ANALYSIS OF IMPULSE TRAIN ILLUMINATED IMAGES FOR 2D VELOCITY MEASUREMENT

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ABSTRACT

Numerous applications require a precise and angle sensitive measurement of the velocity of a textured surface. This article describes a contactless measurement scheme that analyzes the Fourier spectrum of images acquired under time varying illumination. Theoretical considerations and experimental studies suggest employment of impulse train illumination. It is shown that velocity measurement reduces to the estimation of the cycle distance and angle in a periodic line spectrum. Experimental results demonstrate that the method yields robust measurements and is capable of measuring large velocities.

1. INTRODUCTION

Monitoring the velocity of moving surfaces is an essential measurement task in various industrial processes. Since most of the measurements need to be performed in situ, sensors that work contactless are required. Optical sensors work without contacting the object and therefore without any influence on the process to be monitored.

Two dimensional measurements of the velocity can be performed with image based sensors, i.e. the moving surface is observed with a video camera. The analysis of image sequences is one standard method that can determine the 2D velocity of a moving textured surface [1, 2]. In short, the shift between two subsequent image frames yields an estimate for the velocity since the time between the two subsequent frames is known. An overlap of the subsequent image frames is a strict requirement for the determination of the image shift. Consequently, image sequence analysis requires either high frame rates or low velocities.

In contrast, the method presented in the following text is not bound to these restrictions since it yields a measurement for the velocity vector based on a single image frame. The approach based upon analysis of impulse train illuminated images is capable of measuring high velocities requiring comparably low frame rates.

2. BACKGROUND

For the velocity measurement, a video camera observes the moving textured surface (Fig. 1). For the sake of simplicity, we assume that the optical axis of the camera is orthogonal to the surface and that the surface itself moves in the plane orthogonal to the optical axis with velocity $\vec{v}$.

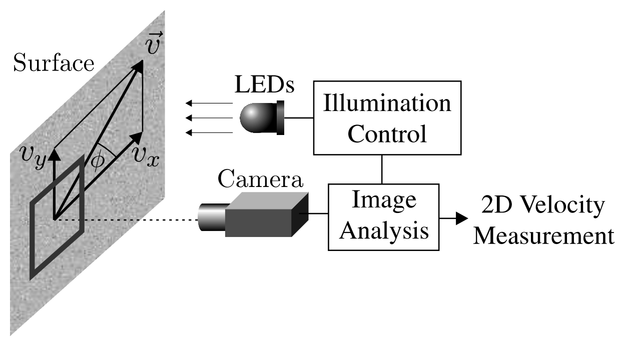


Fig. 1: Camera observing a moving textured surface.

During image acquisition the moving surface, i.e. the object $o(\vec{x}(t))$, is illuminated with a spatially homogeneous, time-varying intensity $l(t)$ so that the camera sensor observes an image intensity $i(\vec{x},t)$ at time instance t:

$$i(\vec{x},t) = o(s \cdot \vec{x}(t)) \cdot l(t). \tag{1}$$

Without loss of generality the scale factor s can be set to 1. Considering the setup shown in Fig. 1 and assuming a constant velocity $\vec{v} = (v_x, v_y)^{\mathrm{T}}$ Eq. (1) can be rewritten as:

$$i(\vec{x},t) = o(\vec{x} - \vec{v}t) \cdot l(t), \text{ with } \vec{x} = \vec{x}(0). \tag{2}$$

Image acquisition integrates the light intensity perceived by the camera sensor over time. Imposing $l(t) = 0$ outside the exposure time interval, we yield

$$i_{\mathrm{Cam}}(\vec{x}) = \int_{-\infty}^{\infty} o(\vec{x} - \vec{v}t) \cdot l(t) \mathrm{d}t. \tag{3}$$

It can easily be shown that the integral for i_{Cam} can be written as a 2D-convolution of the *non-moving* surface intensity $o(\vec{x})$ with an impulse response $h(\vec{x};\vec{v})$.

