SOURCE SEPARATION IN NOISY ASTROPHYSICAL IMAGES MODELLED BY MARKOV RANDOM FIELDS

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ABSTRACT

Astrophysical radiation maps provide images which are superpositions of various cosmological components such as the cosmic microwave background (CMB) radiation, galactic dust, synchrotron, free-free emission and extragalactic radio sources. All these components are of great interest to cosmologists and in particular CMB, in addition to being the picture of the early universe, carries important information that would help us to choose between existing evolution theories of the universe. In this work we present a technique for the separation of these components in the presence of receiver noise. In contrast with most work in the literature, we make use of the spatial information in the images in the form of correlation between pixels which we model using Markov Random Fields. The spatial information is included in the MRF model through a Bayesian estimation framework. We provide comparisons with the results obtained by FastICA.

1. INTRODUCTION

In the field of astrophysics and cosmology an important current problem is the recovery of independent components from a mixture of radiation sources corrupted with receiver noise. The problem is motivated by the availability of the measurements by the satellite Wilkinson Microwave Anisotropy Map (WMAP) [1] and the future ESA mission PLANCK [2] which would provide radiation maps of the sky in unprecedented resolution. Among the components present in the radiation maps, the CMB is of utmost importance: not only it provides a picture of the early universe shortly after the bigbang, but also gives information about cosmological parameters which would help us to choose between competing theories for the evolution of the universe. The other components also are of interest in themselves for understanding the universe.

A number of works in the literature have addressed the problem of the separation of the astrophysical components. In particular, Baccigalupi et al. [3] and Maino et al. [4] used the FastICA algorithm [5] and its noisy version which showed limited success in separating artificial mixture in the presense of significant noise. In both of these works, the techniques employed were completely blind to the sources. In [6], the authors have adopted a generic model for the sources and obtained better results in comparison with the FastICA technique. Alternatively, [7] performs blind source separation via spectral matching, i.e. matching the theoretical spectra with model parameters to the spectra of the observed data. Snoussi et al. [8], utilise an EM algorithm in the spectral domain, making use of some generic priors for the sources. Both of these works assume stationary noise and signals which is not the case for the astrophysical image separation problem, and they both suffer from common drawbacks of the EM algorithm, i.e. local optimality and computational complexity. Moreover, all of these techniques ignore the local spatial information present in the images. However, the image data carries important information in its spatial structure. In particular, foreground sources show significant local correlation and for each source this correlation structure is different from the other ones and especially from CMB. A general technique that models the local correlation structure in the images using Markov Random Fields (MRF) and employs Bayesian estimation to obtain separation was developed in [9], and was showed to be robust against noise. In this work, our objective is to exploit the autocorrelation information in the separation of astrophysical images from noisy data.

The paper is organized as follows. In section 2, we present the technique of Bayesian source separation using MRF image model in its general outline. In section 3, we will present our choices of the Gibbs priors to model the pixel interactions which will function as regularizers, and particular attention will be paid to priors that are adaptive to the true edges in the images. In section 4, based on the convexity of the chosen regularizers, the alternating maximization algorithm employed to perform the MAP estimation of both the sources and the mixing matrix is presented. In section 5, we will show results of simulation studies. The last section is reserved for conclusions and future work.

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2. BAYESIAN SOURCE SEPARATION USING MARKOV RANDOM FIELDS

We model the astrophysical images $\mathbf{x}$ as simple linear instantaneous noisy mixtures of the cosmological radiation sources $\mathbf{s}$:

$$\mathbf{x}(i,j) = \mathbf{A}\mathbf{s}(i,j) + \mathbf{n}(i,j), \ \ i = 1,...,n \ \ j = 1,...,m \quad (1)$$

where $\mathbf{n}(i,j)$ is the noise or measurement error vector at location (i,j), and $\mathbf{A}$ is the unknown mixing matrix.

