TEXTURE SEGMENTATION USING DIRECTIONAL EMPIRICAL MODE DECOMPOSITION

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ABSTRACT

In this paper the technique of Directional Empirical Mode Decomposition (DEMD) and its application to texture segmentation are presented. Empirical Mode Decomposition (EMD) decomposes signals by sifting and then analyzes the instantaneous frequency of the obtained components called Intrinsic Mode Functions (IMFs). As a new form of extending 1-D EMD to 2-D case, DEMD considers the directional frequency and envelope at each point. One type of 2-D Hilbert transform is introduced to compute the analytical functions for the frequency and envelope. The technique of selecting directions for DEMD based on texture's Wold theory is also presented. Experimental results indicate the effectiveness of the method for texture segmentation.

1. INTRODUCTION

Texture segmentation is one of the main questions in perceptual organization for natural images. In the past two decades a large volume of methods have been presented for the question which can be classified into four classes: statistical methods [1], structural methods [2], model based methods [3] and filtering based methods [4]. In recent years, filtering based methods including Gabor filter, wavelet and nonlinear diffusion based techniques have been widely used [4, 5].

In this paper a renovate filtering based approach called Directional Empirical Mode Decomposition (DEMD) is presented for texture segmentation. The reason of introducing EMD to texture processing is as follows: Firstly, according to [6], the obtained Intrinsic Mode Functions (IMFs) by EMD [7] can be seen as results of filter bank for some kinds of signals resembling those involved in wavelet decomposition. Secondly, different from traditional adaptive filtering techniques such as wavelet/wavelet packet and nonlinear diffusion, EMD decomposes signals according to the local scales computed by identifying distance between adjacent extrema. To apply EMD to texture processing, we extend EMD to 2-D case as DEMD. Because the obtained 2-D IMFs are AM-FM components of original signals [7,8], directional frequency and envelopes of IMFs are defined as features for texture segmentation by 2-D Hilbert transform. As another form of 2-D EMD, Bidimentinal EMD (BEMD) [9] only extracts textures by computing envelopes using radial basis functions interpolation so costs much more time. The method of determining directions for DEMD are also presented based on texture's Wold theory [10].

This paper is organized as follows. Section 2 describes the definition and algorithm of DEMD. Then the feature extraction approach, the used 2-D Hilbert transform and the concrete algorithm are given in Section 3. Section 4 presents experimental results. Finally, the conclusions are given in Section 5.

2. DEFINITION AND ALGORITHM OF DEMD

2.1. The Definition of DEMD

EMD is proposed by Huang et al. [7] for processing nonstationary functions. This tool decomposes a signal into components called IMFs satisfying the following two conditions ($*$):

1. The number of extrema and the number of zero-crossings must either equal or differ at most by one.
2. At any point, the mean value of the envelope defined by the local maxima and the envelope by the local minima is zero.

The resultant decomposition can be written as:

$$f(t) = \sum_{i=1}^{N} \mathrm{imf}_i(t) + r_N(t),$$

where $\mathrm{imf}_i(t)$ are IMFs and monotonic function $r_N(t)$ is the residue.

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Similar to the above definition, 2-D IMF and DEMD are defined as follows.

Definition 1. Given $\theta \in [0, \frac{\pi}{2})$, signal $u(x, y)$ is defined as 2-D IMF corresponding to θ, if for any c it satisfies the following condition:

$$v_{1,c}^{\theta}(x) = u(x, [\tan\theta]x + c), \quad 0 \le \theta < \tfrac{\pi}{2}.$$

$$v_{2,c}^{\theta}(x) = \begin{cases} u(x, [\tan(\theta + \frac{\pi}{2})]x + c), & 0 < \theta < \frac{\pi}{2}, \\ u(\cdot, x), & \theta = 0, \end{cases} \qquad (1)$$

and both of $v_{1,c}^{\theta}(x)$ and $v_{2,c}^{\theta}(x)$ meet the conditions $(*)$. $v_{1,c}^{\theta}(x)$ and $v_{2,c}^{\theta}(x)$ are called the 1-D samplings of IMF.

