A TWO-DIMENSIONAL LIFTING SCHEME OF INTEGER WAVELET TRANSFORM FOR LOSSLESS IMAGE COMPRESSION

Yan-kui SUN

Department of Computer Science and Technology
Tsinghua University,Beijing,100084,China
syk@mail.tsinghua.edu.cn

ABSTRACT

For separable two-dimensional wavelet transform, its 2D lifting scheme is implemented by a sequence of one-dimensional wavelet transforms, and the corresponding 2D integer version is built in a similar way. In this paper, by introducing and investigating matrix representation of separable two-dimensional wavelet transform, we propose a new 2D lifting scheme and its integer version, where the lifting steps operate on image directly. It is proved that they have less multiplication number than the classical ones, and the 2D integer wavelet transform is different from the classical 2D integer wavelet transform. Experiments indicate that, for the popular (5-3) filter, the 2D integer wavelet transform can achieves a little better compression than the classical one. The paper also compared JPEG2000 with JPEG-LS by experiments on DEM image lossless compression.

1. INTRODUCTION

Lifting scheme has recently been developed by Ingrid Daubechies and Wim Swedens[1],which has many advantages over the classical wavelet transform. For example, this method has a faster implementation, and it can yield integer-to- integer wavelet transforms[2].For the first generation wavelets, the lifting scheme has been developed for the 1D case and its extension to separable 2D case is easy, so the classical separable 2D lifting scheme reduces itself in successions of 1D lifting scheme. Now for some non-separable case such as quincunx wavelet transform , the quincunx lifting scheme has been investigated and built[3],where the lifting steps are processed in image space directly.In this paper, we will propose a method to build 2D lifting scheme for separable 2D case where the lifting steps are also done in image space, and further develop a novel 2D integer wavelet transform implementation for lossless image compression. This paper is organized as follows. In Section 2, we recall some background. This background is necessary to understand the matrix representations of separable 2D wavelet transforms and its 2D integer wavelet transform implementation in Section 3; some comparisons with other methods for lossless compression by experiments are done in Section 4, to show the efficiency of the 2D integer lifting scheme.

2. BACKGROUND:MATRIX REPRESENTATION OF 1D BIORTHOGONAL WAVELET TRANSFORM

The aim of this section is to make some necessary backgrounds on symmetric factorization and matrix representation of biorthogonal wavelet transform.Let $\{h,\tilde{h},g,\tilde{g}\}$ be a set of biorthogonal filters, where both h and $\tilde{h}$ are symmetric FIR filters, $h(z)$ be z-transform of filter h,$h_e(z)(h_o(z))$ be the even(odd) polyphase component of $h(z)$, and $P(z)$ denote the polyphase matrix of h and g. That is, $h(z) = h_e(z^2)+z^{-1}h_o(z^2)$, and $P(z) = \begin{bmatrix} h_e(z) & g_e(z) \\ h_o(z) & g_o(z) \end{bmatrix}$.Then, we have [4]

Theorem 1 *For any symmetric low pass filters h and $\tilde{h}$,if (h,g) is a complementary filter pair, i.e. $|P(z)| = 1$,then there exists one and only one symmetric factorization such that*

$$P(z) = \prod_{i=1}^{M} \begin{bmatrix} 1 & s_i(z) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ t_i(z) & 1 \end{bmatrix} \begin{bmatrix} K & 0 \\ 0 & 1/K \end{bmatrix}, \quad (1)$$

where $s_i(z) = \sum_{j=0}^{m_i} u_{i,j}(z^j + z^{-j-1})$, $t_i(z) = \sum_{j=0}^{n_i} p_{i,j}$ $(z^{j+1}+z^{-j}),(1 \le i \le M)$, $t_M(z) = 0$, and K is a non-zero constant.

An efficient way to calculate the factorization (1) can be found in [4].

Given a data vector $X = (x_0,x_1,\cdots,x_{N-1})^t$ where the t represents matrix transpose operation, considered for simplicity to be of length N, which can be divisible by 2^L with L non-negative integer. Then, the symmetric factorization leads to a symmetric implementation of biorthogonal wavelet transform:

$$s_l^{(0)} = x_{2l}, d_l^{(0)} = x_{2l+1}$$

This work is supported by the National Natural Science Foundation of China (No. 60203003).

