A NEW FAMILY OF EMBEDDED MULTIPLE DESCRIPTION SCALAR QUANTIZERS

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ABSTRACT

A new family of Embedded Multiple Description Scalar Quantizers (EMDSQ) that supports the progressive transmission of images over variable-bandwidth error-prone channels is proposed in this paper. A control mechanism that allows for tuning the redundancy between the two descriptions for each quantization level is also designed. The employed mechanism enables the control of the tradeoff between the coding efficiency and error-resilience, and provides an increased robustness by improving the error resilience in the most important layers of the embedded bit-streams. Instantiations of the proposed family are incorporated in a wavelet-based embedded coding system, and the redundancy-control mechanism is practically demonstrated. Experimental results show that the proposed EMDSQ outperform the state-of-the-art Multiple Description Uniform Scalar Quantizers (MDUSQ) previously proposed in literature for error-resilient progressive image transmission.

1. INTRODUCTION

The demand for efficient image data delivery over variable-bandwidth error-prone channels – e.g. packet networks, wireless links – has significantly increased in the recent past. In this context, Multiple Description Coding (MDC) addresses the problem of coding a source to be transmitted over a communication network with diversity, which allows for extracting the meaningful information from a subset of the received bit-streams.

The focus of previous MDC research was laid on finding the optimal achievable rates-distortion regions in [1], [2], followed by the design of practical compression systems to meet these theoretical boundaries. In order to produce multiple descriptions of the input data, several MDC systems rely on quantization techniques, as proposed in [3], [4]. The design of multiple description scalar quantizers (MDSQ) was pioneered in [3] under the assumption of fixed-length codes and fixed codebook sizes. Significant improvements are reported in [4], in which the quantizers' design is made under a given entropy-constraint, and not on a given codebook size.

Apart of coding multiple descriptions of the input source, a fine grain scalability of each description is a desirable feature for bandwidth-varying communication environments. A practical MDC system based on multiple-description uniform scalar quantizers (MDUSQ) followed by embedded entropy coders was previously proposed in literature [5]. In the same context, we proposed in [6], [7] a new type of embedded multiple description scalar quantizers, providing a fine-grain refinable (layered) representation of the input data and targeting a high level of redundancy. The embedded quantizer design follows the constraint of constructing double-deadzone central quantizers for each quantization level. Furthermore, in comparison to the state-of-the-art progressive MDC techniques based on the MDUSQ [5], the coding techniques based on the embedded multiple-description scalar quantizers of [6], [7] provide systematically better rate-distortion performances for the central channel reconstruction, as shown in [6], [7].

In order to improve the robustness of each description, the most important layers (the coarser quantizers output) should have a higher redundancy level providing a higher protection than the less important layers (corresponding to the finest quantizers output) [8]. This leads to the requirement of providing a mechanism that, for each quantization level, allows for the control of the redundancy between the two descriptions.

In this paper we extend our initial ideas reported in [6], [7] and propose a generic family of embedded multiple-description scalar quantizers (EMDSQ). The new EMDSQ family is designed to meet the desired features of (1) producing double-deadzone central quantizers at each quantization level, and (2) allowing for the control of the redundancy between the two descriptions at each quantization level as well. The first design constraint ensures optimal (at high rates), and nearly-optimal (at lower rates) rate-distortion performance [9], while the latter enables practical MDC systems to provide embedded descriptions with decreasing level of redundancy from the most important layers to the less important ones.

The paper is structured as follows. In Section 2 the proposed generic family of EMDSQ is introduced and the redundancy control mechanism is explained. In Section 3 instantiations of the proposed EMDSQ family are used in a still-image coding system and the redundancy control at each quantization level is demonstrated. Finally, we conclude our work in Section 4.

2. A NEW FAMILY OF EMDSQ

The embedded multiple description scalar quantizers proposed in [6], [7] are quantizers with connected partitions cells, that target a high redundancy level and provide state-of-the-art coding results. Thus, in the proposed EMDSQ family they can be successfully employed for the coarser quantization levels for an increased resilience of the multiple descriptions. On the other hand, as shown in [3] for the fixed-rate case, quantizers with disconnected partition cells should be employed in order to reduce the descriptions' redundancy. Hence, for the less important layers (corresponding to the finer quantization levels), embedded side quantizers with disconnected partition cells producing double-deadzone central quantizers should be designed. The solution to this problem and the mixture between these two classes of quantizers, yielding eventually the EMDSQ family, is described in the following.

