TIME-FREQUENCY BASED RADAR IMAGE FORMATION

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ABSTRACT

This paper briefly describes the background of synthetic aperture radar imaging and the time-varying behavior of Doppler frequency shifts in radar signals, discusses the conventional Fourier-based image formation and its problems, and introduces the time-frequency based image formation algorithm and its mathematical model, implementation and applications.

1. INTRODUCTION

As an RF (radio frequency) sensor, radar is widely used for imaging both stationary and moving objects (such as aircraft, ships, and ground vehicles) and generating high-resolution maps of earth-resources and terrain. Compared to optical and infrared sensors, radar performs at long distances, in all weather and climatic conditions, and with high accuracy. The radar image is generated from received signals reflected by the object being imaged, and is formed by computing the distribution of the object's reflectivity function in the range and cross-range domains. The range is the dimension along the radar line of sight (LOS) to the object and the cross-range is the dimension transverse to the radar LOS. High range resolution is achieved by using wide bandwidth radar signal waveforms. High cross-range resolution requires a large aperture of the radar antenna, which can be synthesized by using a small real antenna aperture mounted on a moving platform. Such a radar system with a synthetic aperture is called the synthetic aperture radar (SAR).

The most common modes in SAR are strip-map mode and spotlight mode. The strip-map mode can generate a wide-area map of the terrain. Spotlight SAR generates images of smaller scenes at finer resolution. Another mode, called inverse synthetic aperture radar (ISAR), is similar to spotlight SAR, but its data collection is different. In ISAR, the radar can be stationary and the object being imaged is moving. The relative motion between the radar and the object is the key to creating an image of the object. ISAR can be seen as a variation of SAR that images objects by coherently processing the returned radar data collected at different aspect angles.

In SAR and ISAR, a common approach for forming an image is range-Doppler processing, which takes a Fourier transform along the cross-range direction in the range-processed data. An object being imaged is often engaged in complicated motions that combine translation and rotation. Thus, motion compensation must be applied to form a focused image of the object. The challenges in ISAR image formation stem from the unknown nature of the object's motion. Because the only available information is the radar collected data itself, the ISAR motion compensation is a data-driven processing.

2. FOURIER BASED IMAGE FORMATION

A conventional radar image is formed by taking the Fourier transform over the observation time interval. To use the Fourier transform properly, it is assumed that the frequency content of the analyzed signal is time-invariant. However, when the target moves, Doppler frequency shifts are time varying. Thus, the Doppler spectrum becomes smeared, and the radar image becomes blurred.

There are many motion compensation algorithms for solving the Doppler smearing and image blurring problem. Most methods are Fourier-based approaches. To use the Fourier transform properly, during the imaging time interval scatterers on the target must remain in range and their Doppler frequency shifts must be constant. Motion compensation includes range alignment (tracking) and Doppler tracking. The range alignment is accomplished by tracking the movement of a reference point in the range profile across pulses and fitting it to a low-order polynomial. However, the accuracy of the alignment is limited by the range resolution. The range alignment may not be sufficient to overcome the phase errors measured in terms of the radar wavelength. Consequently, Doppler tracking must be carried out in order to align the phase. There are many methods for Doppler tracking. These methods consider the Doppler shifts of the target as a whole, and apply the same correction vector to all of the scatterers in the image.

From the radar received signal, if the target's range as a function of time is known exactly over the imaging time duration, the extraneous range-dependent phase term can be removed by multiplying its conjugate with the received signal. Then, the image of the target can be formed simply by taking the inverse Fourier transform of the motion compensated signal.

The motion-compensated base-band signals are organized into an M-by-N two-dimensional complex array $s_R(r_{m,n})$ where m = 0,1,...,M-1; n = 0,1,...,N-1. Therefore, N range profiles, each containing M range cells, can be obtained. At each range cell, the data across the N range profiles constitutes a new time history series. After applying range tracking and Doppler tracking, the aligned-range profiles becomes $G(r_{m,n})$, (m = 0, 1,…, M-1; n = 0, 1,…, N-1).

