# 3D ORTHOGRAPHIC RECONSTRUCTION BASED ON ROBUST FACTORIZATION METHOD WITH OUTLIERS

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**ABSTRACT**

It is well known that both shape and motion can be factorized directly from the measurement matrix constructed from feature points trajectories under orthographic camera model. In practical applications, the measurement matrix might be contaminated by noises and contains outliers and missing values. A direct SVD(Singular Value Decomposition) to the measurement matrix with outliers would yield erroneous result. In this paper we present a new algorithm for computing SVD by linear $l_1$-norm regression and apply it to structure from motion problem. It is robust to outliers and can handle missing data naturally. The linear regression problem is solved using weighted-median algorithm and is simple to implement. The proposed robust factorization method with outliers can improve the reconstruction result remarkably. Quantitative and qualitative experiments illustrate the good performance of our approach.

## 1. INTRODUCTION

The reconstruction of shape and motion of a rigid object from an image sequence is one of the most studied problems within computer vision community [1] [2]. It has wide applications such as scene modelling, robot navigation, object recognition and virtual reality. Tomasi and Kanade [3] developed a factorization method based on SVD of the measurement matrix for scene reconstruction with an orthographic camera. It was further extended by Poelman and Kanade [4] to weak and para perspective views. Costeria and Kanade [5] proposed the multibody factorization method for reconstructing the motions and shapes of independently moving objects. They used the shape interaction matrix to separate those objects. The shape interaction matrix is also derived from the SVD of the measurement matrix. The factorization algorithm achieves its robustness and accuracy by applying the singular value decomposition to a large number of frames and feature points. The 3D motion of the camera and 3D positions of feature points are recovered by first compute the SVD of the measurement matrix, then the metric constraints are imposed. These factorization algorithms work by linearizing the camera observation model and give good results without an initial guess for the solution. The whole procedure is linear.

The factorization algorithms assume that the problem of finding correspondence between frames has been solved. But the correspondence problem itself is one of the most difficult fundamental problems within computer vision. In practical applications, the tracked feature points trajectories are inevitable not always correct. More recent works on the treatment of outlier trajectories include those by D. Q. Huynh [6] [7], Yasuyuk Sugaya [8]. In [6],A novel robust method for outlier detection in structure and motion recovery for affine cameras is presented. It can also be seen as an importation of the LMedS technique or RANSAC into the factorization framework. In [8], the authors extended the above method to multiple moving objects, they fit an appropriate subspace to the detected trajectories and remove those that have large residuals. In [7], the authors presented an outlier correction scheme that iteratively updates the elements of the image measurement matrix. The magnitude and sign of the update to each element is dependent upon the residual estimated in each iteration.

In this paper, we propose an algorithm for computing SVD robustly when the input data contains outliers. The algorithm is based on the linear $l_1$ norm estimator. The linear regression problem is solved using weighted-median algorithm and is simple to implement. We then apply the proposed algorithm to the problem of 3D orthographic reconstruction based on factorization method. It is robust to outliers and can handle missing data naturally.

This paper is organized as follows: Section 2 briefly describe the problem of the factorization method for 3D reconstruction based on SVD. The definition of measurement matrix and its rank property is also presented. In section 3, we describe in detail the proposed robust SVD with outliers. Section 4 provides the experimental results. In section 5 we draw the conclusion.

## 2. PROBLEM FORMULATION

Given an image sequence, suppose there are a set of $P$ tracked feature points over $F$ frames. We then obtain trajectories

