AN EFFICIENT PHASE RETRIEVAL METHOD USING SNAKES FOR IMAGE RECONSTRUCTION

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ABSTRACT

This paper proposes a novel phase retrieval method using active contour models (snakes) for image reconstruction. The proposed method reconstructs a target image from the magnitude of its Fourier transform and the measured area of the image. In general, the measured area is different from the true area where the target image exists. Thus the snake with an area term, which can extract the shape of the concave target image, is utilized to renew the measured area. By processing this renewal iteratively, the area obtained by the snake converges to the true area and as a result the proposed method can accurately reconstruct a target image even when the measured area is different from the true area. Experimental results show the effectiveness of the proposed method. This method has many applications such as biomedical imaging using X-ray or optical detectors which have lost the phase information.

1. INTRODUCTION

Phase retrieval from the magnitude of the Fourier transform for image reconstruction plays a key role in a number of important applications and has attracted the attention of numerous researchers in the engineering and physics communities [1]. One of the greatest applications is biomedical imaging which uses X-ray or optical detectors which can measure the magnitude of the Fourier transform but not the phase.

Many phase retrieval methods have been proposed [2]-[9]. These methods reconstruct a target image from the magnitude of its Fourier transform and additional information contained in the image. The most common approach for phase retrieval is to use one of the iterative Fourier transform algorithms [6]-[9] based on the Fourier and image constraints, as shown in Fig. 1. The iterative Fourier transform algorithms utilize the magnitude of the Fourier transform as the Fourier constraint and the measured area of the image as the image constraint. However, they cannot accurately reconstruct a target image when the measured area is different from the true area where the target image exists.

To overcome this problem, a novel phase retrieval method using active contour models (snakes) is proposed in this paper. The proposed method reconstructs a target image from the magnitude of its Fourier transform and the measured area of the image. The snake with an area term [10] is utilized to automatically renew the measured area. By processing this renewal iteratively, the proposed method can accurately reconstruct a target image even when the measured area is different from the true area, because the image area renewed by the snake converges to the true area.

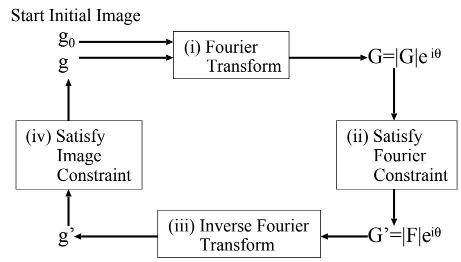


Fig. 1. Block diagram of the iterative Fourier transform algorithms.

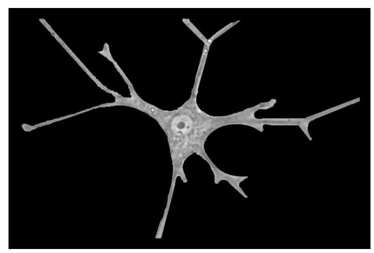


Fig. 2. Concave target image (255 gray levels).

2. ITERATIVE FOURIER TRANSFORM ALGORITHMS

The iterative Fourier transform algorithms reconstruct a target image by using both the Fourier and image constraints, as shown in Fig. 1. They utilize the Fourier transform magnitude of the target image as the Fourier constraint and the measured area of the image as the image constraint. This section explains the error reduction algorithm [8] which is widely used as one of the iterative Fourier transform algorithms.

The *m*th iteration of the error reduction algorithm consists of the following four steps:

Step 1: **Fourier Transform**

The discrete Fourier transform (DFT) of the $m-1$th estimated image $g_{m-1}(x, y)$ is given as follows:

$$\begin{aligned} G_{m-1}(u, v) &= |G_{m-1}(u, v)| \exp[i\theta_{m-1}(u, v)] \\ &= \mathcal{F}[g_{m-1}(x, y)] \end{aligned} \quad (1)$$

where $|G_{m-1}(u, v)|$ and $\theta_{m-1}(u, v)$ are the magnitude and phase of the Fourier transform, respectively.

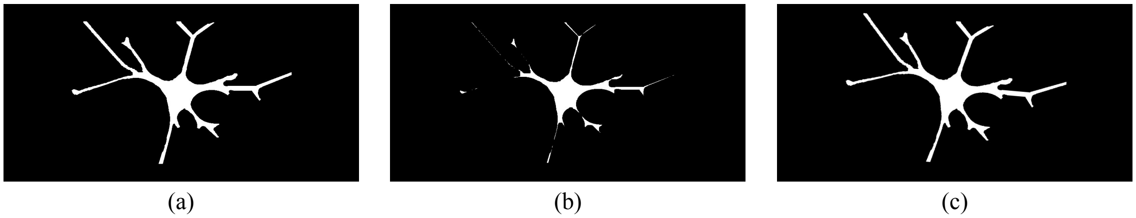


Fig. 3. The image area is indicated in white: (a) The true area where the target image (Fig.2) exists; (b) The measured image area corrupted by erosion; (c) The measured image area rotated 5° to the right of the true area.

