

# SHIFT-INVARIANT WAVELET DENOISING USING INTERSCALE DEPENDENCY

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## ABSTRACT

Using statistical modeling in the wavelet domain, we address the problem of image denoising. Despite being effective, the denoised images can suffer from the Gibbs-like artifacts, like ringing around the edges and speckles in the smooth regions. We employ shift-invariant (SI) wavelet denoising in order to reduce these unpleasant artifacts. Not only is the visual quality greatly improved but also a PSNR gain of about 0.7~0.9 dB is obtained. The proposed approach, siPAB, outperforms siHMT, which is a competitive SI wavelet denoising approach, by 0.1~0.5 dB.

**Keywords:** Shift-invariant wavelet transform, image denoising, interscale statistics.

## 1. INTRODUCTION

In a Bayesian wavelet denoising approach [1-4, 6-10], a prior is first specified for the wavelet coefficients of the unknown image, and then the Bayesian estimate is computed.

However, the simple Gaussian prior is not appropriate. The actual density of the wavelet coefficients usually has a marked peak at zero and heavy tails. The Gaussian mixture model (GMM) [4] and the generalized Gaussian distribution (GGD) [9] are commonly used instead. Although GGD is more accurate, GMM is used in this paper due to its simple form.

Moreover, the wavelet coefficients are not independent [10]. Statistical correlations like interscale dependency [6, 10] and intrascale dependency [8] have been exploited in image denoising. A hidden Markov tree (HMT) was employed by Crowse *et al.* [6] to capture the interscale dependency. However, HMT is computationally costly in the training stage. To overcome this [10] introduced nine meta-parameters to eliminate the training stage, leading to a scheme called uHMT. Alternatively, [3] established that the density of the wavelet coefficients can be well fitted by a 3-mode GMM (section 2). The PAB [3] approach dispenses with the HMM and yet the performance favorably compares with HMM based approaches, like HMT and uHMT.

The PAB approach suffers from some visual artifacts, usually in the form of Gibbs-like ringing around the edges and speckles in smooth regions. This is similar to other traditional (maximally decimated) wavelet denoising approaches. The reason lies in the lack of shift invariance (SI). Coifman and Donoho [5] proposed SI wavelet denoising by “cycle-spinning”. An improvement of about

0.8~1 dB PSNR [2, 10] has been reported for SI wavelet denoising. In this paper, we extend the PAB approach [3] to SI wavelet denoising, which we call siPAB.

## 2. THE PAB APPROACH

In [3], only interscale dependency was employed. Given their parent, the distribution of the child wavelet coefficients is modeled as a 3-mode GMM. The variances for these 3 modes in the GMM are linearly dependent on their parent. This is different from HMM-based approaches, where the variances, predefined in uHMT [10] or obtained by expectation-maximization algorithm in HMT [6], are constant. Fig. 1 is a typical conditional density of the child in the vertical band of level 1. The crux of the method in [3] lies in obtaining this conditional density, which captures the statistics of the interscale dependency between children and their parent. It consists of two steps: variance estimation and Gaussian mixture modeling.

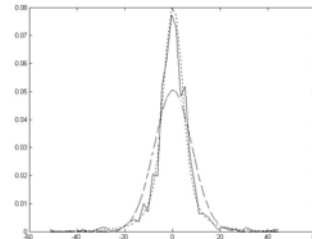


Fig. 1 The conditional density of the child in the vertical band of level 1, the magnitude of whose parent coefficient lies between 10 and 11. The solid line denotes the conditional density, the dotted one for the fixed mixture model, and the dashed one for the Gaussian model.

### 2.1 Variance estimation

The following formula is employed to estimate the variance field:  $\sigma_c = Am_p + B$ , where  $\sigma_c$  is the variance of the child  $c$  and  $m_p$  is the magnitude of the corresponding parent  $p$ . This stems from the intuitive observation that large coefficients persist across scales. Thus, it can be assumed that the children are of large/small variance if their parent has a large/small magnitude. Fig. 2 shows this kind of dependency of level 1 on level 2. Other levels share this near-linear property. Usually, in the same level, the horizontal band and the vertical band show similar statistics, while the diagonal band has a smaller variance assuming the magnitude of

the parent is same, as it can be seen from fig. 2. Thus, the true model becomes  $\sigma_{c,level,band} = A_{level,band}m_p + B_{level,band}$ . For the coarsest level, no parent exists. The parent-on-child dependency is utilized to estimate the variance field for the coarsest level. Table 1 lists the fitted parameters for  $A$  and  $B$  [3].