It was shown in [9] that the problem of estimating the mixing matrix $\mathbf{A}$ and the source samples $\mathbf{s}$ could be stated in a Bayesian setup as the Maximum A Posteriori Estimation problem:

$$\begin{aligned}(\hat{\mathbf{s}}, \hat{\mathbf{A}}) &= arg \max_{\mathbf{s},\mathbf{A}} P(\mathbf{s}, \mathbf{A}, |\mathbf{x}) = \\ &= arg \max_{\mathbf{s},\mathbf{A}} P(\mathbf{x}|\mathbf{s}, \mathbf{A}) P(\mathbf{s}) P(\mathbf{A}) \qquad (2)\end{aligned}$$

where from independence assumption on the sources:

$$P(\mathbf{s}) = \prod_k P_k(\mathbf{s}_k). \qquad (3)$$

Writing the source priors in the Gibbs/MRF formalism, we get:

$$P_k(\mathbf{s}_k) = \frac{1}{Z_k} \exp\{-U_k(\mathbf{s}_k)\} \qquad (4)$$

where Z_k is the normalizing constant and $U_k(\mathbf{s}_k)$ is the prior energy in the form of a sum of potential functions over the set of cliques of interacting locations. The number of different cliques, as well as their shape, is related to the order of the neighborhood system adopted for the MRF. This, in turn, depends on the extent of correlation among the pixels, while the functional form of the potentials determines the correlation strength. In the specific case at hand, we define $U_k(\mathbf{s}_k)$ as:

$$U_k(\mathbf{s}_k) = \alpha_k \sum_{i,j} f_k(s_k(i,j)) + \qquad (5)$$

$$\beta_k \sum_{i,j} \sum_{(m,n)\in N_{i,j}} \phi_k \left(s_k(i,j) - s_k(m,n)\right)$$

where α_k and β_k are two positive weights, the so-called regularization parameters, and $N_{i,j}$ is the first order neighborhood for location (i,j). Functions f_k and ϕ_k are to be chosen according to our expectation about the probability law assigned to each sample of signal $\mathbf{s}_k$, and about the degree of correlation we assign to couples of adjacent samples, respectively.

3. GIBBS PRIORS

Our choices of the priors are led by the characteristics of the images. In particular, the CMB radiation is known to be Gaussian distributed, therefore for f_{CMB} in equation (5) a quadratic error function $||\mathbf{s}||^2$ which is the logarithm of the Gaussian kernel could be considered. This prior has the main function of regularizing against the receiver noise. This is especially important in image patches on the galactic plane where CMB is weaker than the galactic foregrounds and the presence of noise hampers the separation of CMB. The second part of the regularizer, ϕ, models the neighboring pixel interactions. Two features are important to be preserved in this issue: the smoothness of uniform regions and the edges, which are very crucial in image understanding. In order for a regularizer to be edge-preserving, it must increase slower than quadratically. Among the regularizers that possess this characteristic, we have explored specifically two. One was suggested by Shulman and Herve [10] (hereafter SH):

$$\phi(\xi) = \begin{cases} \xi^2 & \text{if } |\xi| \leq \Delta \\ \left(2\Delta|\xi| - \Delta^2\right) & \text{if } |\xi| > \Delta \end{cases} \qquad (6)$$

where parameter Δ is the threshold for the intensity gradient, above which the stabilizer becomes linear. Due to this linear behaviour, the regularizer is adaptive to discontinuities, that is it is able to preserve sharp edges in the images. The second regularizer was proposed by Bouman and Sauer [11] (hereafter BS), and is given by:

$$\phi(\xi) = |\xi|^\eta, \quad 1 \leq \eta < 2. \qquad (7)$$

For choices of η greater than 1, this regularizer is near-quadratic, so that, though still moderately adaptive to discontinuities, it is more suitable for images that are slowly varying rather than piecewise homogeneous. However, note that these two regularizers share the property of being convex.

