# IMAGE DENOISING USING FREBAS MULTI-RESOLUTION IMAGE ANALYSIS


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## ABSTRACT

This paper describes a 2-step image denoising technique based on the FREBAS transform that we have proposed as a new multi-resolution image analysis. Noise removal is performed by applying a Wiener filter function, which is designed using a FREBAS pre-filtered image, in the FREBAS transformed domain. Comparisons of image SNR improvement with previous single FREBAS denoising method and other methods such as nonlinear digital filter and Wiener filtering in the Wavelet domain were made. It was demonstrated that the proposed method provides the highest SNR improvement and the resultant image was free from serious image degradation.


## 1. INTRODUCTION

A wavelet transform, which splits the frequency band of the original image in the image domain while including position information, has been widely applied for image processing. With a wavelet transform, the multi-resolution analysis level of an image can arbitrary set and different processing can be performed for each level or image position. Like the wavelet transform, a multi-resolution image analysis can be performed by using Fresnel transformed signal band-splitting effect(FREBAS)[1, 2].

In the former paper, we have applied the FREBAS method for medical image denoising, and it was shown that the proposed method had a better performance than the Wiener filter in the wavelet domain or nonlinear digital filter in the sense of SNR improvement and edge preservation[1]. In this paper, we propose a new denoising method for natural images based on the FREBAS transform which have a improved performance.

## 2. FREBAS TRANSFORM

FREBAS transform uses two kinds of Fresnel transform successively. The Fresnel integral equation can be solved in different manner using Fourier transform. One method is to expand the quadratic phase term of the Fresnel transform formula and then perform Fourier transform after multiplying by the quadratic phase term. (This method is referred to as [Method 1].) The other method is to obtain the solution according to inverse filtering.(This is referred to as [Method 2].) [1, 2]

If we let $o(x)$ be the image function, $c$ be the coefficient that corresponds to the distance parameter optically, and $u(x')$ be the Fresnel transformed signal, the relationship among these items can be expressed as

$$u(x') = \int_{-\infty}^{\infty} o(x) e^{-jc(x'-x)^2} dx, \qquad (1)$$

[Method 1] solves the Fresnel integrals using the Fourier transform once as follows:

$$o\left(\frac{\Omega_x}{2c}\right) = \frac{c}{\pi} e^{j\frac{\Omega_x{}^2}{4c}} \mathcal{F}\left[u(x') e^{jcx'^2}\right] \qquad (2)$$

where, $\Omega_x = 2cx$, The pixel width of the reconstructed image, $\Delta x_1$, that is given by the Nyquist's theorem as $\Delta x_1 = \pi / cN\Delta x'$. Here, $N$ is a number of sampling data, $\Delta x'$ is a sampling step in the space of $x'$ [1, 2]. The scale of the reconstructed image depends on the distance parameter $c$.

[Method 2] solves the Fresnel integrals by inverse filtering technique. Multiplying the modulation transfer function in the Fourier transformed space, we have the following equation:

$$o(x) = \sqrt{\frac{c}{\pi}} e^{j\frac{\pi}{4}} \mathcal{F}^{-1}\left[\mathcal{F}\left[u(x')\right] e^{-j\frac{\omega_x{}^2}{4c}}\right]. \qquad (3)$$

Pixel width of the reconstructed image $\Delta x_2$ is almost the same as sampling step $\Delta x'$ irrespective of the parameter $c$, since the space where the reconstructed image is obtained is regarded as the same as Fresnel trasform signal $u(x')$ in the inverse filtering technique.

