IMAGE SEGMENTATION VIA BRITTLE FRACTURE MECHANICS

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ABSTRACT

Brittle material like ceramic and rock has the nature that it is hard to crack, but if it is to crack it will crack into not dangling pieces, but separate pieces whose boundaries are contours that link up positions of strong stress in a continuous and often smooth fashion. Image segmentation desires similar results: it is to partition the image so that each partition corresponds to a separate surface or object in the imaged scene, whose boundaries are continuous and often smooth contours with a dense concentration of large intensity gradients. This work explores the adaptation of brittle fracture mechanics model toward image segmentation. An image is regarded as a plate experiencing brittle fracture when external forces are exerted on it according to intensity gradient distribution in the image. Stress concentration due to the external forces, and its change over time due to changing crack distribution are what determine the development of cracks. Real image results indicate that the model is promising.

1. INTRODUCTION

Early vision modules like edge detection to image, and discontinuity detection to raw stereoscopic data, often provide only sparse, noisy information about the imaged scene. Segmentation is about recovering dense description from the raw data, yet partitioning it so that each partition corresponds to a separate surface or object in the scene. It is a problem that is described as ill-posed (i.e., there are too many solution candidates to choose from), and its solution generally requires additional assumption like the smoothness assumption that turns the problem into a well-posed one.

The assumption of uniform smoothness, which is what most of the existing systems adopt, could recover dense description from the raw data, yet falls short of addressing discontinuities in the image data or the imaged scene. For one thing, improper imposition of the uniform smoothness assumption will lead to over-smoothed solution. Discontinuities also represent an indispensable part of the image or scene description and it is important that they are addressed and located when smoothness priors are employed. Segmentation, and in particular image segmentation, with discontinuity detection explicitly addressed is the objective of this work.

Image segmentation has applications to a wide domain of areas including image restoration, data compression, and feature extraction for matching. There has been rich work [2], [3], [5], [8], [9], [10] on it. These methods are all in a broad sense based on the optimization of a function that is related to data compatibility and smoothness of the final solution. Classical models used are the membrane and thin-plate models. More complex schemes with non-convex functions have been proposed to allow discontinuities to be present in the solution.

The above systems do allow discontinuities to develop in the final solution, yet the discontinuities that come out are often fragmented, short, and open. Closed, long discontinuity contours that represent the boundaries of surfaces and objects are often not obtained.

Brittle material like ceramic and rock does not crack easily, but if it is to crack it does not crack partially but all the way to separate pieces, with each piece having a boundary that is closed, continuous, and often smooth. The boundaries of the pieces are also often positions that have a high concentration of stress. In terms of result, if stress concentration could somehow be related to intensity gradient, brittle fracture is exactly what image segmentation desires. This work explores how brittle fracture mechanics could be adapted for image segmentation.

The occurrence of brittle fracture is generally due to stress concentration that is first induced by external force exertion. When the mechanical critical stress condition of the material is offended, cracks will form and propagate along the direction with the highest stress concentration.

In this work, we propose an image segmentation scheme that is adapted from brittle fracture mechanics. An image is regarded as a plate experiencing external forces that are initially set as the intensity gradient distribution in the image. The material of the plate owns a threshold in tensile stress. As soon as the threshold is exceeded, cracks will begin to appear and they will choose the direction of the highest stress to develop. Cracks will keep growing until they meet another crack or the image boundary. The

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stress distribution is not static; a growing crack will induce change in the stress distribution, and the presence of a crack would relieve the stress on its lateral sides. The interplay between the dynamic changes of crack and stress is what determines the final fracture or image segmentation result. Experiment results on real image data show that the model has promising performance.

Though started out from brittle fracture mechanics not from the field concept, the proposed model allows a stress field to emerge in the process. A field concept has been proposed in [6], in which the field is the superposition of the influences of all image features. However, here the focus is not on how the field is formed, but on how the field could be adjusted dynamically under the brittle fracture mechanics model, and how the final segmentation contours could be resulted from the interplay between boundary and field developments.

2. BRITTLE FRACTURE MECHANICS

The initiation and propagation of crack is mainly attributed to the presence of defects and the induced stress concentration [1]. In this work for simplicity we only concentrate on the effect of stress.