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of image coordinates $(u_{fp}, v_{fp}|f = 1, ..., F, p = 1, ..., P)$. We write the horizontal feature coordinates $u_{fp}$ into an $F \times P$ matrix $U$ : we use one row per frame, and one column per feature point. Similarly, an $F \times P$ matrix $V$ is built from the vertical coordinates $v_{fp}$. The combined matrix of size $2F \times P$ $[\frac{U}{V}]$ is called the measurement matrix $W_{2F\times P}$. It is well known that the rank of the measurement matrix is 4. Because the matrix can be written as

$$W_{2F\times P} = M_{2F\times 4}N_{4\times P}$$

where $M_{2F\times 4}$ is the motion matrix and $N_{4\times P}$ is the shape matrix. Using Singular Value Decomposition $W$ is decomposed and approximated as

$$W = USV^T$$

where $S = diag(s_1, s_2, s_3, s_4)$ is a diagonal matrix made of the four biggest singular values which reveal the most important components in the data. Matrices $U \in R^{2F\times 4}$ and $V \in R^{P\times 4}$ are the left and right singular matrices respectively, such that $U^TU = V^TV = I$ ,where $I$ is the $4 \times 4$ identity matrix.

By defining

$$\tilde{M} = US^{1/2}, \tilde{N} = S^{1/2}V^T ,$$

we can write $W = \tilde{M}\tilde{N}$. However, this factorization is not unique, since for any invertible $4 \times 4$ matrix $A$ , $M = \tilde{M}A$ and $N = A^{-1}\tilde{N}$ are also a possible solution because

$$MN = (\tilde{M}A)(A^{-1}\tilde{N}) = \tilde{M}\tilde{N} = W$$

By using the orthogonal property of the rotation matrix and by treat the centroid of the feature points as the origin of the coordinate system, the matrix $A$ can be computed. The detail of the implementation of the factorization method could be found in [3]. If the matrix has no outliers, the motion and shape can be recovered through SVD accurately. But in practical applications, the measurement matrix might be contaminated by noises and contains outliers and missing values. A direct singular value decomposition to the measurement matrix with outliers would yield erroneous result.

## 3. ROBUST SVD USING LINEAR REGRESSION

In [9], the authors proposed a robust method for computing SVD, which is based on the weighted median algorithm. But when the rank of the input matrix is more than one, the algorithm can not give correct result. In [10] the authors proposed a similar algorithm for computing subspaces which is also based on the $l_1$ norm. They optimized the $l_1$ norm optimization problem using weighted median algorithm too. But their method did not give the orthogonal left and right singular matrices explicitly. Also, since they compute the bases one by one, their method is prone to be trapped into some bad local minimum. In this paper we propose a robust algorithm for computing SVD with outliers based on linear $l_1$-norm regression. The statistical linear regression model is defined as

$$y = Ax + \epsilon \qquad (1)$$

where $A$ is a $n \times r$ matrix referred to as the system matrix, $x$ is a $r \times 1$ vector of parameters of the linear model, and $\epsilon$ is a $n \times 1$ vector corresponding to the error terms in the asserted relationship. $y$ is a vector of $n \times 1$ of observations. The objective of linear regression is to find parameter setting $x$ such that some appropriate measure of the resulting residual is as small as possible. It can be treated using either $l_1$ norm or $l_2$ norm. Consider the following linear $l_1$ estimation problem of finding a vector $x \in R^r$ that solves the following minimization problem

$$\min_x \|Ax - y\|_1$$

where $\|z\|_1$ is the $l_1$ norm of $z$ . The linear $l_1$ estimation problem is much more difficult to solve than the linear maximum likelihood estimation problem, where the $l_2$ norm is used instead of the $l_1$ norm. But the linear $l_1$ estimator is more robust than linear squares estimators in the presence of outliers.

In the algorithm 1, we propose a method for computing the above $l_1$ norm regression problem.

**Algorithm 1:**

1) Randomly initialize $x_i, i = 1, ...r$

2) For every element in $x_i, i = 1, ...r$

$\tilde{y} = y - \sum_{k\neq i} A_{\cdot k}x_k$

solve the problem $A_{\cdot i}x_i = \tilde{y}$ using weighted median algorithm. [11]

3) Repeat step (2) until convergence.