Table 1: Parameters for the empirical model.

Level	LH		HL		HH	
	A	B	A	B	A	B
1	3.5	0.26	3.7	0.26	2.3	0.15
2	8.5	0.38	10	0.41	6.5	0.29
3	24	0.35	30	0.35	13.5	0.6
4	60	1.9	62	2.4	37	1.1

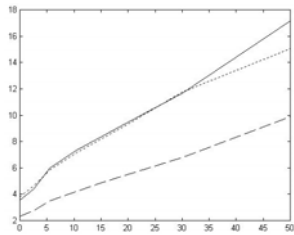


Fig. 2 Variance field estimation in level 1. The solid line denotes the horizontal band of level 1, the dotted one for the vertical band and the dashed one for the diagonal band.

## 2.2 Gaussian mixture model

Like the marginal density of the wavelet coefficients in the whole band, the conditional densities of  $p(x_c | x_p)$  and  $p(x_p | x_c)$  also appear non-Gaussian: marked peak at zero and heavy tails. In PAB, as in Chipman *et al.* [4], a GMM is used to fit this type of non-Gaussian property. If the variance  $\sigma$  for  $x$  is known, the following mixture model is specified to fit this non-Gaussian property [3],

$$x \sim a_1 N(0, \sigma_1^2) + a_2 N(0, \sigma_2^2) + a_3 N(0, \sigma_3^2) \quad (1)$$

where  $\sigma_1 = \sigma / n_1$ ,  $\sigma_2 = \sigma$  and  $\sigma_3 = n_3 \sigma$ .  $a_1 = 0.6$ ,  $a_2 = 0.3$ ,  $a_3 = 0.1$  and  $n_1 = n_3 = 2.5$  were experimentally determined. They work well in [3] and the relation  $a_1 / n_1^2 + a_2 + a_3 n_3^2 = 1$  approximately holds (e.g., Fig. 1).

For a noisy image, the noise-free wavelet coefficients are unknown and it is impossible to obtain the true variances  $\sigma$  in the model mentioned above. In [3], a substitute  $\hat{\sigma}$  for  $\sigma$  was obtained from the denoised parent coefficient for all except the coarsest level or from its four noisy children for the coarsest level. With  $\sigma_m$  known, the following MMSE estimator is used to estimate the noisy coefficients

$$\hat{x}(y) = \sum_{m=1}^3 p(m | y) \frac{\sigma_m^2}{\sigma_m^2 + \sigma_n^2} y \quad (2)$$

where

$$p(m | y) = \frac{a_m g(y, \sigma_m)}{\sum_{i=1}^3 a_i g(y, \sigma_i)}, \quad \text{and}$$

$$g(y, \sigma_m) = \frac{1}{\sqrt{2\pi} \sigma_m} \exp\left(-\frac{y^2}{\sigma_m^2}\right)$$

## 3. EXTENSION TO SI WAVELET TRANSFORM

Here, we extend the PAB approach [3] to SI wavelet denoising in order to reduce the ringing and speckle effects. The idea of SI wavelet denoising [5] is simple: first, to circularly shift the image; second, to denoise all the shifted images; last, to align and average over the denoised images. This strategy aims to ‘‘average out’’ the translation dependency in maximally decimated wavelet transform, and was coined as cycle-spinning [5]. However, the direct implementation of Average [Shift-Denoise-Unshift] will have computational complexity  $O(n^2)$ . In fact, cycle-spinning can be implemented in an undecimated wavelet transform, where the complexity reduces to  $O(n \log n)$ .

The PAB approach [3] can be easily extended to the undecimated wavelet transform. However, the relationship between parent and child is a little different in the undecimated and decimated representations. In the decimated wavelet transform, each parent has four children, while the wavelet trees in an undecimated representation overlap---the same coefficients appear in more than one tree. This redundancy introduces a 1-1 parent-child relationship. The change from a 1-4 parent-child relationship to a 1-1 relationship makes only a very small change in the top-down procedure to estimate variances. In the traditional decimated 1-4 scheme, the variance estimate for a node, in all except the coarsest level, is obtained from its denoised parent, and therefore the 1-1 scheme can retain the same character. However, at the coarsest level, since the node has no parent, its variance is estimated from its four noisy children in the decimated 1-4 scheme. Now, with a 1-1 relationship, only one noisy child node is available for each parent at the coarsest level. In order to reduce the noise effect, the variance for a node  $p$  in the coarsest level is estimated from the average of the magnitudes of five nodes, which consist of its child  $c$  and its four nearest neighbors (denoted by  $\blacktriangle$ ) in the finer level, as in fig. 3.