4. THE ALGORITHM

As it is usual when the joint maximization with respect to two groups of variables with different characteristics is concerned, we propose to solve the problem of equation (2) by means of alternating maximization. In particular, this results in iteratively alternating steps of estimation of A and $\mathbf{s}$, respectively:

$$\begin{aligned} A^{(k)} &= arg \max_A P(\mathbf{x}|\mathbf{s}^{(k-1)}, A) P(A) \qquad (8) \\ \mathbf{s}^{(k)} &= arg \max_{\mathbf{s}} P(\mathbf{x}|\mathbf{s}, A^{(k)}) P(\mathbf{s}) \qquad (9) \end{aligned}$$

When very general assumptions are made about the involved distributions, an implementation of the above scheme which is able to ensure convergence and to limit the computational complexity is based on an overall simulated annealing for the estimation of A according to equation (8), interrupted at the end of each Metropolis cycle, to perform an update of the sources $\mathbf{s}$, according to equation (9). Nevertheless, in our case, we have made three basic assumptions that allow significant simplifications for the alternating maximization scheme. First, the noise is assumed to be white and Gaussian, second, that the regularizer is convex, and third, that no prior information is assumed for the mixing matrix, so that P(A) is the uniform distribution. All together, these assumptions make the problem convex in both the variables, so that the alternating maximization scheme can be implemented without need for stochastic relaxation. Furthermore, a gradient ascent can be used to solve equation (9), while equation (8) results in an analytic updating formula for A.

5. SIMULATION RESULTS

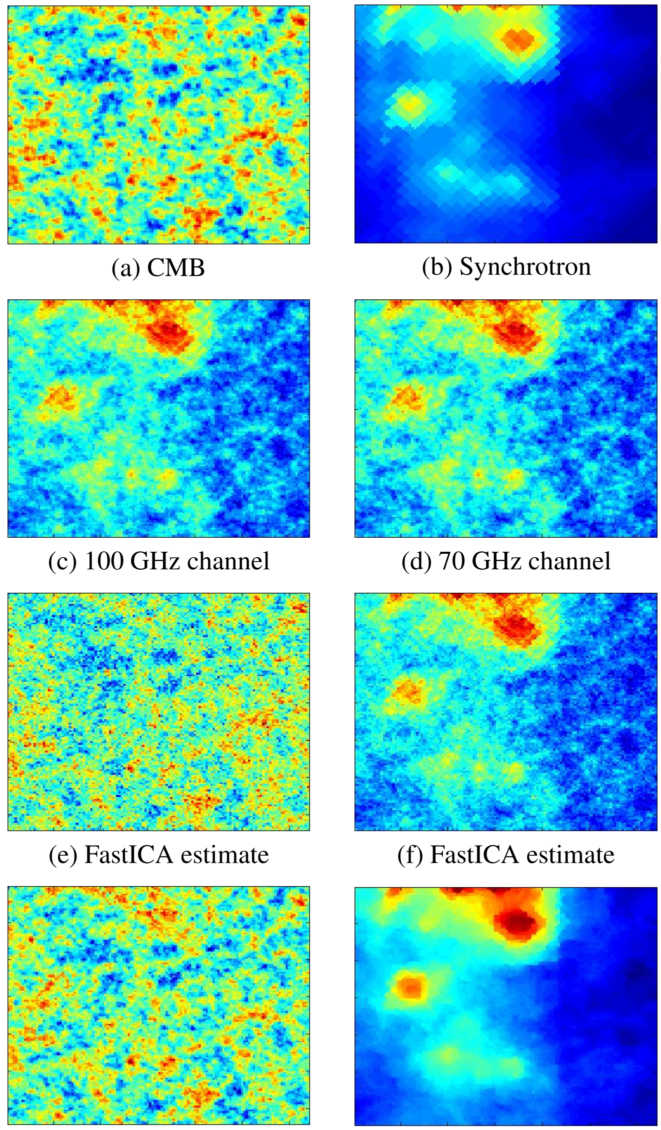
In this section we test our technique on the separation of simulated but realistic astrophysical image mixtures. For simplicity here we give examples for only mixtures of two sources (CMB and synchrotron radiation), but the algorithm is valid for mixtures of more components (the expected number of sources is six: CMB, galactic synchrotron, thermal galactic dust, Sunyaev-Zeldovich effects from clusters of galaxies, free-free emissions, and the effect of the individual radiogalaxies).