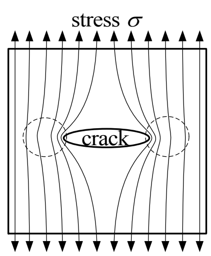


Fig 1. Stress concentration induced by a crack.

Fig. 1 illustrates how stress is induced by a crack to a structure that is under the influence of external forces. Lines with arrows at their ends denote forces exerted on a plate structure that has developed a crack (denoted by an ellipse in the figure) in its middle. The two acute ends and the major axis of the ellipse indicate respectively the tips and the direction of the crack. As summarized in [4], since crack cannot bear load, the lines of force become concentrated around the crack tips (the regions indicated by dash circles), leading to stress concentration in those regions. The stress so induced on the structure by a crack tip is highest at positions that are closest to the crack tip and along the direction of the crack. In other words, the induced stress generally decreases with (1) distance r from the crack tip, and (2) azimuth angle θ with respect to the crack's orientation. More precisely, the stress induced by a crack to any position (r,θ) (polar coordinates with respect to the crack's position and orientation) on the structure consists of 2 orthogonal components σ_{ij}'s, where ij represents the directions $r\theta$, $\theta\theta$ as illustrated by Fig. 2, in the form

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta) \tag{1}$$

Here K is the stress intensity factor which generally increases with the crack length and the magnitude of the external forces, and is dependent on the loading geometry.

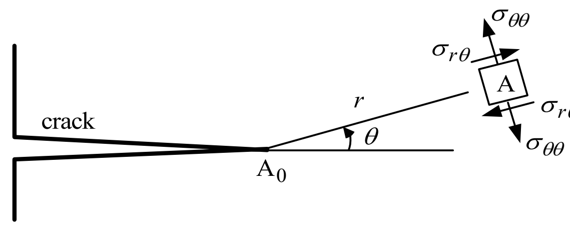


Fig 2. Stress components induced by a crack at position (r, θ).

Although shear stress $\sigma_{r\theta}$ that exerts along the direction of a crack could also generate fracture, most practical examples of fracture involve tensile stress that exerts in the direction orthogonal to that of the crack. Here we only pay attention to the effect of tensile stress $\sigma_{\theta\theta}$, which is often termed as mode I loading in fracture mechanism. This is illustrated in Fig. 3, in which stresses shown in arrows are exerted in the direction orthogonal to that of the crack.

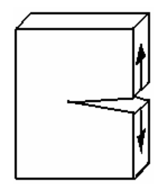


Fig. 3. Mode I loading on a crack.

One asymptotic solution for the stress component $\sigma_{\theta\theta}$ can be expressed as

$$\sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin^2\frac{\theta}{2}\right) + \ldots \tag{2}$$

Ignoring the higher order terms (assuming θ is small), Equation 2 could be approximated as

$$\sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin^2\frac{\theta}{2}\right) \tag{3}$$

In mode I loading,

$$K_I = \sigma\, Y \sqrt{c} \tag{4}$$

where K_I is the stress intensity factor in mode I, Y is a dimensionless parameter depending upon the concerned crack and the loading geometry, σ is the tensile stress exerted on the crack tip, and c is the crack length. With that, we can have a more precise expression for $\sigma_{\theta\theta}$:

$$\sigma_{\theta\theta} = \sigma \frac{Y}{\sqrt{2\pi}} \sqrt{\frac{c}{r}} \cos\frac{\theta}{2}\left(1 - \sin^2\frac{\theta}{2}\right) \tag{5}$$

which could be simplified to

$$\sigma_{\theta\theta} = \sigma \frac{Y}{\sqrt{2\pi}} \sqrt{\frac{c}{r}} \cos^3 \frac{\theta}{2} \qquad (6)$$

Equation 6 shows that the stress $\sigma_{\theta\theta}$ at a position near the crack tip is proportional to the stress σ exerted exactly on the crack tip (in the direction orthogonal to that of the crack), the square root of the crack length c, and the inverse of the square root of the position's distance r from the crack tip. The stress $\sigma_{\theta\theta}$ also decreases with the change of azimuth angle θ from 0 to π or from 0 to -π.

In addition, the formation of a crack will release the stress on its lateral sides and prevents crack nucleation sites from forming there [7]. Such a stress release mechanism is also implemented in our image segmentation system in a simple form: those positions on the lateral sides of a crack are simply masked away from being the nucleation sites of new cracks.

