IMAGE FUSION USING WEIGHTED MULTISCALE FUNDAMENTAL FORM

Tao Chen, Ruosan Guo and Silong Peng

National ASIC Design Engineering Center
Institute of Automation, Chinese Academy of Sciences
Beijing 100080-2728, China
{tao.chen}{ruosan.guo}{silong.peng}@mail.ia.ac.cn

ABSTRACT

Several images to be fused can be taken as a multivalued image. From multiscale fundamental form (MFF), a multivalued image wavelet representation can be obtained. In order to avoid the enlargement of the wavelet coefficients, the weighted multiscale fundamental form (WMFF) is exploited in our fusion process. The mutual information and the conditioned entropy are introduced to evaluate the fused result. Compared with the multiscale fundamental form, the weighted one have a better performance on image fusion.

1. INTRODUCTION

Image fusion is the combination of two or more different images to form a new image using a certain algorithm [1]. Now the application area of image fusion is extensive.

In general, the techniques of image fusion are grouped into three classes: (i) color-related techniques; (ii) numerical/statistical methods; (iii) techniques based on multiscale decomposition. Wavelet, as a promising tool, takes more and more attention. Li et al [2] proposed a choose-max wavelet fusion algorithm. In [3], Núñez et al fused a high resolution panchromatic image (SPOT) with a low resolution multispectral image (Landsat TM) using the AWL algorithm. A novel algorithm based on multiscale first fundamental form was presented by Scheunders et al in [4]. In those papers, a new multivalued image representation , i.e. multiscale fundamental form, was derived from the first fundamental form in [5] and a dyadic discrete wavelet transform in [6].

In this paper, the disadvantages of the first fundamental form are analyzed. It distributes the same weight to all the gradient of the input images. This paper proposes a modified first fundamental form, i.e. the weighted first fundamental form. The pointwise weight depends on the gradient magnitudes of the multi-image components. Correspondingly, the multiscale fundamental form becomes the weighted multiscale fundamental form.

This paper is organized as follows. The next section introduces the concept of MFF and the MFF based fusion algorithm in [4]. In Section 3, the weighted multiscale fundamental form is presented. Section 4 gives the experimental results. Conclusion remarks are shown in Section 5.

2. IMAGE FUSION BASED ON MULTISCALE FUNDAMENTAL FORM

2.1. *The Multiscale Fundamental Form*

In this subsection, the multiscale fundamental form is introduced following [4].

Let $\mathbf{I} = (I_1, I_2, \ldots, I_N) : R^2 \longrightarrow R^N$ be a continuous multivalued image. The differential of $\mathbf{I}$ is given by

$$d\mathbf{I} = \frac{\partial \mathbf{I}}{\partial x}\, dx + \frac{\partial \mathbf{I}}{\partial y}\, dy.$$

Then its squared norm is

$$\| d\mathbf{I} \|^2 = \begin{pmatrix} dx \\ dy \end{pmatrix}^T \begin{pmatrix} \sum_{i=1}^{N} (\frac{\partial I_i}{\partial x})^2 & \sum_{i=1}^{N} \frac{\partial I_i}{\partial x}\frac{\partial I_i}{\partial y} \\ \sum_{i=1}^{N} \frac{\partial I_i}{\partial x}\frac{\partial I_i}{\partial y} & \sum_{i=1}^{N} (\frac{\partial I_i}{\partial y})^2 \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix}, \quad (1)$$

The quadratic form in (1) is called the first fundamental form. For a graylevel image ($N = 1$), the eigenvalue λ^+ of 2×2 matrix is equal to $\| \nabla I_1 \|^2$ and the corresponding eigenvector v^+ is equal to $\nabla I_1 / \| \nabla I_1 \|$. The other eigenvalue is zero. If $N \geq 1$, the eigenvalue is the squared gradient magnitude of the multivalued image $\mathbf{I}$ and the corresponding eigenvector lies in the direction of the gradient.

The first fundamental form reflects only the edge information at a single scale. To describe multiscale information of a multivalued image, the first fundamental form was extended to the multiscale fundamental form [4]. The extension was based on the nonorthogonal wavelet transform introduced by Mallat in [6].