FREBAS transform is executed by applying alternative Fresnel transforms to input image succesively in inverse direction. Firstly, Fresnel transform of input image is performed using inverse [Method 2]. And then image reconstruction from the Fresnel transform signal is done using [Method 1]. Although the equivalent sampling step of the calculated Fresnel transformed signal $\Delta x'$ does not change

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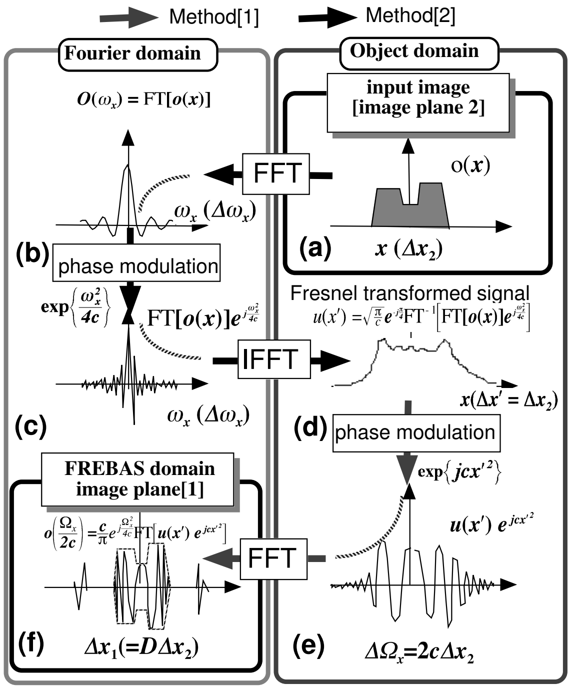



**Fig. 1**. An algorithm of FREBAS transform using dual Fresnel transform pairs.

by the parameter $c$, the pixel width of reconstruction image $\Delta x_1$ changes according to the parameter $c$ ($\Delta x_1 = \pi/cN\Delta x'$). So, we can obtain arbitrary scaled images by giving an adequate parameter $c$ in this algorithm. Figure 1 shows the algorithm of FREBAS transform using inverse [Method 2] and [Method 1].

The down-scaled image $o_{scale}(x)$ is expressed as Eq(4) using continuous Fourier transform equation[1].

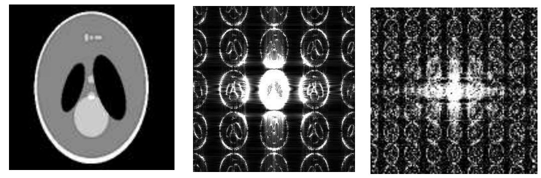
$$o_{scale}(x) = \frac{c}{\pi}\sum_{s=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} o(m,x)\,\mathrm{comb}\left(\frac{x}{\Delta x_1}\right) * \delta(x - sDX_2), \quad (4)$$

$$o(m,x) = e^{jcx^2}\left[\left\{o(x-mX_2)\,e^{-jcx^2}\right\} * \mathrm{sinc}(2cX_2x)\right]$$
$$= \left\{o(x-mX_2)e^{-jc(x-mX_2)^2} * \mathrm{sinc}(2cX_2x)e^{j2cmX_2x}\right\} \times e^{jc(x-mX_2)^2}. \quad (5)$$

where $X_2(=N\Delta x_2)$ is the field of view of original image.

The distance parameter $c$ is written as $c = \tilde{c}/D$ by introducing coefficients $D$ and $\tilde{c} = \pi/N{\Delta x_1}^2$ which gives the condition when the scale of FREBAS transformed image will not change($\Delta x_1 = \Delta x_2$). We find from these relationship that the coefficient $D$ is a scaling parameter which defines the scale of images as $\Delta x_1 = D\Delta x_2$.

When $D$ satisfies the relation $D \geq 1$, ($D \in \boldsymbol{R}$) then the reconstructed image is down-scaled and alias signal components contained in the calculated Fresnel transformed signal



(a) D=1 (b) D=4 (c) D=8

**Fig. 2**. FREBAS multi-resolution images : (a) D=1(original image), (b) D=4, (c) D=8.

appeared in different location representing edge-detected images. Thus, we see that the FREBAS transform does not split the signal in frequency space but spilt in the Fresnel transformed signal space at an equal interval. Viewing the decomposition algorithm from the point of frequency space, Eq.(5) is a convolution integral of phase-modulated image function located at $mX_2$, $o(x-mX_2)\exp\left[-jc(x-mX_2)^2\right]$, and band-pass filter function $\mathrm{sinc}(2cX_2x)\exp(j2cmX_2x)$. So, FREBAS is considered as the system in which quadratically phase modulated images are divided into sub-band filter banks.