For $i = 1$ to M

$$s_l^{(i)} = s_l^{(i-1)} - \sum_{j=0}^{m_i} u_{i,j}(d_{l+j}^{(i-1)} + d_{l-j-1}^{(i-1)})$$

$$d_l^{(i)} = d_l^{(i-1)} - \sum_{j=0}^{n_i} p_{i,j}(s_{l+j+1}^{(i)} + d_{l-j}^{(i)})$$

where $l = 0, 1, \cdots, N/2 - 1$.
For $l = 0$ to $N/2 - 1$

$$s_l = s_l^{(M)}/K, d_l = Kd_l^{(M)}.$$

The symmetric implementation is equivalent to a matrix transform on vector space [4], i.e. if we denote $Y = (s_0, s_1, \cdots, s_{N/2-1}, d_0, d_1, \cdots, d_{N/2-1})^t$,then $Y = TX$, where

$$T = RS \prod_{i=M}^{1} P_i U_i \tag{2}$$

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 & & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & & \cdots & 0 & 0 \\ & & & & \ddots & & & \\ 0 & 0 & 0 & 0 & & \cdots & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ & & & & \ddots & & & \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix},$$

$$S = \begin{bmatrix} K^{-1} & & & & & & \\ & K & & & & & \\ & & K^{-1} & & & & \\ & & & K & & & \\ & & & & \ddots & & \\ & & & & & K^{-1} & \\ & & & & & & K \end{bmatrix},$$

U_i consists of the row vectors $U_{i,n}$,$(n = 0, 1, 2, \cdots, N - 1)$: for $0 \le l \le N/2 - 1$, $U_{i,2l} = (0, 0, \cdots, 0, 0, -u_{i,m_i}, 0, -u_{i,m_i-1}, 0, \cdots, 0, -u_{i,0}, 1, -u_{i,0}, 0, \cdots, 0, -u_{i,m_i-1}, 0, -u_{i,m_i}, 0, 0, \cdots, 0)$;$U_{i,2l+1} = (0, 0, \cdots, 0, 1, 0, \cdots, 0)$ where the element at $(2l + 1)th$ row and $(2l + 1)th$ column equals to 1, all the other entries in $(2l + 1)th$ row are zeroes. Similarly,P_i is composed of the row vectors $P_{i,n}$,$(n = 0, 1, 2, \cdots, N - 1)$,where $P_{i,2l} = (0, 0, \cdots, 0, 1, 0, \cdots, 0)$;$P_{i,2l+1} = (0, 0, \cdots, 0, 0, -p_{i,n_i}, 0, -p_{i,n_i-1}, 0, \cdots, 0, -p_{i,0}, 1, -p_{i,0}, 0, \cdots, 0, -p_{i,n_i-1}, 0, -p_{i,n_i}, 0, 0, \cdots, 0)$.

3. TWO-DIMENSIONAL LIFTING SCHEME FOR INTEGER WAVELET TRANSFORMS

Let $W_0 = (x_{ij})_{N \times N}$ be an image, and T denote transform matrix of a biorthogonal wavelet transform on a signal of length N ,as is defined by (2), then, in terms of matrix operations, one level wavelet transform on all the rows of W_0 generates an intermediate matrix $W_0 T^t$, and the successive one level transform on all its columns leads to a matrix $T(W_0 T^t)$. Thus, if we use matrix W denote the separable 2D wavelet transform, then we have

$$W = T(W_0 T^t) = T W_0 T^t \tag{3}$$

For $0 \le l \le M - 1$, let $W_{2l+1} = U_l W_{2l} U_l^t$ and $W_{2l+2} = P_l (W_{2l+1}) P_l^t$,then, we can derive from (2) and (3) that:

$$\begin{aligned} W &= T W_0 T^t = (RS \prod_{i=M}^{1} P_i U_i) W_0 (RS \prod_{i=M}^{1} P_i U_i)^t \\ &= R(S W_{2M} S^t) R^t. \end{aligned} \tag{4}$$

In the following, we describe how to use the equation (4) to build a 2D lifting scheme for separable 2D wavelet transform. The procedure consists of the following stages:

Firstly, for all $i, j = 0, 1, 2, \cdots, N/2 - 1$,the original image W_0 is split into four parts by (5).

$$\begin{aligned} s_{2i,2j}^{(0)} &= x_{2i,2j}, d_{2i,2j+1}^{(0)} = x_{2i,2j+1}, \\ d_{2i+1,2j}^{(0)} &= x_{2i+1,2j+1}, s_{2i+1,2j+1}^{(0)} = x_{2i+1,2j+1}. \end{aligned} \tag{5}$$

This is equivalent to perform a two-dimensional Lazy wavelet transform on W_0, i.e.

