chi_{\mathbf{R}^c_{\vec{\gamma}_{N-2}}}(\mathbf{x})\chi_{\mathbf{R}^c_{\vec{\gamma}_{N-1}}}(\mathbf{x}) \end{aligned}$$

Here again, since $\Phi_2(\mathbf{x})$ and $\sum_{j\neq 2} \oint_{\vec{\gamma}_j} ds$ have no dependence on $\vec{\gamma}_2$, the functional derivative $\frac{\delta E}{\delta \vec{\gamma}_2}$ is computed as for the standard region competition functional, yielding:

$$\begin{aligned} \frac{\delta E}{\delta \vec{\gamma}_2}(\vec{\gamma}_2(s,t)) &= (\ \chi_{\mathbf{R}^c_1}(\vec{\gamma}_2(s,t))[\xi_2(\vec{\gamma}_2(s,t)) \\ &- \Phi_2(\vec{\gamma}_1(s,t))] + \lambda\kappa_2(s,t)\ )\ \vec{n}_2(s,t) \end{aligned}$$

where $\vec{n}_2$ is the outward unit normal to $\vec{\gamma}_2$, and κ_2 the curvature function of $\vec{\gamma}_2$. Proceeding similarly to compute the functional derivatives $\frac{\delta E}{\delta \vec{\gamma}_j}$ for all j, the minimization of the multiregion competition functional is achieved through the following system of coupled curve evolution equations:

$$\begin{aligned} \frac{d\vec{\gamma}_1}{dt} &= -\left(\xi_1(\vec{\gamma}_1) - \Phi_1(\vec{\gamma}_1) + \lambda\kappa_1\right)\vec{n}_1 \\ \frac{d\vec{\gamma}_2}{dt} &= -\left(\chi_{\mathbf{R}^c_1}(\vec{\gamma}_2)[\xi_2(\vec{\gamma}_2) - \Phi_2(\vec{\gamma}_2)] + \lambda\kappa_2\right)\vec{n}_2 \\ &\ldots \\ \frac{d\vec{\gamma}_j}{dt} &= -(\ \chi_{\mathbf{R}^c_1}(\vec{\gamma}_j)\ldots\chi_{\mathbf{R}^c_{j-1}}(\vec{\gamma}_j) \\ &\quad [\xi_j(\vec{\gamma}_j) - \Phi_j(\vec{\gamma}_j)] + \lambda\kappa_j\ )\ \vec{n}_j \\ &\ldots \\ \frac{d\vec{\gamma}_{N-1}}{dt} &= -(\ \chi_{\mathbf{R}^c_1}(\vec{\gamma}_{N-1})\ldots\chi_{\mathbf{R}^c_{N-2}}(\vec{\gamma}_{N-1}) \\ &\quad [\xi_{N-1}(\vec{\gamma}_{N-1}) - \Phi_{N-1}(\vec{\gamma}_{N-1})] + \lambda\kappa_{N-1}\ )\ \vec{n}_{N-1} \end{aligned} \quad (3)$$

where $\vec{n}_j$ is the outward unit normal to $\vec{\gamma}_j$ and κ_j the curvature function of $\vec{\gamma}_j$, for $j = 1, \ldots, N-1$, and $\Phi_j(\mathbf{x})$ is given by

$$\begin{aligned} \Phi_j(\mathbf{x}) &= \xi_{j+1}(\mathbf{x})\chi_{\mathbf{R}_{\vec{\gamma}_{j+1}}}(\mathbf{x}) \\ &+ \xi_{j+2}(\mathbf{x})\chi_{\mathbf{R}^c_{\vec{\gamma}_{j+1}}}(\mathbf{x})\chi_{\mathbf{R}_{\vec{\gamma}_{j+2}}}(\mathbf{x}) + \ldots \\ &+ \xi_{N-1}(\mathbf{x})\chi_{\mathbf{R}^c_{\vec{\gamma}_{j+1}}}(\mathbf{x})\ldots\chi_{\mathbf{R}^c_{\vec{\gamma}_{N-2}}}(\mathbf{x})\chi_{\mathbf{R}_{\vec{\gamma}_{N-1}}}(\mathbf{x}) \\ &+ \xi_N(\mathbf{x})\chi_{\mathbf{R}^c_{\vec{\gamma}_{j+1}}}(\mathbf{x})\ldots\chi_{\mathbf{R}^c_{\vec{\gamma}_{N-2}}}(\mathbf{x})\chi_{\mathbf{R}^c_{\vec{\gamma}_{N-1}}}(\mathbf{x}) \end{aligned}$$