Step 2: **Application of the Fourier Constraint**

A new Fourier transform is formed by replacing $|G_{m-1}(u,v)|$ in Eq. (1) with the Fourier transform magnitude of the target image $|F(u,v)|$ as follows:

$$G'_{m-1}(u,v) = |F(u,v)| \exp[i\theta_{m-1}(u,v)] \tag{2}$$

Step 3: **Inverse Fourier Transform**

The inverse DFT is applied to the formed Fourier transform $G'_{m-1}(u,v)$ as follows:

$$g'_{m-1}(x,y) = \mathcal{F}^{-1}[G'_{m-1}(u,v)] \tag{3}$$

Step 4: **Application of the Image Constraint**

A new estimated image $g_m(x,y)$ is formed by

$$g_m(x,y) = \begin{cases} g'_{m-1}(x,y) & (x,y) \in D \\ 0 & (x,y) \notin D \end{cases} \tag{4}$$

where D is the set of pixels (x,y) which exist in the measured area of the target image.

The error reduction algorithm reconstructs a target image by processing these steps iteratively. The iterations are started by using an initial image $g_0(x,y)$ which is composed of uniformly distributed random values.

The iterative Fourier transform algorithms utilize the measured area of the target image as the image constraint. In general, the measured area is different from the true area where the target image exists, because of erosion, rotation, as shown in Fig. 3, and so on. In this case, the iterative Fourier transform algorithms cannot accurately reconstruct a target image. To overcome this problem, we present an efficient phase retrieval method.

3. PROPOSED PHASE RETRIEVAL

In this section, a novel and efficient phase retrieval method using the snake with an area term is proposed. The proposed method reconstructs a target image by satisfying both the Fourier and image constraints. The snake with an area term is utilized to iteratively renew the measured area, which is used as the image constraint, and converges to the true area where the target image exists. Therefore, the proposed method can accurately reconstruct a target image even when the measured area is different from the true area.

3.1. Snake with an Area Term

Snakes were first proposed by Kass *et al.* [11] and can extract the shape of a target image. The basic snake is defined as an energy-minimizing spline under the influence of internal and external forces. The internal forces of the snake serve as a smoothness constraint, and the external forces guide the snake towards image features such as lines and edges.

The total energy of the snake with parametric representation $v_i = (x_i, y_i)$ $(i = 1, 2, \ldots, N)$ can be defined as

$$E_{snake} = \sum_{i=1}^{N} \{E_{int}(v_i) + E_{ext}(v_i)\}, \tag{5}$$

where v_i denotes the coordinates of the ith snake point and N is the number of the snake points; $E_{int}(v_i)$ represents the internal energy and $E_{ext}(v_i)$ represents the external energy. The internal energy function $E_{int}(v_i)$ can be given by

$$E_{int}(v_i) = (\alpha\|v_i - v_{i-1}\|^2 + \beta\|v_{i-1} - 2v_i + v_{i+1}\|^2)/2, \tag{6}$$

where α and β are weighting parameters to control the smoothness constraint. The external energy function $E_{ext}(v_i)$, which is usually used as the edge energy function $E_{edge}(v_i)$, can be described as follows:

$$E_{ext}(v_i) = E_{edge}(v_i) = -w_{edge}|\nabla I(v_i)|^2, \tag{7}$$

where w_{edge} is a weighting parameter, ∇ is the gradient operator, and $I(v_i)$ is the intensity value at the position v_i.