Definition 2. DEMD of 2-D image $f(x, y)$ for direction θ is defined as the decomposition such that

$$f(x, y) = \sum_{i=1}^{N} \mathrm{imf}_i^{\theta}(x, y) + r_N^{\theta}(x, y), \qquad (2)$$

where $\mathrm{imf}_i^{\theta}(x, y)$, $i = 1, \ldots, N$, are 2-D IMFs for θ and $r_N^{\theta}(x, y)$ has at least one monotonic 1-D sampling for $v_{1,c}^{\theta}(x)$ or $v_{2,c}^{\theta}(x)$.

2.2. The Decomposition of DEMD

In 1-D EMD, IMFs are obtained by 1-D $sifting$ method [7]. To obtain 2-D IMFs, the following 2-D $sifting$ approach is adopted.

1. Initialize: $r_{i-1}(x, y) = f^{\theta}(x, y)$ $(i = 1)$ where $f^{\theta}(x, y)$ is the clockwise rotated form of $f(x, y)$ by θ.

2. Extract the $\mathrm{imf_i(x, y)}$:

   (a) Initialize: $h_0(x, y) = r_{i-1}(x, y), j = 1$.

   (b) Form the mean envelope of $h_{j-1}(x, y)$:

      i. Extract the local maxima and minima of each row of $h_{j-1}(x, y)$. Then generate the middle upper and lower envelopes by interpolating the maxima and minima of the samplings respectively: $h_{mu}(x, y)$, $h_{ml}(x, y)$;

      ii. Calculate the middle mean envelope as $m_{mid}(x, y) = (h_{mu}(x, y) + h_{ml}(x, y))/2$;

      iii. Compute the upper and lower envelopes of $m_{mid}(x, y)$ by extracting and interpolating local extrema for each column. So calculate the mean envelope of $m_{mid}(x, y)$ as $m_{j-1}(x, y) = (h_u(x, y) + h_l(x, y))/2$.

   (c) Compute $h_j(x, y) = h_{j-1}(x, y) - m_{j-1}(x, y)$, and $j = j + 1$.

   (d) Compute $SD = \sum_{x,y} \left[\frac{|(h_j(x,y) - h_{j-1}(x,y))|^2}{h_{j-1}^2(x,y)} \right]$.

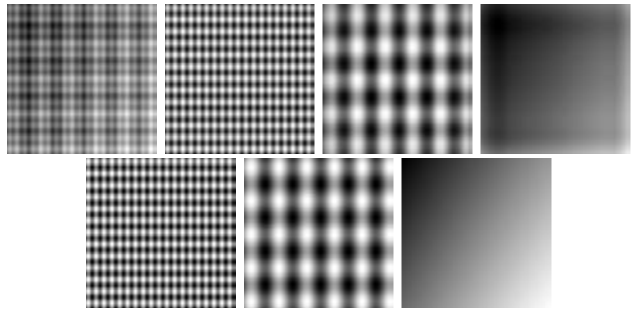

Fig. 1. The four images in the upper row are respectively part of synthetic texture, the first two IMFs and the residue using DEMD ($\theta = 0°$). The three images in the lower row are the three ingredients which constitute the synthetic texture.

   (e) Repeat step (b) to (d) until SD $< \varepsilon$ and put then $\mathrm{imf_i}(x, y) = h_j(x, y)$.

3. Update residua $r_i(x, y) = r_{i-1}(x, y) - \mathrm{imf}_i(x, y)$.

4. Repeat step 2-3 with $i = i+1$ until there exists monotonic 1-D sampling of $r_i(x, y)$ for horizontal or vertical direction.

5. Rotate the imf_j, $j = 1, \ldots, N$ $(N = i)$ and $r_N(x, y)$ anticlockwise by θ to form imf_j^{θ}, $j = 1, \ldots, N$ and $r_N^{\theta}(x, y)$.