Define a 2-D differentiable smoothing function $\theta(x, y)$ whose integral over x and y is 1 and convergence to 0 at infinity and two wavelet functions $\psi^1(x, y)$ and $\psi^2(x, y)$

0-7803-8554-3/04/$20.00 ©2004 IEEE.

such that $\psi^1(x,y) = \frac{\partial\theta(x,y)}{\partial x}, \psi^2(x,y) = \frac{\partial\theta(x,y)}{\partial y}$. Let $f(x,y) \in L^2(R^2)$ and

$$\theta_{2^j}(x,y) = \frac{1}{2^{2j}}\theta(\frac{x}{2^j}, \frac{y}{2^j}),$$
$$\psi_{2^j}^{\{1,2\}}(x,y) = \frac{1}{2^{2j}}\psi^{\{1,2\}}(\frac{x}{2^j}, \frac{y}{2^j}).$$

The wavelet transform of $f(x,y)$ at scale 2^j is defined by

$$W_{2^j}^1 f(x,y) = f * \psi_{2^j}^1(x,y), W_{2^j}^2 f(x,y) = f * \psi_{2^j}^2(x,y),$$

where $*$ denotes the convolution operator. One can simply prove that

$$\begin{pmatrix} W_{2^j}^1 f(x,y) \\ W_{2^j}^2 f(x,y) \end{pmatrix} = 2^j \nabla(f * \theta_{2^j})(x,y). \qquad (2)$$

With a particular class of wavelets, in [6] Mallat proposed two fast algorithms to implement the wavelet transform and the inverse wavelet transform.

Based on (1) and (2), a multiscale fundamental form can be derived for a multivalued image **I**. The squared norm of the differential of $\mathbf{I} * \theta_{2^j}(x,y)$ is given by

$$\| d(\mathbf{I} * \theta_{2^j}(x,y)) \|^2 = 2^{-2j}$$
$$\begin{pmatrix} dx \\ dy \end{pmatrix}^T \begin{pmatrix} \sum_{i=1}^N (W_{i,2^j}^1)^2 & \sum_{i=1}^N W_{i,2^j}^1 W_{i,2^j}^2 \\ \sum_{i=1}^N W_{i,2^j}^1 W_{i,2^j}^2 & \sum_{i=1}^N (W_{i,2^j}^2)^2 \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix},$$

where $W_{i,2^j}^1$ and $W_{i,2^j}^2$ are the detail coefficients of the nth band image at scale 2^j. The expression will be referred to as the jth scale fundamental form and therefore reflects the edge information at scale 2^j. Denote two eigenvalues of the 2×2 matrix by $\lambda_{2^j}^+, \lambda_{2^j}^-$ ($\lambda_{2^j}^+ \geq \lambda_{2^j}^-$) and the corresponding eigenvectors by $v_{2^j}^+, v_{2^j}^-$. For a multivalued image, the edge information is contained in both eigenvalues. The eigenvalues and eigenvectors describe the edge information in a multiresolution way.

The same as in (1), here the eigenvectors are not uniquely specified. In [4], the wavelet transforms of the average of all the bands were used to determine the signs. But based on the properties of the convolution operator, the $v_{2^j}^+$ can be determined as follows:

$$v_{2^j}^+ = \begin{cases} v_{2^j}^+, & \text{if } v_{2^j,x}^+ \sum_{i=1}^N W_{i,2^j}^1 + v_{2^j,y}^+ \sum_{i=1}^N W_{i,2^j}^2 \geq 0, \\ -v_{2^j}^+, & \text{otherwise.} \end{cases}$$

2.2. The image fusion algorithm in [4]

Ignoring the second eigenvector λ^-, in [4] a multivalued image wavelet representation is given. The image fusion algorithm based on MFF is as following. A low resolution image $\bar{L}_{2^d}$ is obtained by averaging the low resolution images $\{L_{i,2^d}, i = 1, \cdots, N\}$ of the original bands or by choosing one low resolution image. The detail image $W_{2^j}^{l,+}$ are derived from the multiplication of $\sqrt{\lambda_{2^j}^+}$ and $v_{2^j}^+$. Then a wavelet representation $\{\bar{L}_{2^d}, W_{2^j}^{l,+}, l = 1,2, j = 1, \cdots, N\}$ of the multivalued image is provided. By applying the inverse wavelet transform in [6] to this representation, one obtains a graylevel image, i.e. the fused image of the original image **I**. The flowchart of the algorithm is shown in Fig. 1.