Based on algorithm 1, we can derive the algorithm for robustly computing the singular value decomposition of a given matrix. The left, right singular matrices and the singular values are all computed explicitly. The original SVD problem can be formulated as a set of $l_1$-norm regression subproblems, which are solved iteratively using algorithm 1. One of $U$ and $V$ is computed while keeping the other fixed. The $U$ and $V$ are orthogonalized in each iteration to ensure convergence. Extensive experiments show that our method is insensitive to the initialization and converges very fast. The procedure is described in algorithm 2 in detail.

**Algorithm 2:**

Input: A rank $r$ matrix $W_{n\times m}$

Output: The columns subspace $U_{n\times r}$ , the rows subspace $V_{m\times r}$ , the singular values $s_i, i = 1, ..r$ , where $U^TU = I_{r\times r}$ , $V^TV = I_{r\times r}$ and $Udiag(s)V^T = W$

1) Randomly initialize $U^{(0)}, V^{(0)}$ , where $U^{(k)}, V^{(k)}$ means the value in the *k-th* iteration;

2) For each iteration step $k$ :

a) For each row of $V$ , which is denoted as $V_i, i = 1, ..., m$ , solve the subproblem $U^{(k-1)}\tilde{V}_i^{(k)T} = W_{\cdot i}$ using algorithm 1;

b) $V^{(k)} = orth(\tilde{V}^{(k)})$ , where $orth(\cdot)$ means computing the orthogonal columns subspace of the given matrix;

c) For each row of $U$ , which is denoted as $U_i, i = 1, ..., n$ , solve the subproblem $\tilde{U}_i^{(k)} V^{(k)T} = W_i$.

d) $U^k = orth(\tilde{U}^{(k)})$

3) Repeat step (2) until convergence

4) Construct matrix $A_{nm \times r}$ , the *i-th* column is defined as $vec(U_{\cdot i} V_{\cdot i}^T)$, where $vec(\cdot)$ means vectorize the given matrix. Construct matrix $B_{nm \times 1} = vec(W)$. Solve the problem of $AS = B$ using algorithm 1.

When the matrix $W$ has missing data, the algorithm 2 can be slightly modified by discarding the corresponding missing item. In each iteration step, the subspace bases are computed all at once and orthogonalized immediately.We compare the results of using the standard SVD and the proposed method for computing the singular values of a random generated matrix of rank 10. The computed singular values are plotted in the figure 1. The two methods give almost the same result. We repeated the experiments many times, and find that the propose method is insensitive to the initial values and converge fairly fast in just several iterations. In the next section, we will demonstrate that the proposed algorithm is much more robust to outliers than the standard SVD.

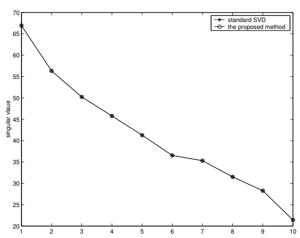
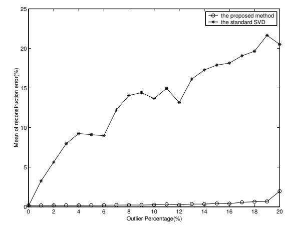


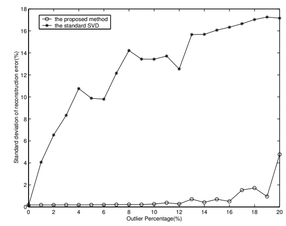

**Fig. 1**. Singular values computed using the standard SVD('*') and the proposed method('o'). The two methods give almost the same result