$$i_{\text{Cam}}(\vec{x}) = o(\vec{x}) * h(\vec{x};\vec{v}), \quad \text{where} \tag{4}$$
$$h(\vec{x};\vec{v}) = \frac{1}{|\vec{v}|}\delta(\vec{e}_{\phi\perp}^{\mathrm{T}}\vec{x})\, l\left(\frac{\vec{e}_{\phi}^{\mathrm{T}}\vec{x}}{|\vec{v}|}\right).$$

In this formulation $\vec{e}_\phi$ is the unit-vector in the direction of the velocity-vector $\vec{v}$ and $\vec{e}_{\phi\perp}$ is the unit-vector orthogonal to $\vec{v}$ [3, 4]. It is a very important fact that the scale of $h(\vec{x};\vec{v})$ depends on the magnitude of $\vec{v}$ and that the orientation of $h(\vec{x};\vec{v})$ depends on the orientation, i.e. the angle ϕ, of $\vec{v}$. See Figs. 2(b) and 3(b) for examples of $h(\vec{x};\vec{v})$.

The images acquired according to Eq. (4) can be efficiently analyzed in the frequency domain. A convolution of $o(\vec{x})$ with $h(\vec{x};\vec{v})$ in the spatial domain results in a multiplication of the spectrum of the textured surface $O(\vec{f})$ with the velocity dependent function $H(\vec{f};\vec{v})$, which denotes the 2D Fourier transform of $h(\vec{x};\vec{v})$:

$$I_{\text{Cam}}(\vec{f}) = O(\vec{f}) \cdot H(\vec{f};\vec{v}). \tag{5}$$

3. IMPULSE TRAIN ILLUMINATION VS. LONG TIME ILLUMINATION

The idea for the velocity-measurement presented in this paper extends from the well known analysis of velocity-dependent motion blur in the frequency domain. In the case of motion blur the camera images are illuminated with a constant intensity for a considerably long time – 'long time illumination'. In this case the lighting-function $l(t)$ is a rectangle,

$$l(t) = \text{rect}\left(\frac{t}{T}\right), \tag{6}$$

where T denotes the shutter-time of the camera. According to Eq. (4) the acquired image is a convolution of an image of the non-moving surface with a line (Fig. 2(b))[1]. The length and orientation of the line depend on the magnitude and the angle of the velocity-vector (Eq. (4)). In the frequency domain we obtain a multiplication of the spectrum of the surface with a sinc-function (Fig. 2(f)). Considering the scaling theorem of the Fourier transform, the scale and the orientation of this sinc-function therefore represent the magnitude and angle information of the velocity-vector.

As can be seen from Fig. 2(f), the extraction of this information may be quite noise sensitive, since the desired information is incorporated in the zero crossings. An extension that improves this finding has been examined in [3] in terms of double-illuminations. In the sequel it will be shown that an impulse train illumination further improves the signal and allows a robust analysis of the images in the frequency domain.

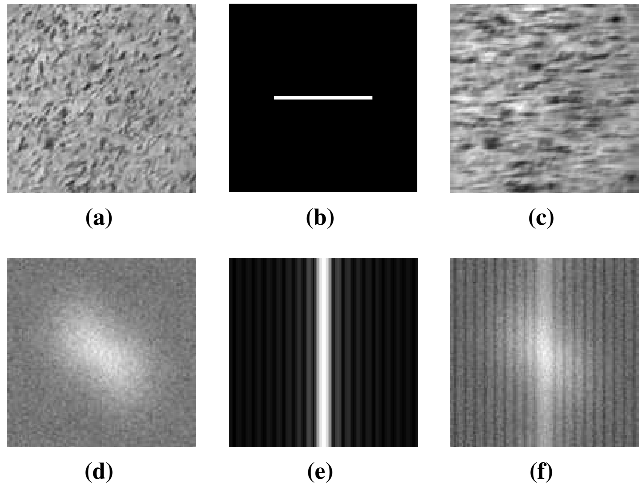


Fig. 2: Long time illumination (motion blur) as a convolution of the non-moving surface with a line, $\vec{v} = (v_x, 0)^{\mathrm{T}}$. (a) Non-moving surface $o(\vec{x})$. (b) Sketch of $h(\vec{x};\vec{v})$, enlarged. (c) Long time illuminated image $i_{\text{Cam}}(\vec{x}) = o(\vec{x}) * h(\vec{x};\vec{v})$. (d), (e), (f) Images (a), (b) and (c) in the frequency domain, respectively.