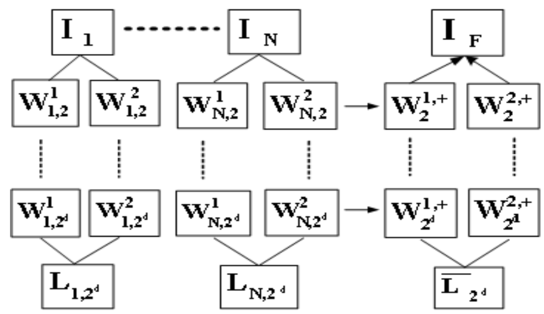


Fig. 1. Flowchart of the image fusion algorithm

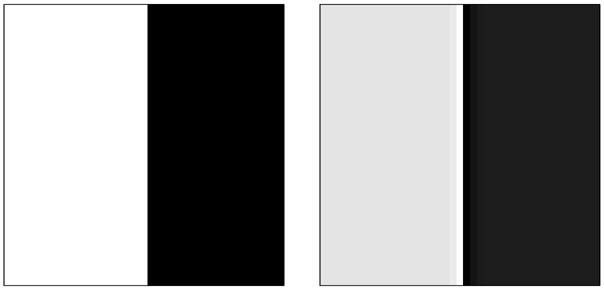
3. THE WEIGHTED MULTISCALE FUNDAMENTAL FORM

For the first fundamental form, the same weight is distributed to the gradients of the input images. The difference between the original images is not considered. By analysis, we think that the same weight is not good in the field of image fusion.

Take $\mathbf{I} = (I_1, I_1)$ as a special example. Two components of **I** are the same image. The expected fused image should be the image I_1. From (1), the eigenvalue λ^+ is equal to $2\| \nabla I_1 \|^2$ and eigenvector v^+ is equal to $\frac{\nabla I_1}{\|\nabla I_1\|}$. Correspondingly, by the multiscale fundamental form, the eigenvalue $\lambda_{2^j}^+$ and eigenvector $v_{2^j}^+$ are given by

$$\lambda_{2^j}^+ = 2\,[(W_{1,2^j}^1)^2 + (W_{1,2^j}^2)^2]$$
$$v_{2^j}^+ = \frac{\nabla(I_1 * \theta_{2^j}(x,y))}{\| \nabla(I_1 * \theta_{2^j}(x,y)) \|} = (\frac{W_{1,2^j}^1}{\lambda_{2^j}^+}, \frac{W_{1,2^j}^2}{\lambda_{2^j}^+}).$$

So to the MFF based image fusion algorithm, the representation of the multi-image becomes $\{L_{1,2^d}, \sqrt{2}W_{1,2^j}^l, l = 1,2, j = 1,2,\cdots,d\}$. The detail images are expanded by $\sqrt{2}$ but the low resolution image doesn't change . The fused image I_F is not the image I_1 but its enhanced version. The enlargement of the detail images results in that noise is expanded and that some artifacts occur in the fused image I_F. Fig. 2 shows an example. Either of the input images is Fig. 2(a). The fused image based on MFF is Fig. 2(b). The noise amount is increased in the fused image. The signal-to-noise (SNR) of (a) is 15.30 but that of (b) is 14.37. At the same time, some artifacts can be seen near the edges in Fig.2(b).



(a) The original image (b) The fused image

Fig. 2. Fusion result based on MFF

From the above, we draw a conclusion that the detail coefficients of the representation $\{\bar{L}_{2^d}, W_{2^j}^{l,+}, l = 1, 2, j = 1, 2, \cdots, d\}$ are too large. To avoid the problem, we proposed a adaptively weighted first fundamental form, i.e.

$$\| d\mathbf{I} \|^2 = \begin{pmatrix} dx \\ dy \end{pmatrix}^T \begin{pmatrix} \sum\limits_{i=1}^{N} \alpha_i (\frac{\partial I_i}{\partial x})^2 & \sum\limits_{i=1}^{N} \alpha_i \frac{\partial I_i}{\partial x}\frac{\partial I_i}{\partial y} \\ \sum\limits_{i=1}^{N} \alpha_i \frac{\partial I_i}{\partial x}\frac{\partial I_i}{\partial y} & \sum\limits_{i=1}^{N} \alpha_i (\frac{\partial I_i}{\partial y})^2 \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix},$$

where $\alpha_i = (\frac{\partial I_i}{\partial x}^2 + \frac{\partial I_i}{\partial y}^2)/(\sum\limits_{i=1}^{N} \frac{\partial I_i}{\partial x}^2 + \frac{\partial I_i}{\partial y}^2)$. Corresponding, the jth scale weighted fundamental form is

$$\| d(\mathbf{I} * \theta_{2^j}(x,y)) \|^2 = 2^{-2j} \begin{pmatrix} dx \\ dy \end{pmatrix}^T \begin{pmatrix} \sum\limits_{i=1}^{N} \beta_i (W_{i,2^j}^1)^2 & \sum\limits_{i=1}^{N} \beta_i W_{i,2^j}^1 W_{i,2^j}^2 \\ \sum\limits_{i=1}^{N} \beta_i W_{i,2^j}^1 W_{i,2^j}^2 & \sum\limits_{i=1}^{N} \beta_i (W_{i,2^j}^2)^2 \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix}$$