For the two-channel EMDSQ we denote the set of embedded side-quantizers as $Q_m^0, \ldots, Q_m^K$, $Q_m^{K+1}, \ldots, Q_m^{K+P+1}$, with $m = 1, 2$, where the quantization level q, $K+1 \le q \le K+P+1$, corresponds to the low-rate quantizers with connected partitions cells, while

$q,\ 0 \le q \le K$ corresponds to the higher-rate quantizers with disconnected partitions cells. In these notations $P+1$ and $K+1$ denote the number of quantization levels for the embedded quantizers with connected and disconnected partition cells respectively. Also, we denote by $Q^0, \ldots, Q^K$, $Q^{K+1}, \ldots, Q^{K+P+1}$ the set of embedded central-quantizers $Q^{q-1}(q_k^1, q_k^2) = Q_1^{q-1}(q_k^1) \cap Q_2^{q-1}(q_k^2)$ for any quantization level $q,\ 0 \le q \le K+P+1$.

2.1. EMDSQ with connected partitions cells

From the analytical expression of the embedded multiple-description scalar quantizers with connected partitions cells presented in [6], [7] one can derive the following index allocation matrix:

$$\mathbf{I}(r) = \begin{bmatrix} 1\ 2 & & & & \mathbf{0} \\ \ \ 3 & & & & \\ & 4\ 5 & & & \\ & \ \ 6 & \ddots & & \\ \mathbf{0} & & & 3N-2 & 3N-1 \\ & & & & 3N \end{bmatrix} \tag{1}$$

where $1 \le r \le 3N$, $3N$ represents the number of partition cells contained in the granular region of the central quantizer at the quantization level $q = K+1$, and $P = \lfloor \log_2 N \rfloor$.

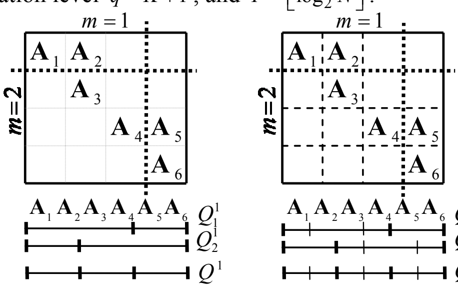


Fig. 1. EMDSQ with connected partitions cells and the corresponding embedded index assignment for two quantization levels ($P=1$).

In the proposed family of EMDSQ, the integer entries in the index allocation matrix $\mathbf{I}(r)$ of (1) are replaced by $S \times S$ square matrices of consecutive indices $\mathbf{A}_r = (a_{ij})_{1 \le i,j \le S}$. In this generic case, the index allocation matrix is given by:

$$\mathbf{I}(\mathbf{A}_r) = \begin{bmatrix} \mathbf{A}_1\ \mathbf{A}_2 & & & & \mathbf{0} \\ \ \ \mathbf{A}_3 & & & & \\ & \mathbf{A}_4\ \mathbf{A}_5 & & & \\ & \ \ \mathbf{A}_6 & \ddots & & \\ \mathbf{0} & & & \mathbf{A}_{3N-2} & \mathbf{A}_{3N-1} \\ & & & & \mathbf{A}_{3N} \end{bmatrix} \tag{2}$$

For any matrix $\mathbf{M}$ we define the operator $nnz(\mathbf{M})$ that determines the number of nonzero elements contained in the matrix $\mathbf{M}$. From (2) it follows that for the central quantizers at level $q = K+1$ the sizes of the bounded partitions cells are of the form $nnz(\mathbf{A}_r) \cdot \Delta_C$, where $\Delta_C \in \mathbb{R}_+^*$ is the central-channel's step size at the highest rate. Imposing the design constraint of obtaining a double-deadzone central quantizer for the level $q = K+1$ implies equal size for all bounded partition-cells. This necessarily implies an equal number of nonzero elements for all the index matrices $\mathbf{A}_r$, i.e. $nnz(\mathbf{A}_r) = const., \forall r, 1 \le r \le 3N$. Therefore, the cell size for the central quantizer at level $q = K+1$ is $\Delta_C^{(K+1)} = nnz(\mathbf{A}_r)\Delta_C$. From the abovementioned and the EMDSQ cell size formula presented in [6], [7] we deduce that the central channel quantizer is a double-deadzone embedded quantizer with the cell size of the form:

$$\Delta_C^{(q)} = 2^{q-K-1} nnz(\mathbf{A}_r)\Delta_C \tag{3}$$

for any level $q,\ K+1 \le q \le K+P+1$. Consequently, from the above presented and from [6], [7], one can deduce that the cell size for the side channels is of the form $\Delta^{(q)} = 2^{q-K-1}\Delta^{(K+1)}$ where $\Delta^{(K+1)} = nnz(\mathbf{A}_r)\Delta_C$ or $\Delta^{(K+1)} = 2 \cdot nnz(\mathbf{A}_r)\Delta_C$.

The example depicted in Fig. 1 illustrates an instantiation of the EMDSQ with connected partitions cells for two quantization levels and the corresponding embedded index assignment strategy. The negative side of the quantizers is a mirrored version of the positive side shown in Fig. 1. We notice that if $nnz(\mathbf{A}_r)$ is constant, then Q^0 and Q^1 are double-deadzone scalar quantizers indeed.

2.2. EMDSQ with disconnected partitions cells

Starting with the quantization level $q = K$, quantizers with disconnected partition cells are employed in order to reduce the redundancy between the two descriptions. This corresponds to a reduced error-robustness for the finer quantization levels; however, this reduces the rate in comparison to the rate that would be obtained with quantizers with connected partition cells.

At the level $q = K$ we consider each of the $S \times S$ square matrices $\mathbf{A}_r$ as block matrices of the form $\mathbf{A}_r = [\mathbf{B}_{ij}^K]_{1 \le i,j \le L_K}$ defined by the contained blocks $\mathbf{B}_{ij}^K$, $1 \le i,j \le L_K$ and represented as follow:

$$\mathbf{A}_r = \begin{bmatrix} \mathbf{B}_{11}^K & \mathbf{B}_{12}^K & \cdots & \mathbf{B}_{1L_K}^K \\ \vdots & \vdots & & \vdots \\ \mathbf{B}_{L_K 1}^K & \mathbf{B}_{L_K 2}^K & \cdots & \mathbf{B}_{L_K L_K}^K \end{bmatrix} \tag{4}$$

Moreover, each of the blocks $\mathbf{B}_{ij}^K$ are square matrices of the same size $S_K = S / L_K$. Similarly, at the finer quantization level $q = K-1$, each block matrix $\mathbf{B}_{ij}^K$ is again of the form: $\mathbf{B}_{ij}^K = [\mathbf{B}_{mn}^{K-1}]_{1 \le m,n \le L_{K-1}}$ for any $1 \le i,j \le L_K$, and the contained blocks $\mathbf{B}_{mn}^{K-1}$ are of the size $S_{K-1} = S/(L_K \cdot L_{K-1})$ Recursively, $\mathbf{B}_{ij}^q = [\mathbf{B}_{mn}^{q-1}]_{1 \le m,n \le L_{q-1}}$ for any $1 \le i,j \le L_q$, and for all the quantization levels $q, 0 \le q \le K$ (see Fig. 2).

From $nnz(\mathbf{A}_1) = nnz(\mathbf{A}_2) = \ldots = nnz(\mathbf{A}_{3N}) = const.$ and imposing the design constraint that for any quantization level one obtains a double-deadzone central quantizer, yields: $nnz(\mathbf{B}_{ij}^q) = nnz(\mathbf{B}_{mn}^q)$, $\forall i,j,m,n \ \ 1 \le i,j,m,n \le L_q$ for any $[\mathbf{B}_{ij}^q] \ne [\mathbf{0}]$ and $[\mathbf{B}_{mn}^q] \ne [\mathbf{0}]$, where by $[\mathbf{0}]$ one denotes the zero matrix.

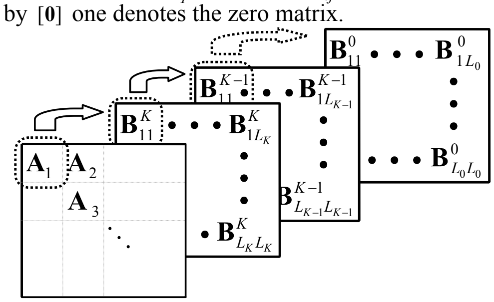


Fig. 2. Recursive block-matrix decomposition for each quantization level $q, 0 \le q \le K$.