Thus arbitrary 2-D signal $f(x, y)$ can be decomposed into the form (2). The upper row of Fig 1 shows one synthetic texture, its first two IMFs and the residue obtained by DEMD respectively. Three components of the texture are given in the lower row. Although similar results can be given by BEMD [9], complexity of DEMD is $O(N_1)$ while that of BEMD is $O(N_2^3)$ which can be reduced to $O(N_2 \log N_2)$ using fast algorithm (N_1 is the total point number, N_2 is the larger one among the numbers of maxima and minima which is proportional to N_1 in natural textures).

2.3. Determination of the Directions for DEMD

From the Definition 1 and 2, it is known that one key point of DEMD is the determination of decomposition directions. Based on 2-D Wold theory of textures [10], 2-D regular and homogeneous random field can be uniquely represented by three orthogonal components: (i) purely indeterministic and regular random field $w(x, y)$ with absolutely continuous spectrum; (ii) half-plane deterministic random field $p(x, y)$ with 2-D singular spectrum; (iii) generalized evanescent random field $g(x, y)$ with 1-D singular spectrum. The three parts are called 'randomness', 'directionality' and 'periodicity' components, respectively. The later two parts both reflect the effective directions for texture processing. So in our algorithm multiple directions corresponding to the two parts are extracted to improve the classification ability.

Among the multiple decompositions, the decomposition along directions corresponding to the 'directionality' parts are to process the nonstationarity along the directions. While the decomposition along directions corresponding to the 'periodicity' components are to distinguish the corresponding textures from others. To extract the 1-D singular spectrum corresponding to evanescent components, Randon transform of spectrum is utilized. The concrete algorithm is in the following:

1. Obtain the Fourier spectrum $F(u,v)$ of the multi-texture image $f(x,y)$;

2. Locate all the local maximum points $P_k, k=1,\dots,K$, of $F(u,v)$'s Radon transform with values exceeding C_1, then add the corresponding directions to the α_j, $j=1,\dots,K$;

3. Delete the values around P and do inverse Radon transform to compute $F^{'}(u,v)$. All the points around which the energy exceeds C_2 in $F^{'}(u,v)$ are extracted. Their respective directions are ordered by the respective energy as $\alpha_j, j=K+1,\dots,K+M$.

In the above process C_1 and C_2 are constants determined beforehand. So DEMD can be computed for the directions $\alpha_j, j=1,\dots,K+M$.

3. TEXTURE SEGMENTATION BY DEMD

Before presenting feature extraction for segmentation, the used 2-D Hilbert transform will be introduced.

3.1. 2-D Hilbert Transform

As one of the essential features for 1-D IMFs, instantaneous frequency is obtained through the corresponding analytical signals defined by 1-D Hilbert transform. But there has been no uniform definition for 2-D Hilbert transform [11]. Because of the directionality of DEMD one of the definitions is adopted in the following [11] after presenting the corresponding analytical function:

Definition 3. Given $f(x,y)$, its Fourier transform $F(u,v)$ and the angel θ, define $F_A(u,v)$ as

$$F_A(\mathbf{u}) = \begin{cases} 2F(\mathbf{u}), & \mathbf{u}\cdot\mathbf{e} > 0, \\ F(\mathbf{u}), & \mathbf{u}\cdot\mathbf{e} = 0, \\ 0, & \mathbf{u}\cdot\mathbf{e} < 0. \end{cases}$$

where $\mathbf{u}=(u,v)$ is the coordinate vector and $\mathbf{e}=(\cos\theta,\sin\theta)$. Then the inverse Fourier transform $f_A(x,y)$ of $F_A(u,v)$ is called the analytical function of $f(x,y)$ for θ.

Definition 4. The Hilbert transform $f_H(x,y)$ of $f(x,y)$ for θ is defined as the imaginary part of $f_A(x,y)$.

3.2. Feature Vector

Assume $\theta_j, j=1,\dots,K+M$ are the extracted directions in Section 2. So DEMD are computed according to directions θ_j in the following:

$$f(x,y) = \sum_{i=1}^{N} \mathrm{imf}_i^j(x,y) + r_N^j(x,y),\ j=1,\dots,K+M,$$

where $\mathrm{imf}_i^j(x,y)$ denotes the i^{th} IMF of DEMD for directions θ_j.