In the spatial domain the impulse train illumination can be modeled as a convolution of the surface with a sequence of impulses (Fig. 3(b)). Each impulse represents one light-flash while the shutter of the camera is open. In the spatial domain the distance of the impulses $|\Delta\vec{x}|$ and therefore the scale of the impulse train is directly connected to the magnitude of the velocity-vector by the simple formula:

$$|\vec{v}| = \frac{|\Delta\vec{x}|}{\Delta t}, \tag{7}$$

where Δt is the time between the impulses. The orientation of the impulse train $h(\vec{x};\vec{v})$ depends on the angle of the velocity-vector. From Eq. (4) and (5) follows that the spectrum of the impulse train illuminated moving surface is a multiplication of the spectrum of the non-moving surface with a sequence of parallel lines (Fig. 3(f)). A sequence of impulses transforms into a sequence of parallel lines when a 2D Fourier transform is applied. The distance and the angle of the lines depend directly on the magnitude and the angle of the velocity-vector.

The angle of $\vec{v}$ can consequently be measured by measuring the angle of the lines in the spectrum of the impulse train illuminated image of the moving surface. The magnitude of $\vec{v}$ can be measured by measuring the distance of these lines, since the time Δt between the impulses (=light flashes) is known. Hence, in fundamental contrast to the case of long time illumination, the information of $\vec{v}$ is now incorporated in the distinct signal maxima rather than in the zero crossings.

[1]The sketches of $h(\vec{x};\vec{v})$ are enlarged for visualization.

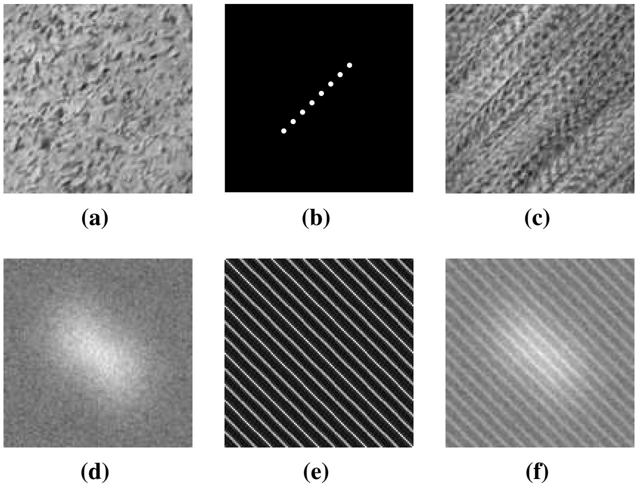


Fig. 3: Impulse train illumination as a convolution of the non-moving surface with a sequence of impulses, $\vec{v} = (v_x, v_y)^{\mathrm{T}}$, $v_x = v_y$. (a) Non-moving surface $o(\vec{x})$. (b) Sketch of $h(\vec{x};\vec{v})$, enlarged. (c) Impulse train illuminated image $i_{\mathrm{Cam}}(\vec{x}) = o(\vec{x}) * h(\vec{x};\vec{v})$. (d), (e), (f) Images (a), (b) and (c) in the frequency domain, respectively.

4. RETRIEVING ANGLE AND MAGNITUDE

As shown in Section 3, the parameters of parallel impulse lines in the spectrum of an impulse train illuminated image represent magnitude and angle of $\vec{v}$. The parameters of these collinear structures can be retrieved by performing a Hough transform after converting the grayscale image to a binary image by a simple threshold operation [5]. Figure 4 shows two simulated impulse train illuminated images of a moving textured surface in the frequency domain. The Hough transforms of the corresponding binary images are also shown.