We summarize the approach below:

1. Non-decimated wavelet transform.
2. From coarsest to finest, compute the denoised coefficients
  - a. Linearly estimate the variance based on its denoised parent (five noisy children for the coarsest level) according to table 1.
  - b. Compute denoised coefficients from (2).

3. Reconstruct by the undecimated IDWT.

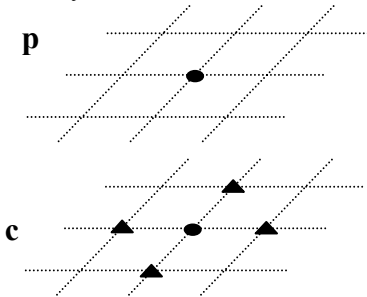


Fig. 3 Five coefficients for the estimation of variance  $\sigma_p$  in the coarsest level.

#### 4. EXPERIMENTAL RESULTS AND CONCLUSION

As in [3, 10], the Daubechies' length-8 wavelet D4 [7] is employed to decompose the images into four levels. In table 2, the results of PSNR for 11 images [10] of size 256x256 are listed. Compared with the PAB approach,

this SI-wavelet denoising approach gains an improvement of 0.7-0.9 dB, similar to the gains in other SI wavelet approach [2, 10]. It also outperforms siHMT with a gain of 0.1~0.5 dB.

A visual display of the image "bridge" can be found in figure 4. A noticeable improvement of visual quality in the denoised images by siPAB can be easily observed, over those denoised by PAB, because both the rings and the speckles are greatly eliminated. To compare siPAB and siHMT, in figure 5 we particularly focus on the bridge image to compare the visual effect because the PSNR indexes for two approaches, used to denoise this image, are almost same. Note: it is a little difficult to compare the visual effects, because of the similarity as reflected in the near identical PSNR indexes for two approaches. By taking a detailed look at the images, we can see that siPAB performs slightly better than siHMT in preserving the straight lines, while the siHMT works slightly better than siPAB on the texture.

Table 2: Comparison of PSNR for different approaches with  $\sigma_n = 0.05 / 0.1 / 0.2$

	SiPAB	si-HMT [10]	PAB [3]	uHMT [10]
<b>Baby</b>	33.0/30.0/26.9	33.1/29.6/26.3	32.0/28.8/25.9	32.4/28.9/25.8
<b>Birthday</b>	30.9/28.1/25.6	29.6/26.4/23.7	30.3/27.4/24.9	28.9/25.8/23.1
<b>Boats</b>	31.8/28.2/25.0	31.4/27.4/24.1	31.0/27.3/24.1	30.4/26.4/23.3
<b>Bridge</b>	28.8/25.4/22.7	28.9/25.3/22.7	28.1/24.8/22.0	28.1/24.6/22.0
<b>Buck</b>	33.6/29.8/26.4	33.7/29.6/25.8	32.8/28.8/25.2	32.5/28.4/24.7
<b>Building</b>	30.5/27.2/24.0	30.4/26.6/23.5	29.7/26.3/23.0	29.7/25.9/22.8
<b>Camera</b>	31.0/27.4/24.2	31.1/27.0/23.7	30.2/26.5/23.3	30.3/26.2/23.1
<b>Clown</b>	31.5/28.0/24.6	31.7/27.8/24.5	30.7/27.0/23.6	30.6/26.8/23.7
<b>Fruit</b>	33.1/29.8/26.5	33.3/29.7/26.4	32.4/28.8/25.5	32.2/28.5/25.3
<b>Kgirl</b>	32.4/29.3/26.3	32.6/29.3/26.4	31.8/28.8/25.4	31.6/28.3/25.4
<b>Lena</b>	31.2/27.7/24.9	31.1/27.6/24.5	30.4/26.9/24.1	30.4/26.7/23.8
<b>Average</b>	31.41/28.05/25.01	31.28/27.61/24.51	30.66/27.22/24.11	30.43/26.76/23.76