Let $\mathrm{imf}_{iH}^j(x,y)$ be the 2-D Hilbert transform of $\mathrm{imf}_i^j(x,y)$ for θ_j and $f_{iA}^j(u,v)$ be the corresponding analytical function. Then the envelope and phase are defined as follows:

$$A_i^j = \sqrt{(\mathrm{imf}_i^j)^2 + (\mathrm{imf}_{iH}^j)^2}, \tag{3}$$

$$\varphi_i^j = \arctan(\frac{\mathrm{imf}_{iH}^j}{\mathrm{imf}_i^j}), \tag{4}$$

where $i=1,\dots,N, j=1,\dots,K+M$.

The directional frequency is computed by directional derivative along θ_j in the following:

$$\omega_i^j = \frac{d}{d\vec{j}}\varphi_i^j = \frac{\mathrm{imf}_i^j\cdot\frac{d}{d\vec{j}}\mathrm{imf}_{iH}^j - \mathrm{imf}_{iH}^j\cdot\frac{d}{d\vec{j}}\mathrm{imf}_i^j}{(\mathrm{imf}_i^j)^2 + (\mathrm{imf}_{iH}^j)^2}. \tag{5}$$

From the above process, two groups of features ((**??**) and (**??**)) corresponding to each pixel are obtained like this: $V(m,n) = \{A_i^j(m,n), \omega_i^j(m,n)\}_{1\le i\le N, 1\le j\le K+M}$.

3.3. Concrete Algorithm

The following process is adopted to do texture segmentation by the extracted features mentioned above:

1. Firstly, decompose textures by DEMD along the $K+M$ directions computed from Section 2.3;

2. Secondly, extract two directional frequency and two directional envelopes from each IMF as features;

3. Thirdly, k-mean clustering is done using the above obtained features;

4. Finally, morphological dilation and erosion [12] are used to optimize segmentation results.

4. EXPERIMENTS

In this section experimental evaluation for DEMD to do texture segmentation is described. Standard textures in Brodatz set [13] are used.

The images in figure 2 and 3 are respectively the original image, the used mosaics, the segmentation result and misclassified points. Figure 2 shows the segmentation result of 200×200 image composed of three textures. Only 5% of the pixels in the image are misclassified. Considering the clustering of coordinates it is not surprising that the narrow part of the texture image have more misclassified points. Segmentation result of image with size 512×512 composed of 8 textures in Brodatz set is shown in figure 3. Since the image contains texture of large scale and textures difficult to differentiate, the ratio of pixels being misclassified increases to 9%.

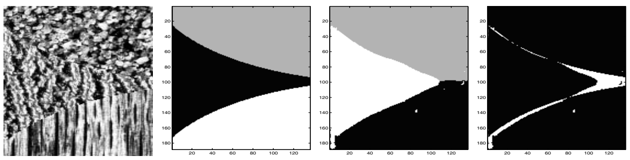

Fig. 2. Images of textures, the textures' mosaic, segmentation result and misclassified pixels.

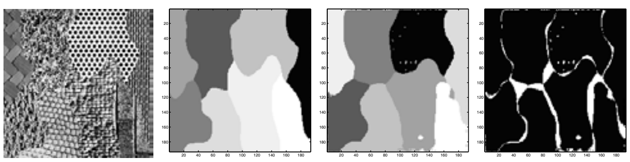

Fig. 3. Images of textures, the textures' mosaic, segmentation result and misclassified pixels.

5. CONCLUSIONS

In this paper the method of DEMD is presented and applied to texture segmentation. Different from other adaptive filtering techniques such as wavelet and nonlinear diffusion, DEMD computes local scale by identifying distance between adjacent extrema. It has two virtues compared with existing image processing algorithms using EMD: directionality and ease of extracting features. Experiments indicate the effectiveness of DEMD to texture segmentation.

Our future plans involve improving the segmentation results of DEMD and applying it to other texture processing problems such as texture recognition. Method of computing number of texture classes should also be considered for unsupervised segmentation using this technique.

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