## 4. EXPERIMENTS

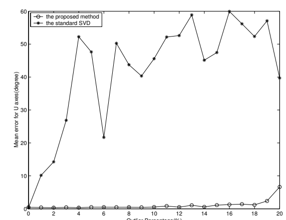
We have tested the proposed robust SVD algorithm quantitatively and qualitatively. In quantitative experiments, we randomly generate a rank 4 matrix $W$ of dimension $30 \times 45$ ,which means that $45$ feature points are tracked through $15$ frames. Small uniform random noises are added to each element of $W$ . Outliers are then added to a percentage range from $0 - 20\%$ , every columns may contain outlier coordinates. The matrix with outliers and noises is denoted as $\tilde{W}$ . First the singular decomposition of the true measurement matrix is computed. Then we apply the standard SVD and the proposed robust SVD to the matrix $\tilde{W}$ and compare them to the groundtruth ,respectively. At each outlier percentage level, the procedure repeat 20 times and the mean error is computed. Fig.2 shows the results: (a) is the means of reconstruction errors(in $\%$ ). (b) is the standard deviation of reconstruction errors(in $\%$ ).(c) is the means of the errors between the computed column subspaces and the true column subspaces. It is computed as the mean of angle error between corresponding subspace vectors. (d) is the means of the errors between the computed row subspaces and the true row subspaces. It can be seen clearly that the proposed robust SVD gives much better performance when the input data contains outliers.



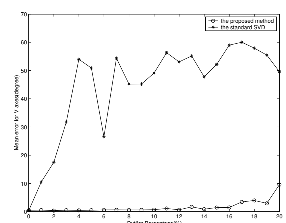

(a)




(b)




(c)



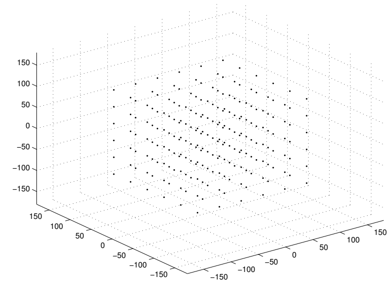

(d)

**Fig. 2**. Performance comparison between standard SVD and the proposed method. a) the mean of reconstruction errors(%); b)the standard deviation of reconstruction errors(%); c) the mean of the angle errors between the computed column subspaces and the true column subspaces ; d) the mean of the angle errors between the computed row subspaces and the true row subspaces.

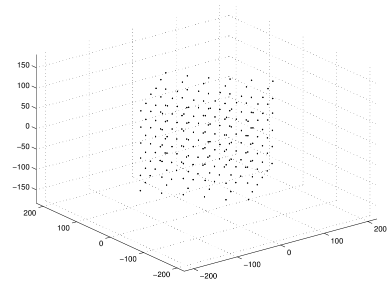
In experiment two, we synthesized a 3D cube which contains 216 points and projected these points orthographical to 20 frames. Next the size $40 \times 216$ measurement matrix is constructed. Then noises and outliers are added. The original cube is shown in Fig.2 (a). Then we apply the original factorization method [3] to compute the 3D metric reconstruction result, with standard SVD and the proposed robust SVD,respectively. Fig2.(b) is the reconstruction result using the proposed SVD method. Fig2.(c) is the reconstruction result using the standard SVD. The estimation of the shape improves greatly by using the proposed factorization method based on robust SVD.

## 5. CONCLUSION

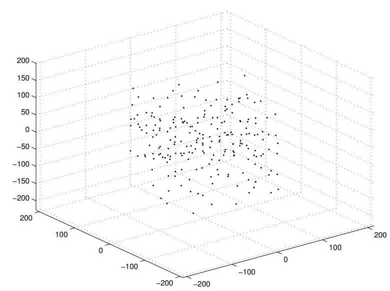
This paper proposes a singular value decomposition method based on the $l_1$ -norm linear regression, which is solved using the simple weighted median algorithm. The proposed method is robust to outliers and can handling missing data naturally. This algorithm is well fitted to the problem of 3D orthographic reconstruction using factorization method. Since in practical applications, the measurement matrix is often contaminated by noises and contains outliers and missing data. The proposed method is simple to implement. The iterative procedure is insensitive to the initialization and converges fast. Extensive experiments prove the good performance of our approach.



(a)



(b)



(c)

**Fig. 3**. Reconstruction result of the cube sequence. a) the original cube shape b) reconstruction using the proposed SVD method c) reconstruction using the standard SVD

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