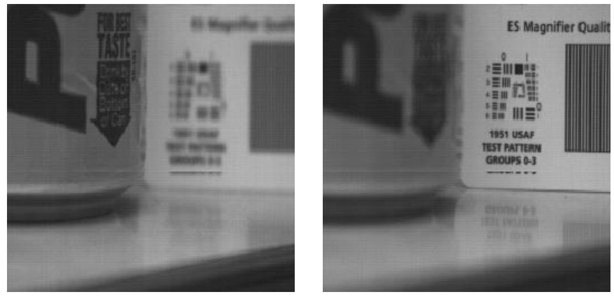
where $\beta_i = ((W_{i,2^j}^1)^2 + (W_{i,2^j}^2)^2)/(\sum\limits_{i=1}^{N} (W_{i,2^j}^1)^2 + (W_{i,2^j}^2)^2)$.

Let $m = \max\limits_{i \in \{1,\cdots,N\}} \{(\frac{\partial I_i}{\partial x})^2 + (\frac{\partial I_i}{\partial y})^2\}$. For the weighted first fundamental form, there are the following two properties: (i) the gradient magnitude given from it is less than the gradient magnitude of I_m; (ii) the gradient direction is more approximate to $(\frac{\partial I_m}{\partial x}, \frac{\partial I_m}{\partial y})$.

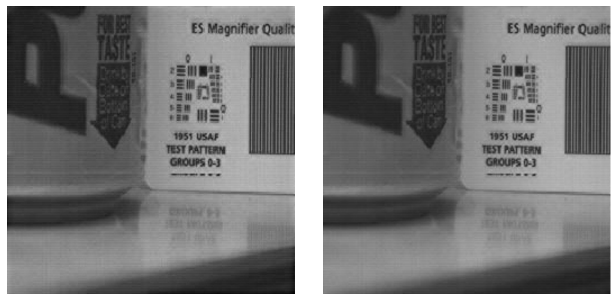
Using the weighted multiscale fundamental form, the fused image is I_1 for $\mathbf{I} = (I_1, I_1)$ or $\mathbf{I} = (I_1, 0)$. Obviously, the fused result is consistent with the expectation. The result also shows that the weighted multiscale fundamental form is robust to the different input images.

4. EXPERIMENTS

Firstly, we take two-focus images as the testing images. In Fig.3, (a) and (b) are two-focus images, taken by the digital camera, which have different focus. In Fig.3 (a), the focus is on the Pepsi can. In Fig.3 (b), the focus is on the testing card. Using the fusion algorithm in Section 2.2, the fused images based on WMFF and MFF are given in Fig.3 (c) and (d), respectively.



(a) The image focus on the can (b) The image focus on the card



(c) Result based on MFF (d) Result based on WMFF

Fig. 3. Fusion results for two-focus images

In vision two fused images are similar. In order to obtain a quantitative evaluation, we use an information theoretic quality measure based on the mutual information and the conditional entropy. In [7], Rockinger used them to describe the stability and consistency and the instability and inconsistency of the fused sequences. Suppose two random variables, A and B, with marginal probability distribution, $p_A(a)$ and $p_B(b)$ and joint probability distribution, $p_{AB}(a, b)$. The Mutual Information (MI) of A and B is defined as follows [8]:

$$MI(A,B) = \sum_{a,b} p_{AB}(a,b) \log_2 \frac{p_{AB}(a,b)}{p_A(a)p_B(b)}. \quad (3)$$

MI is related to entropy by the following equation

$$\begin{aligned} MI(A,B) &= H(A) + H(B) - H(A,B) \\ &= H(A) - H(A|B) \\ &= H(B) - H(B|A), \end{aligned}$$

with $H(A)$ and $H(B)$ being the entropy of A and B, respectively, $H(A,B)$ their joint entropy, and $H(A|B)$ and $H(B|A)$ the conditional entropy.