Following the recursive decomposition of each block matrix $\mathbf{B}_{ij}^{q+1}$, $1 \le i,j \le L_{q+1}$, one obtains L_q^2 square blocks of size:

$$S_q = \frac{S}{\prod_{k=q}^{K} L_k} \tag{5}$$

Notice that for the highest-rate quantization level $q=0$ all blocks $\mathbf{B}_{ij}^{0}$ $1 \le i,j \le L_0$ are matrices of dimension $S_0 = 1$, i.e. they contain a single element.

Similar to (3) one obtains the cell size at any level $q, 1 \le q \le K$ of the double-deadzone central quantizer:

$$\tilde{\Delta}_C^{(q)} = nnz(\mathbf{B}_{ij}^{q})\Delta_C \qquad (6)$$

for any $1 \le i,j \le L_q$ and $[\mathbf{B}_{ij}^{q}] \ne [\mathbf{0}]$.

2.3. Central-channel EMDSQ analytical expression

Consider a block matrix $\mathbf{M} = [\mathbf{M}_{ij}]_{1\le i\le I, 1\le j\le J}$. The number of blocks $[\mathbf{M}_{ij}] = [\mathbf{0}]$ contained in $\mathbf{M}$ is determined via the nonzero-blocks operator $nzb(\mathbf{M}, \mathbf{M}_{ij})$. The operator $nzb(\mathbf{M}, \mathbf{M}_{ij})$ is similar to the operator $nnz(\mathbf{M})$ in the case of unitary size blocks $\mathbf{M}_{ij}$. With this definition, for any of the block matrices $\mathbf{B}_{ij}^{q} = [\mathbf{B}_{mn}^{q-1}]_{1\le m,n\le L_{q-1}}$, $1 \le i,j \le L_q$, the number of blocks $\mathbf{B}_{mn}^{q-1}$ different from the zero matrix is given by $N_{q-1} = nzb(\mathbf{B}_{ij}^{q}, \mathbf{B}_{mn}^{q-1})$. Also, we denote by $N_K = nzb(\mathbf{A}_r, \mathbf{B}_{ij}^{K})$ where $1 \le i,j \le L_K$.

The total number of indices mapped in each $\mathbf{B}_{ij}^{q}$ block matrix at quantization level q is $nnz(\mathbf{B}_{ij}^{q}) = \prod_{k=0}^{q-1} N_k$, for any r, $1 \le r \le 3N$ and $1 \le i,j \le L_q$. Moreover, $nnz(\mathbf{A}_r) = \prod_{q=0}^{K} N_q$. Therefore, the total number of indices mapped in the index assignment matrix $\mathbf{I}(A_r)$ is $N_r = 3 \cdot 2^P \cdot \prod_{q=0}^{K} N_q$.

It can then be shown that the analytical expression of central EMDSQ for any quantization level q, $0 \le q \le K+P+1$ is:

$$Q_C(x) = sign(x)\left\lfloor \frac{|x|}{\Delta_C \cdot 2^{l(q)H(l(q))} \cdot \prod_{k=0}^{q-1-l(q)H(l(q))} N_k} \right\rfloor \qquad (7)$$

where the level function $l(q) = q-K-1$, $H(x)$ denotes the Heaviside unitary step function and $sign(x) = 2H(x)-1$.

2.4. Redundancy control

Denote by R_m the rates and by $D_m(R_m)$ the corresponding side description distortion, where $m = 1,2$. Also, denote by D_0 the central distortion. The standard source coder, i.e. the single-description coder (SDC) minimizes D_0 for a given rate R_0. Intuitively, the redundancy is the bit-rate sacrificed by an MDC compared to the SDC in order to achieve the same central D_0 distortion:

$$\rho = \sum_{m=1}^{2} R_m - R_0 \qquad (8)$$

The redundancy expression for the quantization levels $q, K+1 \le q \le K+P+1$ is derived from [6], [7] for the two-channel case. Hence, the SDC rate is $R_0 = \log_2(3 \cdot 2^{K+P+2-q} - 1)$ and the corresponding side-channel rate is $R_m = \log_2(2 \cdot 2^{K+P+2-q} - 1)$. Thus, formula (8) for $K+1 \le q \le K+P+1$ is given by:

$$\rho_q = 2 \cdot \log_2(2 \cdot 2^{K+P+2-q} - 1) - \log_2(3 \cdot 2^{K+P+2-q} - 1) \qquad (9)$$