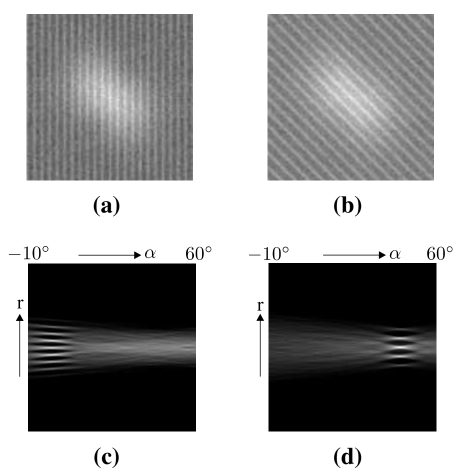


Fig. 4: Simulated impulse train illuminated images of a moving surface. (a) $v = (v_0, 0)^{\mathrm{T}}, \phi = 0°$. (b) $v = (v_1, v_1)^{\mathrm{T}}, \phi = 45°$. (c), (d) Hough transforms of (a) and (b), respectively.

As a first step, the angle of the velocity can be calculated by finding the maximum of the variances of the columns of the Hough transforms. Figure 5 shows the variances of the columns (=variances in r-direction) σ_r^2 of the Hough transforms shown in Fig. 4(c) and 4(d).

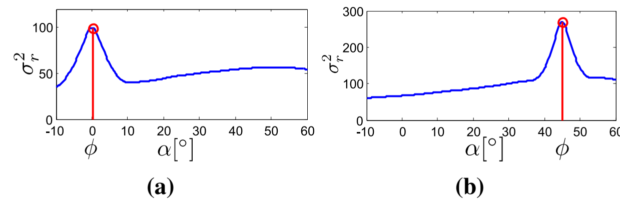


Fig. 5: Variances of the columns of the Hough transform for angle detection. (a) Column-variances of Fig. 4(c). (b) Column-variances of Fig. 4(d).

With the detected angle ϕ as an a priori knowledge, the magnitude of $\vec{v}$ can be retrieved by calculating the distance of the 1D-pulses Δr in the column of the Hough transform corresponding to the angle ϕ. Figure 6 shows the columns of the Hough transforms $H(r, \alpha = \phi)$ corresponding to the detected angle ϕ for Fig. 4(c) and 4(d).

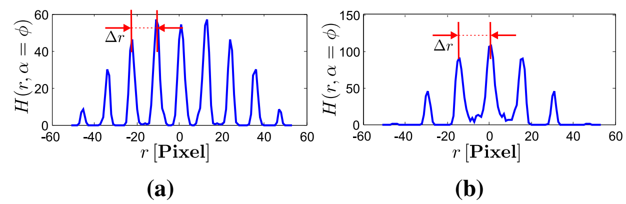


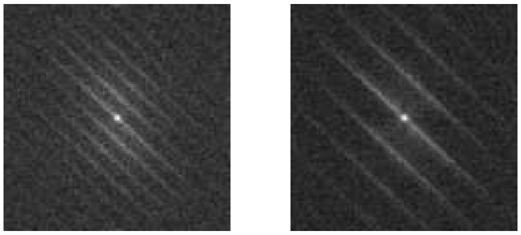
Fig. 6: Columns of the Hough transform corresponding to the angle ϕ. (a) Column of Fig. 4(c) for $\alpha = \phi = 0°$. (b) Column of Fig. 4(d) for $\alpha = \phi = 45°$.

The distance of the 1D-pulses Δr in $H(r, \alpha = \phi)$ and therefore the magnitude of $\vec{v}$ may be measured by a further Fourier-analysis of the 1D-signal $H(r, \alpha = \phi)$.

5. EXPERIMENTAL RESULTS

The presented algorithm has been tested on a real moving textured surface. The surface used for the experiments is the same as for the simulations in section 3 (Fig. 3(a)). Figure 7 shows results for a constant magnitude and angle of $\vec{v}$ using two different frequencies f_{light} for the impulse train illumination.