3. ADAPTATION TO SEGMENTATION

An image is regarded as a brittle thin plate that is under the influence of some external forces, whose magnitudes and directions and application points are the same as those of the intensity gradients in the image.

We suppose the magnitudes of the external forces are large enough to trigger the fracture process. On this, the position with the largest magnitude of the external force (i.e., of the intensity gradient) is selected as the one where the first crack appears, and this crack nucleation has an orientation orthogonal to the intensity gradient there.

The crack will induce stress concentration around its tips. On this we use a form generalized from Equation 6. The stress $\sigma_{\theta\theta}$ induced at position A by a crack tip at position A_0 (as shown in Fig. 4) is:

$$\sigma_{\theta\theta} = \sigma \sqrt{\frac{c}{r}} \left(\cos \frac{\phi - \phi_0}{2}\right)^k \qquad (7)$$

where

$$\sigma = \|\mathbf{g}\| \sin(\alpha - \phi_0) \|\mathbf{g}_0\| \sin(\alpha_0 - \phi_0) \qquad (8)$$

Here k is the exponential parameter (in Equation 6 it is of value 3), c is the length of the crack, r is the distance of A from A_0, ϕ_0 is the orientation of the crack tip, ϕ is the orientation of the vector A_0A, $\mathbf{g}$ and $\mathbf{g}_0$ are the intensity gradients at positions A and A_0 respectively, and α and α_0 denote the directions of $\mathbf{g}$ and $\mathbf{g}_0$.

Each crack tip will then pick the direction of maximum stress to grow by one position (In our implementation we have a threshold $\sigma_{\theta\theta T}$ for this maximum stress to exceed, but we view this threshold as merely a scale parameter). Once the crack attains new length it would in turn perturb the stress distribution around it in two ways: (1) the renewed stress field as expressed by Equations 7 and 8, and (2) the renewed stress release over the crack's lateral sides as outlined previously. The interplay between the growth of the crack and the re-distribution of the stress allows the crack to attain longer and longer length. The crack will keep on growing until it either hits another crack to form a closed contour, or it meets the image boundary.

Then the next crack will start to form. First, the position that is not embedded by any previously developed cracks and that has the largest intensity gradient is chosen as its starting point. The crack then grows and the stress distribution readjusts in the same way as outlined above.

The iterative process for crack development goes on until a preset number of cracks for the image have been acquired (or no position has maximum stress exceeding $\sigma_{\theta\theta T}$). In the process, if any crack so developed has a final length that is regarded as too short (shorter than a threshold c_T), the crack would be regarded as negligible and the crack development process will be reversed.

In calculating the stress distribution around each crack tip a window of size $2n \times 2n$ pixels is used. In our implementation, we set n to be 1/4 of the crack length, with its lower bound being 1 and the upper bound being N.

N is an important control parameter of the entire fracture mechanism. In the experimental results we present later in this paper, we simply used various global values of N so as to explore its effect. However, N could be set locally and adaptively for each crack in accordance with the respective crack length and strength (the average intensity gradient on it), so as to encourage long and dominant cracks to grow further, yet discouraging short and minor cracks from forming.

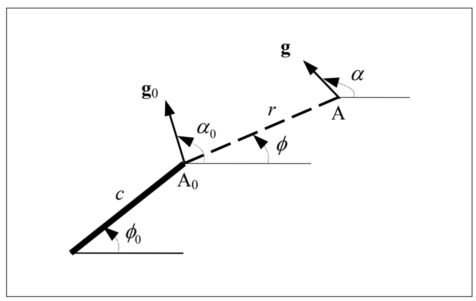


Fig. 4. Stress calculation in image segmentation.

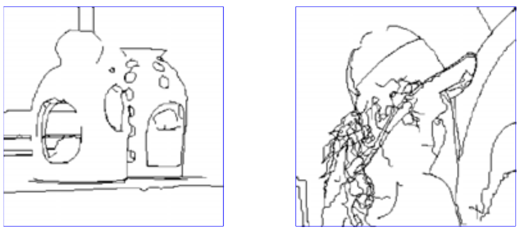
4. EXPERIMENT RESULTS

Fig. 5 shows the segmentation results under different choice of values for the various control parameters: the exponent k (in Equation 7), the upper bound N for the stress calculation window size, and stress threshold $\sigma_{\theta\theta T}$ which is only a scale parameter.