3.4. Level Set Implementation

We refer the reader to [7] for a discussion on level sets and related numerical schemes. We represent the curve $\vec{\gamma}_j$ *implicitly* by the zero level set of a function $u_j : \mathbb{R}^2 \to \mathbb{R}$ (with $j = 1, \ldots, N-1$), with the region inside $\vec{\gamma}_j$ corresponding to $u_j > 0$. The level set evolution equations corresponding to (3) are then given by the following system of coupled partial differential equations:

$$\begin{aligned} \frac{\partial u_1}{\partial t}(\mathbf{x},t) &= -\left(\xi_1(\mathbf{x}) - \Phi_1(\mathbf{x}) + \lambda\kappa_1\right)\|\vec{\nabla}u_1(\mathbf{x},t)\| \\ &\ldots \\ \frac{\partial u_j}{\partial t}(\mathbf{x},t) &= -(\ \chi_{\{u_1(\mathbf{x},t)\le 0\}}\ldots\chi_{\{u_{j-1}(x,t)\le 0\}} \\ &\quad [\xi_j(\mathbf{x}) - \Phi_j(\mathbf{x})] + \lambda\kappa_j\ )\ \|\vec{\nabla}u_j(\mathbf{x},t)\| \\ &\ldots \\ \frac{\partial u_{N-1}}{\partial t}(\mathbf{x},t) &= -(\ \chi_{\{u_1(\mathbf{x},t)\le 0\}}\ldots\chi_{\{u_{N-2}(\mathbf{x},t)\le 0\}} \\ &\quad [\xi_{N-1}(\mathbf{x}) - \Phi_{N-1}(\mathbf{x})] \\ &\quad +\lambda\kappa_{N-1}\ )\ \|\vec{\nabla}u_{N-1}(\mathbf{x},t)\|, \end{aligned}$$

where $\chi_{\{u_k(\mathbf{x},t)\le 0\}} = 1$ if $u_k(\mathbf{x},t) \le 0$ and 0 otherwise, and $\Phi_j(\mathbf{x})$ is given by

$$\begin{aligned} \Phi_j(\mathbf{x}) &= \xi_{j+1}(\mathbf{x})\chi_{\{u_{j+1}(\mathbf{x},t)>0\}}(\mathbf{x}) \\ &+ \xi_{j+2}(\mathbf{x})\chi_{\{u_{j+1}(\mathbf{x},t)\le 0\}}(\mathbf{x})\chi_{\{u_{j+2}(\mathbf{x},t)>0\}}(\mathbf{x}) \\ &+ \ldots \\ &+ \xi_{N-1}(\mathbf{x})\chi_{\{u_{j+1}(\mathbf{x},t)\le 0\}}(\mathbf{x})\ldots\chi_{\{u_{N-2}(\mathbf{x},t)\le 0\}}(\mathbf{x}) \\ &\quad \chi_{\{u_{N-1}(\mathbf{x},t)>0\}}(\mathbf{x}) \\ &+ \xi_N(\mathbf{x})\chi_{\{u_{j+1}(\mathbf{x},t)\le 0\}}(\mathbf{x})\ldots\chi_{\{u_{N-2}(\mathbf{x},t)\le 0\}}(\mathbf{x}) \\ &\quad \chi_{\{u_{N-1}(\mathbf{x},t)\le 0\}}(\mathbf{x}) \end{aligned}$$

and κ_{u_j} is the curvature of the level set of u_j, with κ_u being given as a function of u by the following expression:

$$\kappa_u = -\vec{\nabla} \cdot \left(\vec{\nabla} u / \|\vec{\nabla} u\|\right)$$