#### 5. REFERENCES

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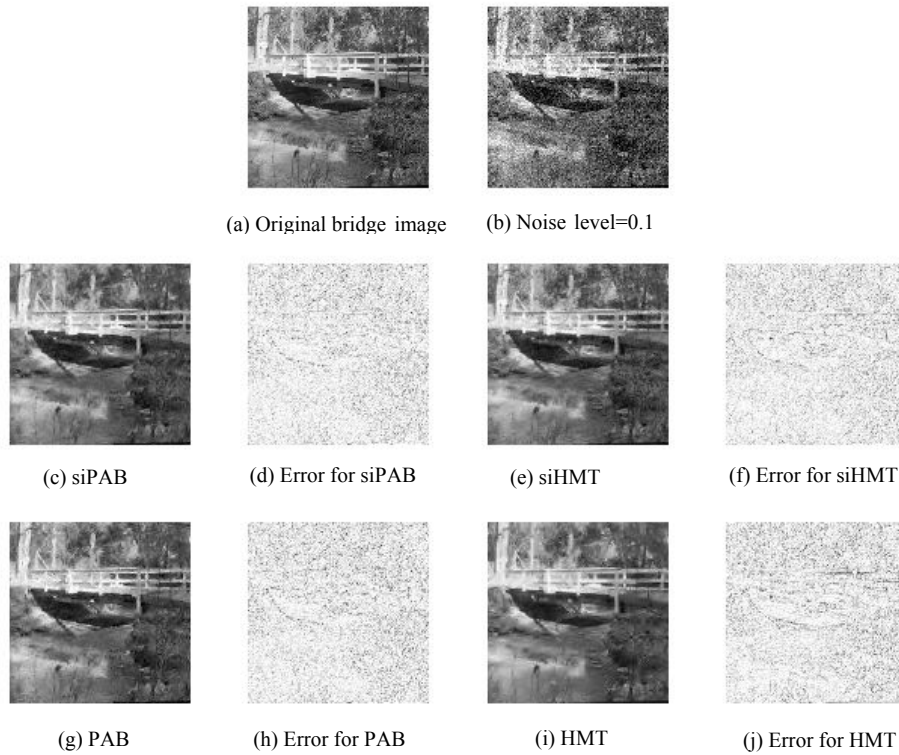


Fig. 4, Image bridge (a), noisy version (b) with level of 0.1, denoised copies by (c) siPAB, (e) siHMT, (g) PAB, and (j) uHMT, and their error images. In order to have a visible scene, the error images have been scaled at a same ratio. The darker the pixel, the bigger the error magnitude. The SI wavelet denoising has an improved visual effect, which can be seen by comparing siPAB with PAB and by comparing siHMT with HMT. As siPAB and siHMT are concerned, it is a little difficult to compare the visual effect, especially for the printed images, because the PSNR indexes are almost same, where PSNR for siPAB is 25.42 and siHMT has an index of 25.36.

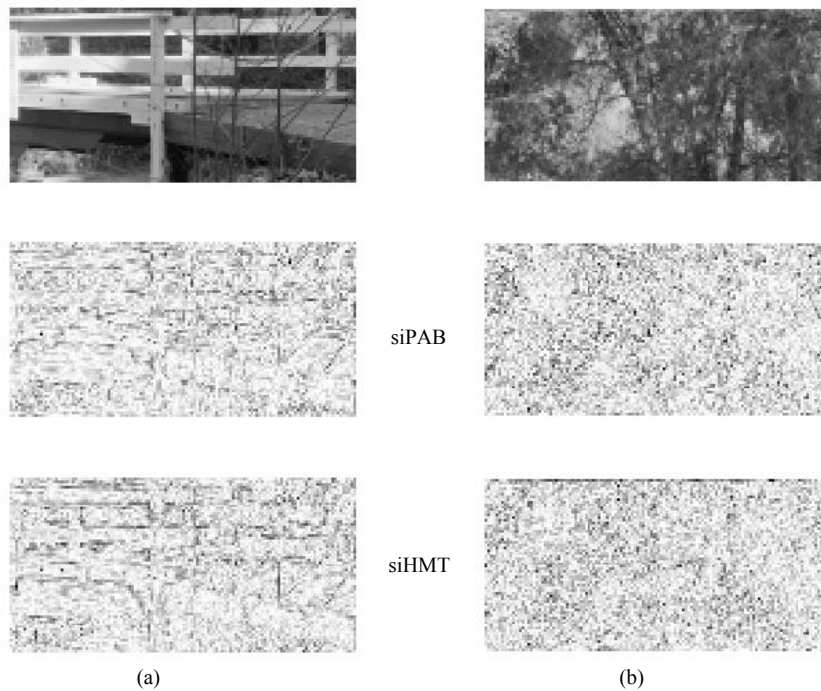


Fig. 5, Two different portions of the bridge image and their error images for siPAB (the second row) and siHMT (the third row). (a) siPAB performs a little better in preserving the structures, like the straight lines of the balustrade of the bridge, than siHMT. (b) siHMT works slightly better than siPAB on the textured regions.