$$W_0 = \begin{bmatrix} \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & s_{2i,2j}^{(0)} & d_{2i,2j+1}^{(0)} & s_{2i,2j+2}^{(0)} & \cdots \\ \cdots & d_{2i+1,2j}^{(0)} & s_{2i+1,2j+1}^{(0)} & d_{2i+1,2j+2}^{(0)} & \cdots \\ \cdots & s_{2i+2,2j}^{(0)} & d_{2i+2,2j+1}^{(0)} & s_{2i+2,2j+2}^{(0)} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}.$$

Secondly, starts from W_0 , compute $W_{2l+1} = U_l W_{2l} U_l^t$and $W_{2l+2} = P_l W_{2l+1} P_l^t$ alternately,$l = 0, 1, 2, \cdots, M - 1$.

For each $0 \le k \le 2M$, denote

$$W_k = \begin{bmatrix} \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & s_{2i,2j}^{(k)} & d_{2i,2j+1}^{(k)} & s_{2i,2j+2}^{(k)} & \cdots \\ \cdots & d_{2i+1,2j}^{(k)} & s_{2i+1,2j+1}^{(k)} & d_{2i+1,2j+2}^{(k)} & \cdots \\ \cdots & s_{2i+2,2j}^{(k)} & d_{2i+2,2j+1}^{(k)} & s_{2i+2,2j+2}^{(k)} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}.$$

then,through matrix multiplication operation,we can conclude that the matrix $W_{2l+1} = U_l W_{2l} U_l^t$ can be computed sequentially in the following 2D lifting steps,where each lifting step computes for all $i, j = 0, 1, \cdots, N/2 - 1$, and periodizing symmetric boundary extensions are used.

Step1. $d_{2i+1,2j}^{(2l+1)} = d_{2i+1,2j}^{(2l)} - \sum_{k=0}^{m_i} u_{l,k}(s_{2i+1,2j-2k-1}^{(2l)} + s_{2i+1,2j+2k+1}^{(2l)})$.

Step 2. $s_{2i,2j}^{(2l+1)} = s_{2i,2j}^{(2l)} - \sum_{k=0}^{m_i} u_{l,k}(d_{2i,2j-2k-1}^{(2l)}$
$+ d_{2i,2j+2k+1}^{(2l)} + d_{2i-2k-1,2j}^{(2l+1)} + d_{2i+2k+1,2j}^{(2l+1)})$.

Step3. $d_{2i,2j+1}^{(2l+1)} = d_{2i,2j+1}^{(2l)} - \sum_{k=0}^{m_i} u_{l,k}(s_{2i-2k-1,2j+1}^{(2l)} + s_{2i+2k+1,2j+1}^{(2l)})$.

Spep4. $s^{(2l+1)}_{2i+1,2j+1} = s^{(2l)}_{2i+1,2j+1}$.

Similarly, $W_{2l+2} = P_l W_{2l+1} P_l^t$ can be computed sequentially as follows:

Step1. $d^{(2l+2)}_{2i,2j+1} = d^{(2l+1)}_{2i,2j+1} - \sum_{k=0}^{n_i} p_{l,k}(s^{(2l+1)}_{2i,2j-2k} + s^{(2l+1)}_{2i,2j+2k+2})$.

Step 2. $s^{(2l+2)}_{2i+1,2j+1} = s^{(2l+1)}_{2i+1,2j+1} - \sum_{k=0}^{n_i} p_{l,k}(d^{(2l+2)}_{2i-2k,2j+1} + d^{(2l+2)}_{2i+2k+2,2j+1} + d^{(2l+1)}_{2i+1,2j-2k} + d^{(2l+1)}_{2i+1,2j+2k+2})$.

Step3. $d^{(2l+2)}_{2i+1,2j} = d^{(2l+1)}_{2i+1,2j} - \sum_{k=0}^{n_i} p_{l,k}(s^{(2l+1)}_{2i-2k,2j} + s^{(2l+1)}_{2i+2k+2,2j})$.

Spep4. $s^{(2l+2)}_{2i,2j} = s^{(2l+1)}_{2i,2j}$.

Thirdly,calculate $W_{2M+1} = SW_{2M}S^t$:

$$s^{(2M+1)}_{2i,2j} = s^{(2M)}_{2i,2j}/K^2, s^{(2M+1)}_{2i+1,2j+1} = K^2 s^{(2M)}_{2i+1,2j+1},$$

$$d^{(2M+1)}_{2i,2j+1} = d^{(2M)}_{2i,2j+1}, d^{(2M+1)}_{2i+1,2j} = d^{(2M)}_{2i+1,2j}.$$

Finally, obtain

$$W = RW_{2M+1}R^t = \begin{bmatrix} LL & LH \\ HL & HH \end{bmatrix},$$

where $LL = \left(s^{(2M+1)}_{2i,2j}\right), LH = \left(d^{(2M+1)}_{2i,2j+1}\right), HL = \left(d^{(2M+1)}_{2i+1,2j}\right)$, and $HH = \left(s^{(2M+1)}_{2i+1,2j+1}\right)$.