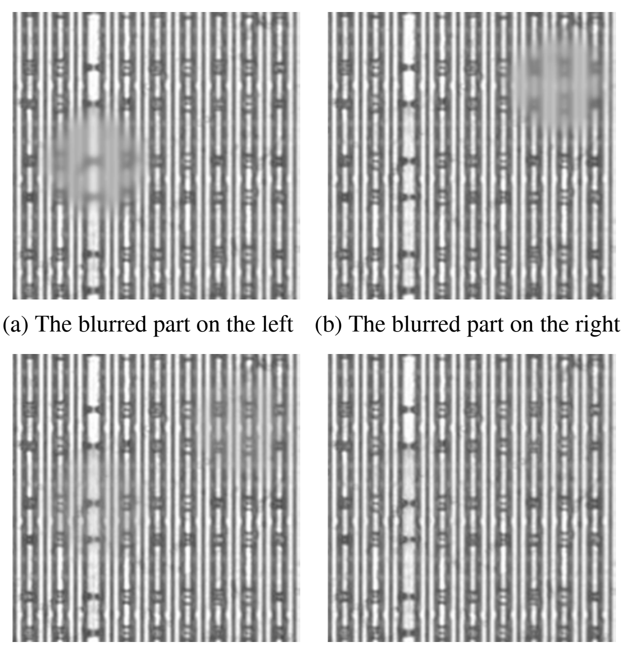
Denote two original images by I_1, I_2 and the fused image by I_F. To evaluate the fused images, $H(I_1, I_2|I_F)$,

$H(I_F|I_1, I_2)$ and $MI((I_1, I_2), I_F)$ are calculated. High $MI((I_1, I_2), I_F)$ corresponds to more relevant information between the original image and the fused image. Low conditional entropy $H(I_1, I_2|I_F)$ and $H(I_F|I_1, I_2)$ indicate less inconsistency and instability. The results are shown in Table 1. We see from Table 1 that the weighted multiscale fundamental form appears to outperform the multiscale fundamental form. For the fused image based on WMFF, more relevant information in I_1 and I_2 is preserved and less artifacts or inconsistencies are introduced.

	$H(I_1, I_2\|I_F)$	$H(I_F\|I_1, I_2)$	$MI((I_1, I_2), I_F)$
$WMFF$	5.2772	1.4368	4.0290
MFF	5.5409	1.7563	3.7654

Table 1: The information theoretic quality measure of Fig.3

In Fig.4, (a) and (b) are the original integrated circuit (IC) images which are partially blurred. (c) and (d) are the fused images based on WMFF and MFF, respectively. The quantitative evaluation is in Table 2.



(a) The blurred part on the left (b) The blurred part on the right

(c) Result based on MFF (d) Result based on WMFF

Fig. 4. Fusion results for partly blurred IC images

	$H(I_1, I_2\|I_F)$	$H(I_F\|I_1, I_2)$	$MI((I_1, I_2), I_F)$
$WMFF$	3.9930	1.0518	4.2236
MFF	4.1739	1.1834	4.0428

Table 2: The information theoretic quality measure of Fig.4

5. CONCLUSION REMARKS

In this paper, a weighted multiscale fundamental form is proposed. Using this form, the detail images will not be enlarged in the fusion process. The mutual information and the conditioned entropy are used as the evaluation criteria to asses the fused image. By the fusion experiments of two-focus images and partially blurred IC images, the weighted multiscale fundamental form are demonstrated to outperform the multiscale fundamental form on image fusion.

6. REFERENCES

[1] C. Pohl and J. L. Genderen, "Multisensor image fusion in remote sensing: concepts, methods and application," *International Journal of Remote Sensing*, Vol. 19, No. 5, pp. 823-854, 1998.

[2] H. Li, B. S. Manjunath, S.K. Mitra, "Multisensor image fusion using the wavelet transform," *Graphical Models and Image Processing*, Vol. 57, pp. 235-245, 1995.

[3] J. Núñez, X. Otazu, O. Fors, A. Prades, V. Pala, and R. Arbiol, "Multiresolution based image fusion with additive wavelet decomposition," *IEEE Tran. Geosci. and Remote Sensing*, Vol. 37 , No. 3, pp. 1204-1211, 1999.

[4] P. Scheunders, "A multivalued image wavelet representation based on multiscale fundamental form," *IEEE Trans. Image Processing*, Vol. 11, No. 5, pp. 568-575, 2002.

[5] S. Di Zeno, "A note on the gradient of a multi-image," *Comput. Vis. Graph. Image Process.*, Vol. 33, pp. 116-125, 1986.

[6] S. Mallat, Ed., *A wavelt tour of signal processing*, 2nd ed., New York: Academic, 1999.

[7] O. Rockinger, T. Fechner, " Pixel-level image fusion: the case of image sequences," *SPIE Processings*, Vol. 3374, pp. 378-388, 1998.

[8] F. Maes, A. Collignon, D. Vandermeulen, G. Marchal and P. Suetens, "Mutimodality image registration by maximization of mutual information," *IEEE Trans. Medical Imaging*, Vol. 16, No. 2, 1997.