The Fourier based image formation takes the inverse Fourier transform for the new time history series and generates an N-point Doppler spectrum. By combining the M Doppler spectra at M range cells, finally, the M-by-N image is formed

$$I(r_m, f_n) = FFT_n\{G(r_{m,n})\}$$

where FFT_n denotes the fast Fourier transform operation with respect to the variable n. Therefore the radar image $I(r_m,f_n)$ represents the reflectivities of the target mapped onto the range-Doppler plane.

If the coherent processing interval is long or when the target exhibits fast maneuvers, the phase error due to the non-uniform rotational motion is often not negligible and must also be properly compensated. The time-frequency transform is an effective method to perform the Doppler tracking [1, 2].

3. TIME-FREQUENCY BASED IMAGE FORMATION

Fig. 1 illustrates the time-frequency based image formation. Standard motion compensation is used prior to image formation. The Fourier-based image formation generates only one image frame from an M×N complex (in-phase and quadrature phase) data set. The data consists of M time series histories, each having length N. However, time-frequency based image formation takes the time-frequency transform for each time series and generates an N×N time-Doppler distribution. By combining the M time-Doppler distributions at M range cells, the N×M×N time-range-Doppler cube $Q(r_m,f_n,t_n)$ can be formed as

$$Q(r_m, f_n, t_n) = TF\{G\{r_m(n)\}\}$$

where TF denotes the time-frequency operation with respect to the variable n. At a particular time instant t_i, only one range-Doppler image frame $Q(r_m, f_n, t_n = t_i)$ can be extracted from the cube. There are a total of N image frames available, and every one represents a full range-Doppler image at a particular time instant. Therefore, by replacing the Fourier transform with the time-frequency transform, a 2-D range-Doppler Fourier image frame becomes a 3-D time-range-Doppler image cube. The integration of the N time frames is equivalent to the Fourier image. By sampling the image cube in time, a time sequence of 2-D range-Doppler images can be viewed. Each individual time-sampled frame from the cube provides not only a clear image with superior resolution but also temporal change properties from one time to another.

4. RADAR IMAGING OF MANUEVERING TARGETS

To demonstrate radar imaging of maneuvering targets, we use simulated radar data, and compare the Fourier based and the time-frequency based image formation with the data. In the simulation, the radar is assumed operating in X-band at a center frequency of f_0 = 9,000 MHz and transmits a stepped-frequency waveform. Any other waveform, such as linear frequency-modulated and chirp-pulse waveforms can also be used. A total of M = 64 stepped frequencies are used with a frequency step of 8 MHz to cover a 500 MHz bandwidth or achieve 0.29 m range resolution. Each pulse transmits waves of a single carrier frequency. After transmitting a group of 64 pulses at 64 stepped frequencies, called a burst, the radar transmits another burst. In our simulation, the radar pulse repetition frequency (PRF) is 20,000 pulses/sec, which is at least 64 times higher than the burst repetition frequency needed to generate an image covering the entire target. The image observation time should be long enough to achieve the desired cross-range resolution. In the simulation, a coherent image processing time T = M N /PRF = 1.64 sec with N = 512 samples of the time history series is used. Thus, the radar image consists of 64 range-cells and 512 Doppler frequencies or cross-range cells.

An aircraft (MIG-25) is simulated in terms of a 2-D reflectivity density function ρ(x,y) characterized by 120 point-scatterers having equal reflectivity. These 120 point-scatterers are distributed along the edge of the 2-D shape of the aircraft. The simplified point scatterer model is very simple compared to the electromagnetic prediction code simulation. Although the point-scatterers do not represent the actual distribution of the reflectivity, it is convenient for displaying the shape of the formatted image of the target. It is good enough for testing and comparing different motion compensation and image formation algorithms.

The aircraft is initially located at a range of 3,500 m and has a fast rotation rate of 10°/s, which is much higher than the normal rotation rate for producing a clear image of a target. We assume that the target's translation motion can be perfectly compensated. However, due to the fast rotation and relatively longer image observation time, even after standard motion compensation, the uncompensated phase error is still large. Thus, the image formed by using the Fourier transform is still blurred as shown in Fig. 2 (a).