Theorem 2 *Every separable two-dimensional biorthogonal wavelet transform can be obtained as the two-dimensional Lazy wavelet transform followed by a finite alternating primal and dual lifting steps and a scaling, where the primal and dual lifting steps are all operated on image directly.*

The following Theorem (proof omitted here) shows that, for separable two-dimensional biorthogonal wavelet transform, the above 2D lifting scheme is more efficient in computation than the classical one:

Theorem 3 *Under the hypotheses of Theorem 1, for separable two-dimensional biorthogonal wavelet transform, the number of multiplications of the new 2D lifting scheme, compared to that of the classical lifting scheme, is decreased at least 25%.*

By using "$\lfloor\rfloor$" operator and ignore the scaling transform of the 2D lifting scheme, we can get an integer-to-integer wavelet transform.The procedure is as follows:

1. Perform a two-dimensional Lazy wavelet transform on image W_0 according to (5);
2. Compute the integer versions of $W_{2l+1} = U_l W_{2l} U_l^t$ and $W_{2l+2} = P_l W_{2l+1} P_l^t$ alternately,$l = 0, 1, 2, \cdots, M-1$.Taking $W_{2l+1} = U_l W_{2l} U_l^t$ for example. Its 2D integer lifting steps are:

step 1. $d^{(2l+1)}_{2i+1,2j} = d^{(2l)}_{2i+1,2j} - \left\lfloor \sum_{k=0}^{m_i} u_{l,k}(s^{(2l)}_{2i+1,2j-2k-1} + s^{(2l)}_{2i+1,2j+2k+1}) + \frac{1}{2} \right\rfloor$.

step 2. $s^{(2l+1)}_{2i,2j} = s^{(2l)}_{2i,2j} - \left\lfloor \sum_{k=0}^{m_i} u_{l,k}(d^{(2l)}_{2i,2j-2k-1} + d^{(2l)}_{2i,2j+2k+1} + d^{(2l+1)}_{2i-2k-1,2j} + d^{(2l+1)}_{2i+2k+1,2j}) + \frac{1}{2} \right\rfloor$.

step 3. $d^{(2l+1)}_{2i,2j+1} = d^{(2l)}_{2i,2j+1} - \left\lfloor \sum_{k=0}^{m_i} u_{l,k}(s^{(2l)}_{2i-2k-1,2j+1} + s^{(2l)}_{2i+2k+1,2j+1}) + \frac{1}{2} \right\rfloor$.

step 4. $s^{(2l+1)}_{2i+1,2j+1} = s^{(2l)}_{2i+1,2j+1}$.

where $\lfloor x \rfloor$ stands for the integer part of x.

The following fact shows that the 2D integer lifting scheme is different from the classical integer lifting scheme. Considering the computation of W_{2l+1} again,according to classical separable two-dimensional integer lifting scheme, the corresponding $s^{(2l+1)}_{2i,2j}$ is calculated by:

$$\begin{aligned} s^{(2l+1)}_{2i,2j} &= s^{(2l)}_{2i,2j} \\ &- \left\lfloor \sum_{k=0}^{m_i} u_{l,k}(d^{(2l)}_{2i,2j-2k-1} + d^{(2l)}_{2i,2j+2k+1}) + \frac{1}{2} \right\rfloor \\ &- \left\lfloor \sum_{k=0}^{m_i} u_{l,k}(d^{(2l+1)}_{2i-2k-1,2j} + d^{(2l+1)}_{2i+2k+1,2j}) + \frac{1}{2} \right\rfloor. \end{aligned}$$

But, in general

$$\begin{aligned} &\left\lfloor \sum_{k=0}^{m_i} u_{l,k}(d^{(2l)}_{2i,2j-2k-1} + d^{(2l)}_{2i,2j+2k+1}) + \frac{1}{2} \right\rfloor + \\ &\left\lfloor \sum_{k=0}^{m_i} u_{l,k}(d^{(2l+1)}_{2i-2k-1,2j} + d^{(2l+1)}_{2i+2k+1,2j}) + \frac{1}{2} \right\rfloor \\ \neq &\left\lfloor \sum_{k=0}^{m_i} u_{l,k}(d^{(2l)}_{2i,2j-2k-1} + d^{(2l)}_{2i,2j+2k+1} + d^{(2l+1)}_{2i-2k-1,2j} + \right. \\ &\left. d^{(2l+1)}_{2i+2k+1,2j}) + \frac{1}{2} \right\rfloor. \end{aligned}$$