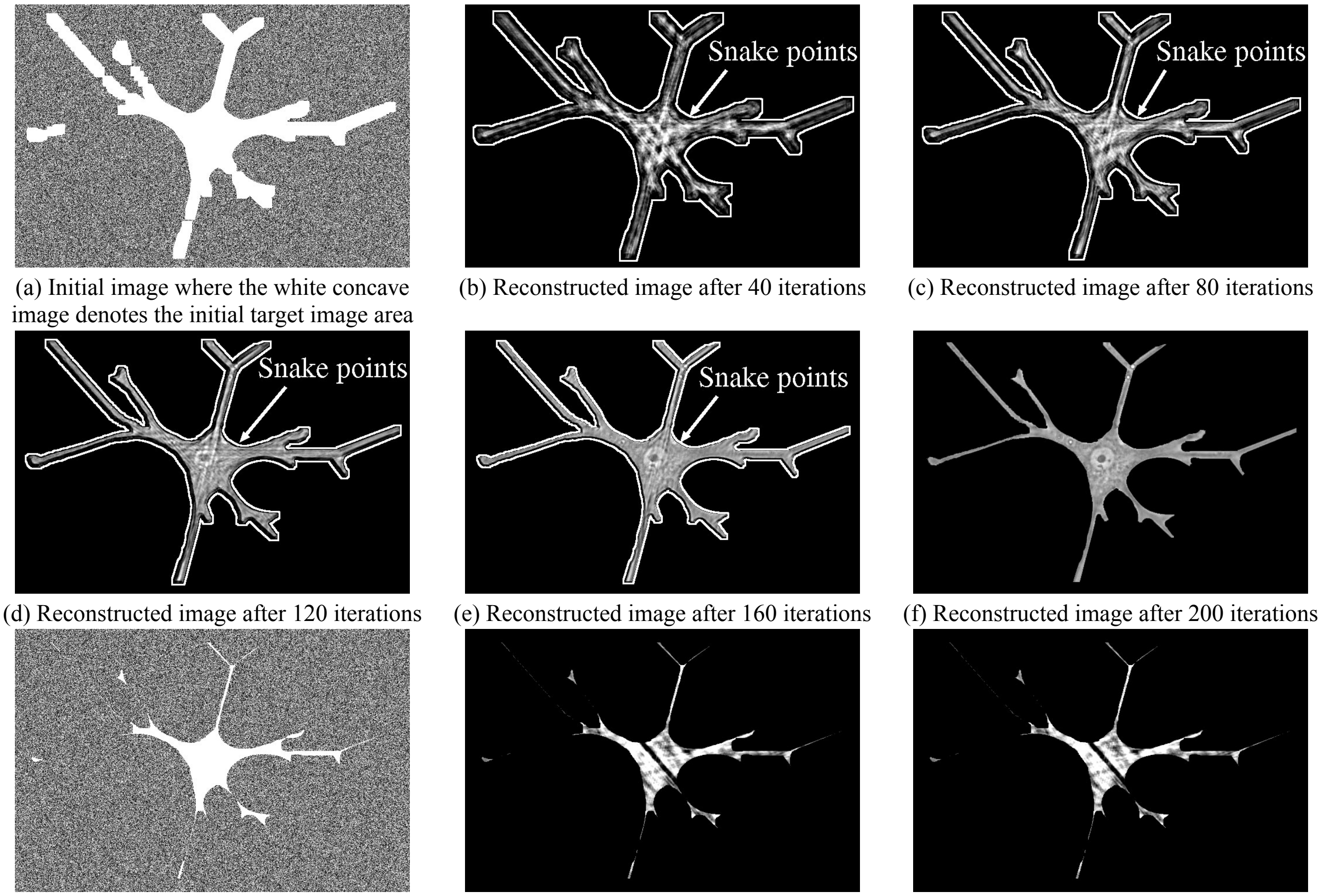
By minimizing E_{snake} in Eq. (5), the shape of the target image can be extracted. However, the basic snake using E_{snake} cannot extract the shape of concave target images such as those used in the proposed method, as shown in Fig. 2. This problem arises due to the fact that the second-order term of the internal energy function, as defined in Eq. (6), forces the snake to maintain the smoothness of the image. To address this problem, a method called the *snake with an area term* was proposed [10]. This method introduced the following energy function $E_{area}(v_i)$ into Eq. (5) as follows:

$$E'_{snake} = \sum_{i=1}^{N} \{E_{int}(v_i) + E_{edge}(v_i) + E_{area}(v_i)\}, \tag{8}$$

The energy function $E_{area}(v_i)$, which is proportional to the area surrounded by the snake points, is defined by

$$E_{area}(v_i) = \frac{1}{2} \sum_{i=1}^{N} [x_i(y_{i+1} - y_i) - (x_{i+1} - x_i)y_i] \tag{9}$$

By minimizing E'_{snake} in Eq. (8), the force of the snake works for the area surrounded by the snake points, and thus the snake with an area term can extract the shape of a concave target image.



(a) Initial image where the white concave image denotes the initial target image area
(b) Reconstructed image after 40 iterations
(c) Reconstructed image after 80 iterations
(d) Reconstructed image after 120 iterations
(e) Reconstructed image after 160 iterations
(f) Reconstructed image after 200 iterations
(g) Initial image where the white concave image denotes the measured image area
(h) Reconstructed image after 80 iterations
(i) Reconstructed image after 200 iterations

Fig. 4. Reconstructed images using the measured area shown in Fig. 3(b): (a)-(f) show the images reconstructed by the proposed method using the snake with an area term; (g)-(i) show the images reconstructed by the error reduction algorithm.

3.2. Iterative Fourier Transform Algorithm Based on the Snake with an Area Term

The iterative transform using the snake with an area term is described as follows:

Procedure 1: The target image area, used as the image constraint, is initially given by processing the measured area with dilation in order to obtain an image area containing the true area where the target image exists.