In the measurement shown in Fig. 7 we observe the parallel impulse lines similar to the simulations shown before. Moreover, we can see that a multiplication of f_{light} by 2 results in a multiplication of the distance of the impulse lines by 2 for a constant magnitude of $\vec{v}$ as expected. Calculation of the angle and magnitude of $\vec{v}$ according to

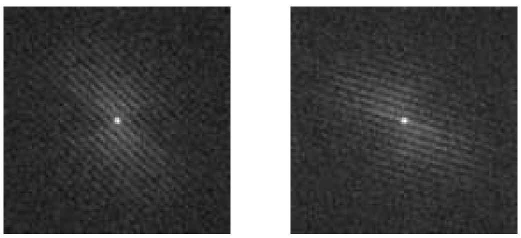


(a) (b)

Fig. 7: Magnitude of the spectrum of the impulse train illuminated moving textured surface. (a) f_{light}=5000 1/min. (b) f_{light}=10000 1/min.

section 4 yields: $|\vec{v}|$=0.507 m/s, $\phi = 46.8°$ for 7(a) and $|\vec{v}|$=0.512 m/s, $\phi = 47.0°$ for 7(b).

Figure 8 shows results for two different angles and a constant magnitude of $\vec{v}$. The magnitude of $\vec{v}$ is doubled in comparison to the measurements shown in Fig. 7.



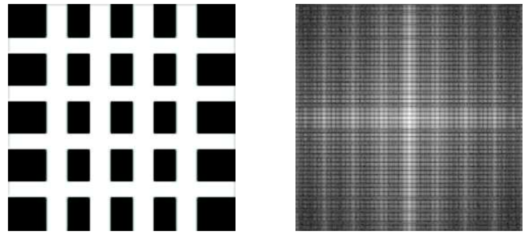
(a) (b)

Fig. 8: Magnitude of the spectrum of an impulse train illuminated moving textured surface. (a) $\phi = \phi_1$, f_{light}=5000 1/min. (b) $\phi = \phi_2$, f_{light}=5000 1/min.

Figure 8 confirms that the angle of the velocity can be detected by measuring the angle of the parallel impulse lines. Furthermore, the distance of the impulse lines is only half of the distance in Fig. 7(a), since the magnitude of $\vec{v}$ has been doubled while f_{light} is constant. Calculation of the angle and magnitude of $\vec{v}$ according to section 4 yields: $|\vec{v}|$=1.083 m/s, $\phi = 47.0°$ for 8(a) and $|\vec{v}|$=1.078 m/s, $\phi = 73.0°$ for 8(b).

6. LIMITATIONS OF THE APPROACH

According to section 3 the spectrum of the moving textured surface is a multiplication of the texture spectrum by a sequence of parallel lines. Therefore, a perfect texture spectrum for the proposed approach would be constant for all spatial frequencies. In contrast, a texture with prominent periodic structures (Fig. 9(a)) would derange the detection of the parallel lines described in section 4. The vertical parallel lines in Fig. 9(b) are hard to detect. Nevertheless experimental studies in our laboratory have shown that the proposed technique works well with a high variety of technical surfaces.



(a) (b)

Fig. 9: (a) Textured surface with prominent periodic structures (spatial domain). (b) Texture in (a) convolved with a horizontal impulse train (magnitude of the spectrum).

7. SUMMARY AND CONCLUSION

In this paper an efficient and highly precise method for measuring the magnitude and angle of the velocity vector $\vec{v}$ has been presented. In contrast to the widely spread analysis of image sequences the presented method can retrieve a measurement for $\vec{v}$ out of a single image of the moving surface. Analysis of impulse train illuminated images in the frequency domain is proposed as a highly sensitive method. It has been shown that the desired velocity information is placed directly into the distance and orientation of the parallel lines in the spectrum. The correct functionality has been verified based upon measurements performed on a real moving textured surface. With state-of-the-art computer hardware the method can easily be implemented in real-time so that an on-line measurement of $\vec{v}$ can be performed even for large $\vec{v}$. Therefore, the method offers itself for a robust, reliable and cost-efficient solution.

8. REFERENCES

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