The precise calculation steps of FREBAS transform are described in former paper[1]. The characteristics of FREBAS transform are 1) Directional multiresolution images are produced only phase modulation and FFT, 2) Image of any scale can be obtained 3) muletiresolution images are decomposed in a manner that Fresnel transformed signal space is divided into $D^2$ pieces. Figure 2 shows an example of FREBAS transformed image with $D = 4$ and $D = 8$.

## 3. 2-STEP DENOISING USING FREBAS TRANSFORM

### 3.1. 2-step denoising algorithm

We have demonstrated that Wiener filtering in the FREBAS transformed domain offers a excellent denoising effect[1]. In the former paper, we used practical Wiener filter function given as:

$$\mathrm{Wf} = \frac{|\mathrm{Freb_D}[\mathrm{o_{noise}}]|^2 - \mathrm{P_{ns}}}{|\mathrm{Freb_D}[\mathrm{o_{noise}}]|^2}, \quad (6)$$

where $\mathrm{o_{noise}}$ means a noisy image composed of original image $o$ and noise components $\mathrm{ns}$, $\mathrm{Freb_D}[\mathrm{o_{noise}}]$ means a FREBAS transform of noisy input image $\mathrm{o_{noise}}$ by a parameter $D$, and $P_{ns}$ is a mean noise power estimated from the noisy image. Wiener filter is an optimal filter in the sense of mean square error of output image. The filter, written as $\mathrm{Wf} = \mathrm{P_{sig}}/(\mathrm{P_{sig}} + \mathrm{P_{ns}})$, requires the power spectrum of original image, $P_{sig}$, free from noise contamination, but we can not know the spectrum. So, we approximate the Wiener filter function from the noisy power spectrum as Eq.(6).

In this paper, we present a new design of Wiener filter function using the FREBAS denoised image to improve the

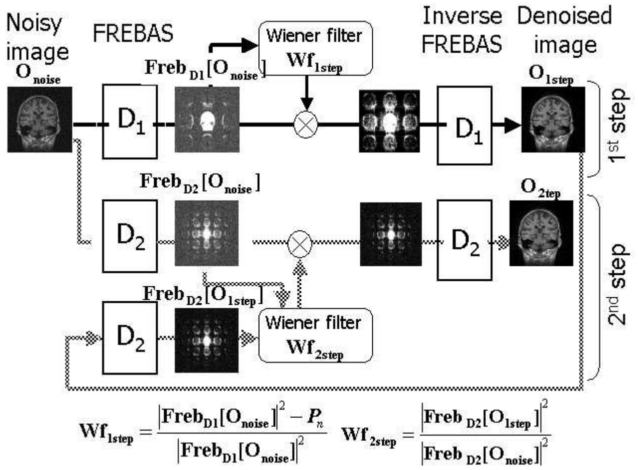



**Fig. 3**. An algorithm of 2-step FREBAS denoising method

filter performance. Since the output image of FREBAS denoising filter is much closer to the original noise free image, we designed a 2-step denoising process as 1) FREBAS denoising using a Wiener filter function written as Eq.(6) by a scale-parameter $D_1(o_{1step})$, 2) Designing an Wiener filter function for the 2nd-step denoising by a scale-parameter $D_2$,using $o_{1step}$, and $P_{ns}$, 3) Performing 2nd-step FREBAS denoising by a scale-parameter $D_2$ using the refined Wiener filter $\mathrm{Wf_{2step}}$:

$$\mathrm{o_{1step}} = \mathrm{Freb}_{\mathrm{D}_1}^{-1}[\mathrm{Wf_{1step}} \cdot \mathrm{Freb}_{\mathrm{D}_1}[\mathrm{o_{noise}}]], \tag{7}$$

$$\mathrm{o_{2step}} = \mathrm{Freb}_{\mathrm{D}_2}^{-1}[\mathrm{Wf_{2step}} \cdot \mathrm{Freb}_{\mathrm{D}_2}[\mathrm{o_{noise}}]], \tag{8}$$

$$\mathrm{Wf_{1step}} = \frac{|\mathrm{Freb}_{\mathrm{D}_1}[\mathrm{o_{noise}}]|^2 - \mathrm{P_{ns}}}{|\mathrm{Freb}_{\mathrm{D}_1}[\mathrm{o_{noise}}]|^2}, \tag{9}$$

$$\mathrm{Wf_{2step}} = \frac{|\mathrm{Freb}_{\mathrm{D}_2}[\mathrm{o_{1step}}]|^2}{|\mathrm{Freb}_{\mathrm{D}_2}[\mathrm{o_{1step}}]|^2 + \mathrm{P_{ns}}}. \tag{10}$$