It could be observed that while the value of k has only limited impact on the segmentation result, N has a dominant role to play. In general, a smaller N discourages cracks from extending. For example, in Fig 5(b) where N

is small, the silhouette boundary of the object on the left of the image is incomplete. Yet in Fig 5(e) where N is larger, the boundary is complete but in the company of many more spurious contours in other places of the image. The parameter $\sigma_{\theta\theta T}$ only influences how much details one wants to extract from the image.

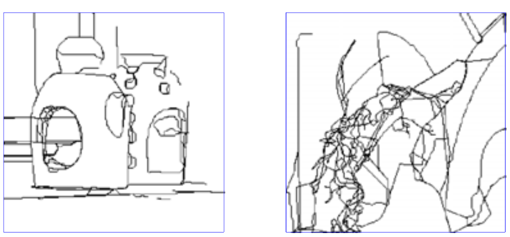
(a) Original image (256×256)



(b) *k*=3, *m*=100, *N*=4, c_T=10, $\sigma_{\theta\theta T}$=200

(c) *k*=3, *m*=100, *N*=4, c_T=10, $\sigma_{\theta\theta T}$=20

(d) *k*=4, *m*=100, *N*=4, c_T=10, $\sigma_{\theta\theta T}$=20



(e) *k*=3, *m*=100, *N*=30, c_T=10, $\sigma_{\theta\theta T}$=200

Fig. 5. Experiment results on "Lenna" image

The experiments were conducted on a computing platform with Pentium IV 1.6GHz CPU and 256M RAM. For images of resolution 256×256, the processing time t was respectively about 60, 70, 70, 600 seconds for subfigures (b) to (e) in Fig. 5.

5. CONCLUSION AND FUTURE WORK

We have presented an image segmentation scheme that is adapted from brittle fracture mechanics. Experimental results show that elongated and closed boundaries could be resulted. Processing speed is also desirable. Testing with various choices of values for the control parameters indicates that the key parameter that controls the result is N: the upper bound in the window size for stress calculation around crack tip.

Future work includes the exploration of how N could be adjusted locally and adaptively in accordance with crack length and strength, to encourage dominant cracks to make full circle, yet minor cracks to retract.

6. REFERENCES

[1] Abhijit Dasgupta, Michael Pecht, "Material Failure Mechanisms and Damage Models", *IEEE Transactions On Reliability*, Vol. 40, No. 5, 1991 Dec.

[2] A. Blake, A. Zisserman, *Visual Reconsturction*, Cambridge, Mass. MIT Press, 1987.

[3] C. Koch, J.Marroquin, A. Yuille, "Analog neuronal networks in early vision", *Proc. Nat'l Academic Science*, USA, Vol. 83, 1986.

[4] Dominique P. Miannay, "Fracture Mechanics", New York: Springer, pp. 5-9, c1998.

[5] Figueiredo M.A.T., Leitao J.M.N., "Simulated tearing: an algorithm for discontinuity-preserving visual surface reconstruction", *Computer Vision and Pattern Recognition*, pp. 28-33, 1993.

[6] Gideon Guy, Gerard Medioni, "Perceptual Grouping Using Global Saliency-Enhancing Operators", *Proc. 11th IAPR Int. Conf. on Pattern Recognition*, I: 99-103, 1992.

[7] O. Miller, L. B. Freund and A. Needleman, "Modeling and simulation of dynamic fragmentation in brittle materials", *International Journal of Fracture* 96: 101-125, 1999.

[8] Sarvajit S. Sinha & Brian G. Schunck, "Discontinuity Preserving Surface Reconstruction", *IEEE Computer Vision and Pattern Recognition,* pp. 229-234, 1989.

[9] S. Geman, D. Geman, "Stochastic relaxation, Gibbs distribution and the Bayesian restoration of images", *IEEE Trans. Pattern Analysis and Machine Intelligence*, Vol. 6, No. 6, pp. 721-724, Nov, 1984.

[10] Shih-Chii Liu, John G. Harris, "Generalized Smoothing Networks in Early Vision", *IEEE Computer Vision and Pattern Recognition*, pp. 184-191, 1989.