In Figures 1.a and 1.b we give the original CMB and synchrotron radiation maps, respectively, that are used to create the noisy synthetic mixtures in Figures 1.c and 1.d, corresponding to observations at the 100GHz and 70GHz channels. These original maps were taken from a simulated data set that is a simplified version of the one expected from the Planck surveyor satellite [2]. The mixing matrix used is:

$$A = \begin{bmatrix} 1.00 & 1.00 \\ 2.81 & 1.14 \end{bmatrix}$$

which again is an estimate, given in [3], of the one expected for Planck at the frequencies above. Figures 1.e and 1.f show the results obtained by the FastICA algorithm and Figures 1.g and 1.h the ones by the technique we have presented in this paper. In this case, the SH regularizer was used for both the two sources. The superiority of our technique is apparent. While in the FastICA results we see that a strong shadow of CMB is left on the synchrotron map, our technique has largely succeeded in the separation as well as in denoising. These performances are reflected also in the estimated matrices: the one estimated by the FastICA is:

$$\hat{A}_{FastICA} = \begin{bmatrix} -1.00 & 0.48 \\ -0.48 & 1.14 \end{bmatrix}$$



(a) CMB (b) Synchrotron

(c) 100 GHz channel (d) 70 GHz channel

(e) FastICA estimate (f) FastICA estimate

(g) MRF+ICA estimate (h) MRF+ICA estimate

Fig. 1. Comparison between FastICA and MRF+ICA source estimates from mixtures corrupted with 20dB of noise.

SIR	MRF+ICA	FastICA
CMB	9.23	-7.07
Synchrotron	16.52	5.99

Table 1. comparison of SIR performances of MRF+ICA and FastICA

while the one estimated by our technique is:

$$\hat{A}_{MRF+ICA} = \begin{bmatrix} 1.00 & 1.15 \\ 3.34 & 1.14 \end{bmatrix}$$

From a quantitative point of view, the signal to interference ratio (SIR) given in table 1 also shows a very significant difference (around 10dB) between the performances of MRF+ICA and FastICA.

6. CONCLUSIONS

In this paper, we have discussed a novel method for the separation of independent components in astrophysical images. The novelty of the method is in utilising the local correlation information in images. We modelled the images using an MRF formulation, where the Gibbs priors are chosen considering special properties of the astrophysical images. In a Bayesian set up, we formulated the blind source separation problem as a MAP estimation. The simulation results demonstrate the superiority of the technique to the FastICA algorithm. Furthermore, we have observed that for foreground components such as synchrotron radiation and galactic dust, edge-preserving regularizers, e.g. the Shulman and Herve regularizer [10], are successful since these images contain large smooth areas with sharp edges. For the CMB radiation, which is characterized by small homogeneous areas and not very pronounced edges, a finer tuning of the threshold parameter Δ is necessary for obtaining good results with this regularizer. Extra experiments are currently in course, aimed at verifying whether the Bouman and Sauer regularizer [11], could outperform the SH regularizer for the CMB estimation. We are also looking into automatic selection of the regularization parameters and for the analytical modelling of the correlation structure in astrophysical images. This is particularly important also for managing non-stationary noise, as expected for the Planck measurements. Although in principle our method is directly applicable to mixtures affected by space-variant noise, this would make the choice of the regularization parameters particularly cumbersome, since they would be pixel-dependent.

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