With the time-frequency based image formation, at each time the range and the Doppler frequency shift of each scatterer can be determined. Thus, without knowing the initial kinematic parameters and re-sampling the data, a blurred Fourier image due to a smeared Fourier spectrum will become a sequence of clear range and instantaneous Doppler images.

The desired time-frequency transform should satisfy the following requirements: (i) it should have high resolution in both the time and frequency domains, and (ii) it should accurately reflect the instantaneous frequencies of the analyzed signal. Time-frequency transforms include linear transforms such as the short-time Fourier transform (STFT), and bilinear transforms such as the Wigner-Ville distribution (WVD). The joint time-frequency resolution of the STFT is limited by the uncertainty principle. With a time-limited window function, the resolution of the STFT is determined by the window size. A larger window has a higher frequency resolution but lower time resolution; a smaller window has a lower frequency resolution but higher time resolution. Unlike the STFT, in which the time and frequency resolution is determined by the selection of the window function, there is no short window involved in the WVD. The WVD not only has a higher frequency resolution close to that of the full-size windowed Fourier transform, but also provides a higher time resolution. Because of the high resolution and the accuracy of the time-frequency representation, the WVD can be a candidate for time-frequency based image formation. However, there is cross-term interference associated with the WVD. When the signal contains more than one component, the WVD will generate cross-term interferences between components that occur at spurious locations of the time-frequency plane. The cross-term possesses a limited energy that reflects the correlation between the two related auto-terms and is highly oscillatory. Although the cross-term has a limited contribution to signal energy, it often obscures the useful time-varying spectrum. To reduce the cross-term interference, the filtered WVD can be used to preserve the useful properties of the time-frequency transform with slightly reduced time-frequency resolution and largely reduced cross-term interference. The WVD with linear low-pass filter is characterized as a Cohen's class distribution, such as the smoothed pseudo Wigner-Ville distribution [3]; and the distribution with a non-linear low-pass filter is characterized as the time-frequency distribution series (TFDS) [4]. The TFDS can have higher resolution and lower cross-term interference depending on its order. When the zero-order is selected, the TFDS is equivalent to the spectrogram of the STFT, and as the order goes to infinity, the TFDS converges to the WVD. In most applications, the order may be selected to be 3 or 4.

Comparing the energy concentration, the instantaneous frequency and the instantaneous bandwidth for the STFT, the TFDS and the WVD, the WVD has the highest time-frequency energy concentration or lowest instantaneous bandwidth, and the instantaneous frequency accurately reflects the true instantaneous frequency of the signal. Depending on the order of the distribution, the TFDS has a slightly lower time-frequency energy concentration than the WVD, and can also accurately reflect the true instantaneous frequencies of the signal. But the STFT has a lower time-frequency energy concentration and a deviation from the true instantaneous frequencies. In the example described in [2], the instantaneous bandwidth in normalized frequency is 0.007 for the WVD, 0.012 for the 4th-order TFDS, and 0.03 for the STFT. Thus, the time-frequency energy concentration of the STFT is about 4.3 times lower than that of the WVD and about 2.4 times lower than that of the 4th-order TFDS.

Since high time-frequency energy concentration and low cross-term interference are desired for the time-frequency based image formation, in our simulation we choose the TFDS for its higher time-frequency energy concentration, lower cross-term interferences, and easier implementation.

Fig.2 (b) shows the image frame no.7 from the sequence of 16 frames using the time-frequency based image formation. The blurred image caused by the target’s fast rotation is now re-focused without applying the polar reformatting. Because the time-varying spectrum is well represented, the smeared Fourier image is resolved into a sequence of time-varying range and instantaneous Doppler images. These images not only have superior resolution, but also show the Doppler change from one image frame to another and range walk from time to time.

5. SUMMARY

We have introduced the time-frequency based image formation for ISAR imaging and demonstrated it with simulated radar data. The same time-frequency based image formation can also be applied to SAR data.

Theoretically, any image formation system based on the Fourier transform, such as medical imaging (CT and MRI), can also apply the time-frequency based image formation method if the Fourier image is blurred due to a time-varying spectrum.