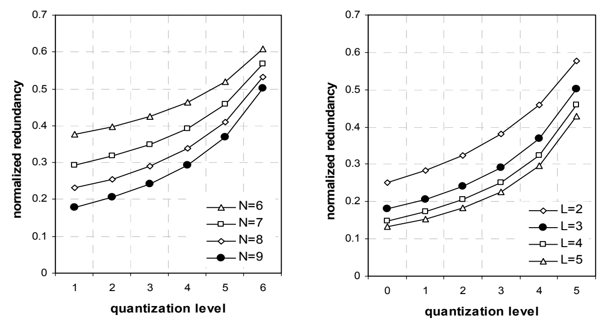
For the quantization levels $q, 0 \le q \le K$ employing quantizers with disconnected partitions cells, the SDC rate is $R_0 = \log_2(3 \cdot 2^{P+1} \cdot \prod_{k=q}^{K} N_k - 1)$ and the corresponding side-channel rate is of the form $R_m = \log_2(2 \cdot 2^{P+1} \prod_{k=q}^{K} L_k - 1)$. Thus, formula (8) for $0 \le q \le K$ is:

$$\rho_q = 2 \cdot \log_2\left(2 \cdot 2^{P+1} \prod_{k=q}^{K} L_k - 1\right) - \log_2\left(3 \cdot 2^{P+1} \prod_{k=q}^{K} N_k - 1\right) \qquad (10)$$

From (10) one can deduce the analytical expression of the normalized redundancy:

$$\rho_q = \frac{2 \cdot \log_2\left(2 \cdot 2^{P+1} \prod_{k=q}^{K} L_k - 1\right)}{\log_2\left(3 \cdot 2^{P+1} \prod_{k=q}^{K} N_k - 1\right)} - 1 \qquad (11)$$

One can conclude that for any instantiation of the generic EMDSQ family, the redundancy is directly dependent on the quantization level. In addition, for all the quantization levels $q, 0 \le q \le K$, the redundancy can be controlled via the L_q and N_q parameters, with $N_q \le L_q^2$.



(a) (b)

Fig. 3 Redundancy ρ versus quantization level $q, 0 \le q \le 5$ for (a) $L_q = 3$, $L_q^2 - 3 \le N_q \le L_q^2$; (b) $2 \le L_q \le 5$, $N_q = L_q^2$.

This mechanism is shown in Fig. 3 which depicts the normalized redundancy versus the quantization level (as expressed by (11)), for practical instantiations of the L_q and N_q parameters. It is noticeable that the descriptions' redundancy decreases from the higher to the lower quantization levels, and moreover, by changing the parameters L_q and N_q we can speed the redundancy's rate-of-decay.

3. EXPERIMENTAL RESULTS

In order to demonstrate the redundancy control mechanism we assess the rate-distortion behavior for several instantiations of the generic family of EMDSQ. The EMDSQ instantiations employ one ($K=0$), two ($K=1$) and three ($K=2$) quantization levels with disconnected partition-cell quantizers. In all the cases $L_q = 2$ and $N_q = 4$ for any $q, 0 \le q \le K$. The corresponding index assignment is illustrated in Fig. 4.

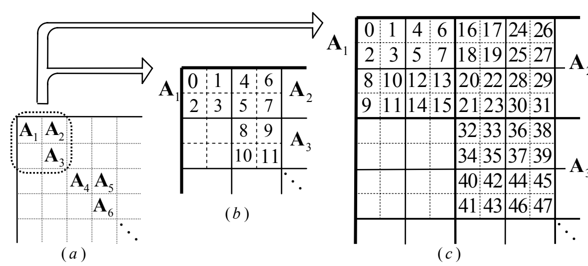


Fig. 4. EMDSQ index assignment strategy for (b) one and (c) two quantization levels employing disconnected partition-cell high-rate quantizers.