The Wiener function $\mathrm{Wf_{2step}}$ is much closer to ideal filter function $\mathrm{Wf}$ than practical filter written as Eq.(6), and the improvement of denoising effect is expected.

It should be noted that the input image used in the 2nd-step denoising is not the 1st-step FREBAS denoised image, but the noisy image, $\mathrm{o_{noise}}$. The signal-to-ratio may be improved when the denoised image is used as the 2nd-step input image, however, the spatial resolution of the output image will be degraded since the Wiener filter is applied twice to the input image. In addition, the scale parameter in the 2nd-step $D_2$ should be different from 1st-step value $D_1$, since the choice of different scale parameter corresponds to the change of basis function of multi-resolution analysis. Figure 3 shows an algorithm of 2-step FREBAS denoising procedure.

### 3.2. Performance evaluation

Performance comparison with other filtering methods using 256 × 256 pixels images were performed. In the preliminary evaluation, we have found that the highest SNR improvement can be obtained when scale parameters are set to $D_1 = 6$, $D_2 = 8$, respectively. From 10 standard images, 3 images were created so that the SNR become 10, 20, 40(20, 26, 32 dB) respectively, by adding Gaussian white noise. The filtering method we used are Wiener filtering in the wavelet space[3](wavelet-Wiener filter) and nonlinear digital filter[4] which is a smoothing digital filter using the local properties. The signal-to-noise ratio of filtered image is defined in this paper as follows. Since noise component is known, the noise can be considered as the residual obtained by subtracting the signal component, from the processed image. The magnitude of the noise is represented by its standard deviation. The Results of SNR improvement are shown in Table 1. Experiments showed that significant SNR improvements were performed by using 2-step FREBAS denoising method and that proposed method showed the better performances than other methods. Figure 4 shows the examples using an input image having SNR of 20dB. All denoised images preserve edge information well, however the manner of remained noise is different. Nonlinear digital filter and Wavelet-Wiener filter showed good denoising effects, but spike-like noise remained. The advantage of FREBAS denoising is that decomposed images have larger amplitude than that of Fourier-transform-based muitiresolution images except for the main image component. This feature results in small suppression of amplitude by the Wiener filtering in the FREBAS transformed signal space and provides images with small loss of local information.

## 4. CONCLUSION

We have introduced a new algorithm for denoising images using FREBAS multiresolution image analysis. To improve the accuracy of Wiener filter function, we used a pre-filtered image using FREBAS transform. As a result, we showed that the proposed approach has a better performance and of-

**Table 1**. Performance comparison of denoising methods: the value shows the mean SNR of 10 standard images.

| SNR of input image | FREBAS 2-step (proposed) | FREBAS single step | wavelet Wiener | nonlinear digital filter |
|---|---|---|---|---|
| 20dB | 26.8 | 24.9 | 24.1 | 25.2 |
| 26dB | 30.3 | 29.5 | 27.0 | 29.6 |
| 32dB | 34.7 | 34.2 | 30.1 | 33.9 |

(a) noisy image : 20dB

(b) single-FREBAS denoised image (D=10): 24.7dB

fers more natural images than previous single step FREBAS denoising and other denoising methods. This denoising filter significantly improves the SNR of noisy images that allows to use it as a practical tool for many applications.

## 5. REFERENCES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(c) 2step-FREBAS denoised image:26.8dB

(d) Wavelet-Wiener denoised image:23.7dB

(e) nonlinear digital filterd image:24.3dB

**Fig. 4**. Examples of denoised images(barbara).