6. REFERENCES

[1] V.C. Chen, "Reconstruction of inverse synthetic aperture radar image using adaptive time-frequency wavelet transform," *SPIE Proc. on Wavelet Applications*, vol. 2491, pp.373-386, 1995.

[2] V.C. Chen and S. Qian, "Joint time-frequency transform for radar range-Doppler imaging," *IEEE Trans. Aerospace and Electronic Systems*, vol. 34, no. 2, pp.486-499, 1998.

[3] L. Cohen, *Time-Frequency Analysis*, Englewood Cliffs, NJ: Prentice-Hall, 1995.

[4] S. Qian and D. Chen, *Joint Time-Frequency Analsysis – methods and applications*, Englewood Cliffs, NJ: Prentice-Hall, 1996.

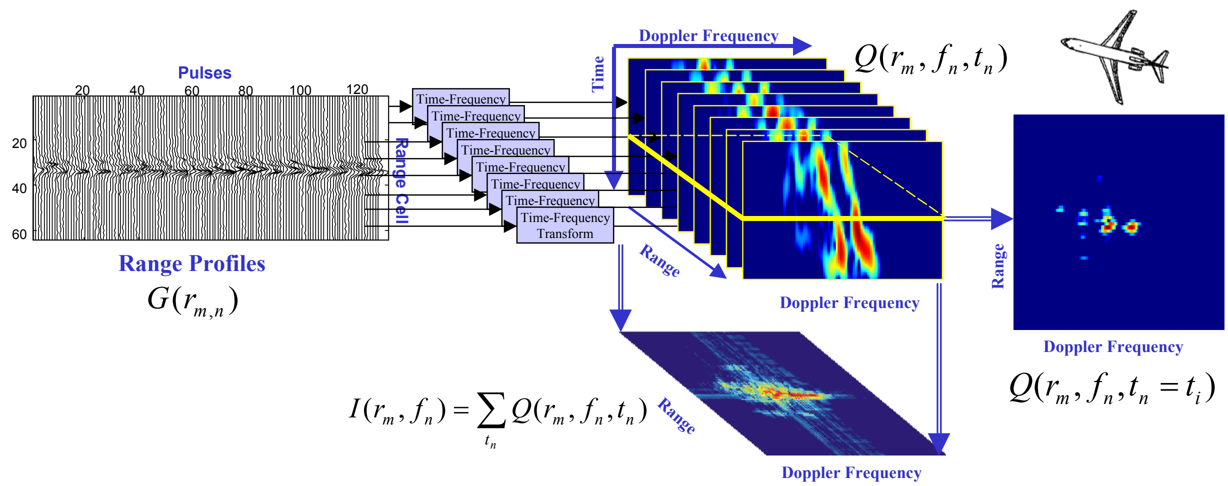

Figure 1 Time-frequency based image formation

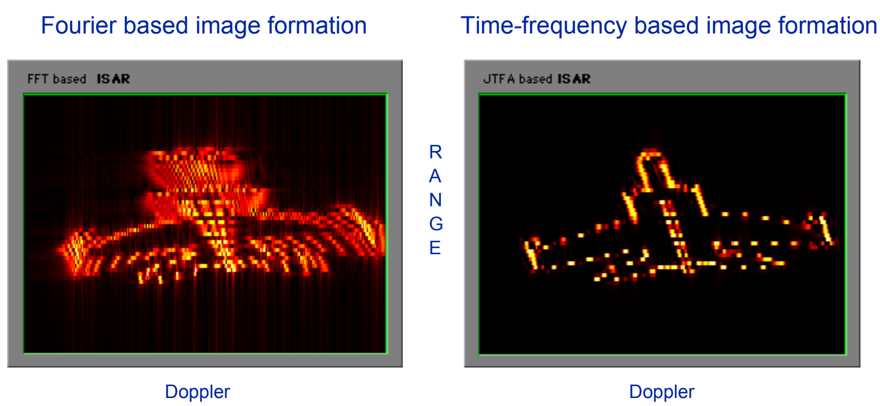

Figure 2 Comparison of the Fourier image and the time-frequency image.