Procedure 2: An estimated image is generated by calculating from Eq. (1) to Eq. (4), based on $|F(u,v)|$ as the Fourier constraint and the target image area as the image constraint.

Procedure 3: The snake with an area term is applied to the image estimated in Procedure 2 and the shape of the image is extracted.

Procedure 4: The image shape extracted in Procedure 3 is renewed as a new target image area of the image constraint.

By applying these procedures from 2 to 4 iteratively, the proposed method accurately reconstructs a target image from an initial image based on the Fourier and image constraints.

4. EXPERIMENTAL RESULTS

This section shows the effectiveness of the proposed method. The target image used for the experiments are shown in Fig. 2. In the experiments, the proposed method utilizes the following conditions:

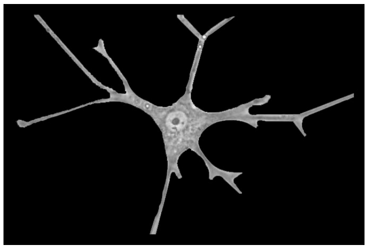
(i) The magnitude of the Fourier transform calculated from the target image (Fig. 2) is used as the Fourier constraint.

(ii) The measured area shown in either Fig. 3(b) or Fig. 3(c) is used as the image constraint.

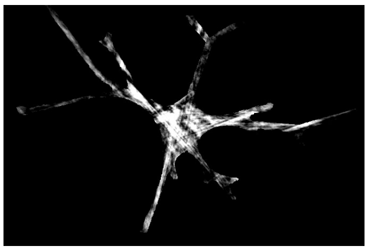
(iii) The initial image is composed of uniformly distributed random values.

(iv) The parameters α, β, w_{edge}, used in the snake with an area term, are fixed at 1.0, respectively,

Fig. 4(a)-(f) show the images reconstructed from the initial image by the proposed method after 40, 80, 120, 160, and 200 iterations, respectively, using the measured area shown in Fig. 3(b) as condition (ii). For comparison, Fig. 4(g)-(i) show the images reconstructed from the initial image by the error reduction algo-



(a)



(b)

Fig. 5. Reconstructed images using the measured area shown in Fig. 3(c): (a) and (b) show the images reconstructed by the proposed method and the error reduction algorithm, respectively, after 200 iterations.

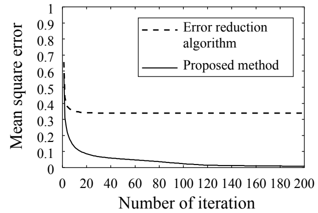
rithm under the same iterations and constraints used in the proposed method. From Fig. 4, it is obvious that the proposed method has much better performance than the error reduction algorithm after the same number of iterations. The reason comes from the fact that the area obtained by the snake with an area term, which is used as the image constraint, converges to the true area (Fig. 3(a)) as shown in Fig. 4(b)-(e) and as a result the proposed method accurately reconstructs the target image (Fig. 2).

Fig. 5 shows the images reconstructed by the proposed method and the error reduction algorithm using the measured area shown in Fig. 3(c) as condition (ii). The results indicate that the proposed method can successfully reconstruct the target image (Fig. 2) preserving more features of the target image than the error reduction algorithm.

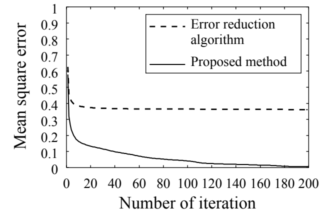
In order to show the relative reconstruction performance between the proposed method and the error reduction algorithm, the mean square error, which is generally used for calculating reconstruction accuracy, is denoted at each iteration in Fig. 6. The mean square error is defined as

$$E_l = \sqrt{\frac{\sum_{u,v}\{|G_l(u,v)| - |F(u,v)|\}^2}{\sum_{u,v}|F(u,v)|^2}}, \tag{10}$$

where $|G_l(u,v)|$ and $|F(u,v)|$ are the estimated magnitude of the Fourier transform for the lth iteration and the Fourier transform magnitude of the target image, respectively. From Fig. 6, it is seen that the proposed method can reconstruct the target image with a higher accuracy rate than the error reduction algorithm in all of the results. Therefore, based on these experiments, it can be recognized that the proposed method accurately reconstructs a target image even when the measured area is different from the true area where the target image exists.



(a)



(b)

Fig. 6. Comparison of the mean square error of the proposed method and the error reduction algorithm: (a) The result used the the measured area shown in Fig. 3(b); (b) The result used the the measured area shown in Fig. 3(c).

5. CONCLUSIONS

This paper has presented a phase retrieval method using snakes for image reconstruction. The proposed method can accurately reconstruct a target image from the magnitude of its Fourier transform and the measured area of the image. This method has many applications such as biomedical imaging using X-ray or optical detectors which cannot measure the phase.

6. REFERENCES

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