Defining

$$\mathbf{R}_{u_i}(t) = \{\mathbf{x} \in \Omega | u_i(\mathbf{x}, t) > 0\}, \quad i = 1, \ldots, N-1,$$

the desired segmentation is then given as $t \to \infty$ by the family:

$$\{\mathbf{R}_{u_1}(t), \mathbf{R}_{u_1}(t)^c \cap \mathbf{R}_{u_2}(t), \mathbf{R}_{u_1}(t)^c \cap \mathbf{R}_{u_2}(t)^c \cap \mathbf{R}_{u_3}(t) \\ \ldots (\cup_{j=1}^{N-1} \mathbf{R}_{u_j}(t))^c\}$$

4. EXPERIMENTAL RESULTS

To validate the algorithm, we ran a series of tests with conclusive results. We tested on real grey level and color images, as well as image sequences for motion based segmentation. We show here one challenging example (we show only one example for lack of space). The image is shown in Figure 2a. Visual examination indicates four regions: highway and areas around buildings; parking and buildings roof; river; woods. Figure 2a contains the three initial curves on the original image. 2b shows the final segmentation where each region is painted with its average grey scale. Figures 2c and 2d show two of the four regions of the final partition, on black background. The last two figures show the other two regions on white background. The obtained partition is very well in agreement with our expectation.

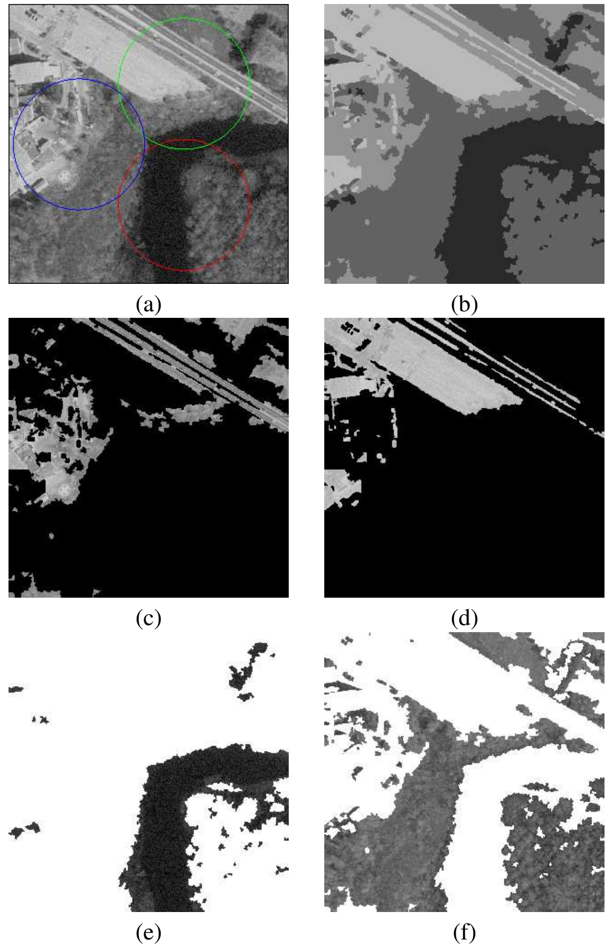

Fig. 2. Image segmentation: (a) original image with initialization; (b) final segmentation represented by region means; segmented regions: (c) highway and regions around buildings; (d) parking and buildings roof; (e) river; (f) wood.

5. CONCLUSION

We investigated a novel representation of a partition of an image domain into a fixed but arbitrary number of regions, and the use of this representation in the context of region competition to provide an extensional level set *multiregion competition* algorithm. We provided a common statement of the multiregion competition algorithm for intensity, motion, and disparity based segmentation. The multiregion competition formulation led to a system of coupled curve evolution equations the resolution of which via level sets guaranteed an unambiguous segmentation. The algorithm and its implementation have been validated on real grey scale and color images, as well as image sequences for motion based segmentation.

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