The EMDSQ instantiations are applied on a memoryless Laplacian source of random generated numbers with zero mean and $\sigma = 44.7$, modeling a wavelet subband. Fig. 6 shows that it is possible to speed the distortion's rate-of-decay for the central channel by decreasing, after a quantization level, the redundancy between the two descriptions. Similar experimental results were obtained by varying the standard deviation within a broad range of values ($12 < \sigma < 90$).

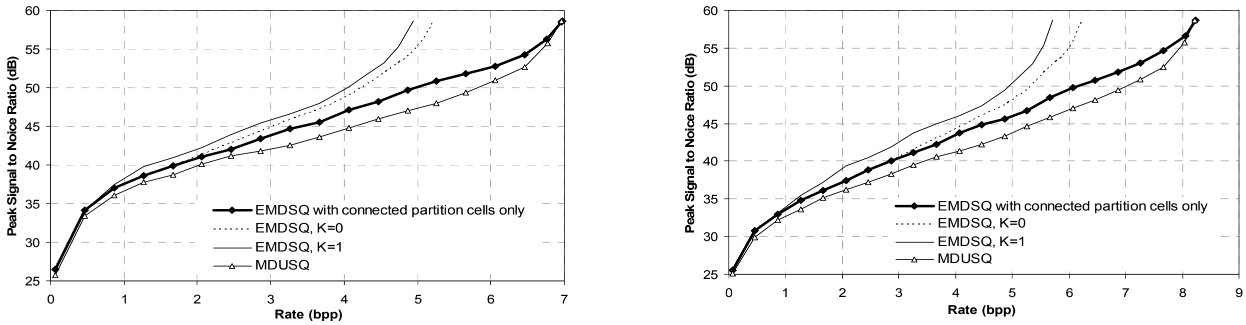


Fig. 5. Comparative central-channel rate-distortion performance for EMDSQ, without disconnected partition cells and with one ($K = 0$) and two ($K = 1$) quantization levels with disconnected partition-cell quantizers. The results are reported for the Lena 512x512 (left) and Goldhill 512x512 (right) images.

The considered EMDSQ instantiations are incorporated in a practical wavelet coding system that entropy codes the quantizer indices using the QuadTree coding algorithm of [10]. The central-channel rate-distortion performances obtained with the different EMDSQ instantiations applied on a common data set are compared, as shown in Fig. 5 for Lena and Goldhill images.

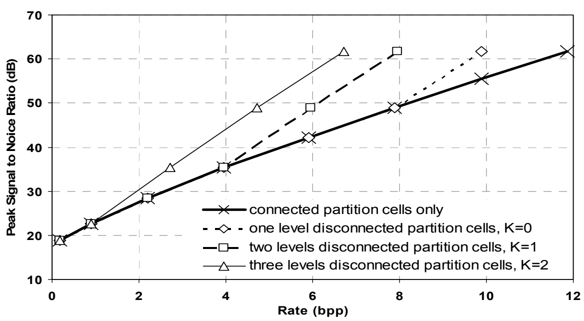


Fig. 6. Comparative central-channel rate-distortion performance for EMDSQ, without disconnected partition-cells and with one ($K = 0$), two ($K = 1$) and three ($K = 2$) quantization levels with disconnected partition-cell quantizers.

In [6], [7] it was demonstrated that the EMDSQ employing quantizers with connected partition cells systematically outperforms the state-of-the-art MDUSQ of [5] on a broad set of images. This also results from Fig. 5. It is also important to notice that the proposed EMDSQ family allows for controlling the redundancy between the two descriptions at each quantization level, while the MDUSQ does not feature this important control mechanism.

4. CONCLUSIONS

A new family of EMDSQ has been proposed in this paper. The EMDSQ provide a fine-grain refinable representation of the input data and are designed under the constraint of producing double-deadzone central quantizers at each quantization level. Moreover, EMDSQ allow for a control mechanism that tunes the descriptions' redundancy for each quantization level. The employed mechanism enables (1) to control the tradeoff between coding efficiency and resilience to errors, and (2) to improve the resilience by increasing the redundancy in the important layers of the bit-streams. Finally, the experimental results show that instantiations of the proposed EMDSQ outperform the state-of-the-art.

6. ACKNOWLEDGEMENTS

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