Example: (5-3) wavelet filter

As is well known that the (5-3) filter has been adopted in JPEG2000 for lossless image compression. For the popular wavelet filter, its polyphase matrix has the following symmetric factorization:

$$P(z) = \begin{bmatrix} 1 & 0 \\ -\tau(1+z) & 1 \end{bmatrix} \begin{bmatrix} 1 & -\nu(1+z^{-1}) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\omega & 0 \\ 0 & \omega \end{bmatrix}$$

where $\tau = -0.5, \nu = 0.25, \omega = \sqrt{2}$. In this case, there are $M = 2, u_{1,0} = 0, p_{1,0} = -\tau, u_{2,0} = -\nu, p_{2,0} = 0, K = 1/\omega$. It can be verified that, compared to the classical method, the number of multiplications of its 2D lifting scheme is decreased 1/2, and the number of multiplications of its 2D integer lifting scheme decreased 1/4 when scaling transform is ignored.

4. APPLICATIONS:LOSSLESS COMPRESSION OF DEM IMAGES

In this section, we discuss the lossless compression performance of the 2D integer lifting scheme by comparing it with the classical separable integer lifting scheme and JPEG-LS, using the well-known (5-3) filter and JPEG2000 Verification Model through experiments on DEM(Data Elevation Model) images. The DEM images and their format specifications are available at the USGS site http://edcftp.cr.usgs.gov/pub/data/DEM/250 and http: //edcwww.cr.usgs.gov/glis /hyper/guide/usgs_dem.supplement#typea. The JPEG2000 Verification Model used here is available at http://www.ece.uvic.ca /∼mdadams/jasper. In this JPEG2000 VM, each component of image pixel is represented as 8 bits, here we modifies it slightly so that it is suitable for processing any integer bit component, still called JPEG2000, while we call its modified version S-JPEG2000 when the wavelet transform is implemented by using the 2D integer lifting scheme instead of the classical separable algorithm. The DEM wavelet encoder block diagram is shown in Figure 1. Experiments on the five image files that have been used as tested images in [5] are done by using JPEG2000 and S-JPEG2000, the comparison of compression ratio(CR) is shown in Table 1.The experiments indicate that, for all the five tested images, S-JPEG2000 achieves better compression than JPEG-2000 while the JPEG2000 achieves better compression than JPEG-LS.

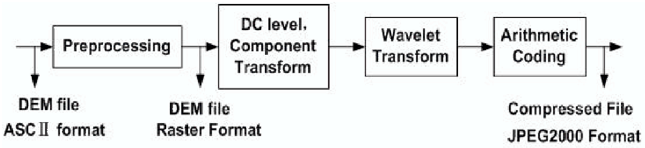

Fig. 1. DEM wavelet coder block diagram

We tested another 100 16-bit DEM images from the USGS data set by using S-JPEG2000 and JPEG2000, the result is that S-JPEG2000 achieves better compression than JPEG2000 for 98 images.

5. CONCLUSION

For separable 2D biorthgonal wavelet transform, this paper proposes a novel 2D lifting scheme and its integer version, where the lifting steps are done in image space directly. The 2D lifting scheme has the same wavelet transform as the classical separable lifting scheme, so they should have the same compression performance, its advantage over the classical one is its higher computation efficiency. The 2D integer lifting scheme has not the same wavelet transform as the classical separable integer version, it also has higher computation efficiency over the classical one. Experiments indicate that, for the popular (5-3) wavelet filter, the 2D integer lifting scheme can achieve a little better compression than the classical one; and some experiments demonstrate that S-JPEG2000 and JPEG2000 may have a little better compression performance than JPEG-LS for DEM image lossless compression.

Table 1. Comparison of compression ratio(CR) on DEM raster format (16-bit per pixel)

DEM image	JPEG2000 CR	S-JPEG2000 CR	JPEG-LS CR[5]
Richmond-e	48.657	50.979	45.15
Grand-rapids-e	16.950	17.723	13.76
Dallas-e	14.148	14.825	11.69
Puerto-rico-c	11.266	11.359	9.75
Denver-w	5.466	5.490	4.12

6. REFERENCES

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