ANALYSIS/SYNTHESIS SYSTEMS FOR PROGRESSIVE-TO-LOSSLESS EMBEDDED WAVELET IMAGE CODING

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ABSTRACT

In this paper, we investigate analysis/synthesis systems for progressive-to-lossless embedded wavelet image coding. We propose a system which does not use an inverse lossless wavelet transform (ILWT) but an inverse wavelet transform (IWT). In this system, we must correct the mean value of rounding errors of each band. We also investigate a mixed-type system which is obtained by applying repeatedly a 4-band non-separable forward lossless wavelet transform (FLWT) for the lowest frequency band. Our simulation shows that the PSNR of the reconstructed image of the non-separable FLWT followed by the IWT is 2.7dB higher than that of the separable FLWT followed by the ILWT at a bit rate and that the mixed-type system switching inverse transforms depending on the bit rate has good performance at all bit rates.

1. INTRODUCTION

A progressive-to-lossless embedded wavelet image coding (PLEWIC) has wide applicability, since lossy and lossless images can be reconstructed from a part and the whole of data encoded, respectively [1], [2]. Specifically, it does not cover only natural images, but also medical, satellite and art images. The PLEWIC can be realized by using a separable two-dimensional (2D) forward lossless wavelet transform (FLWT) and its corresponding inverse lossless wavelet transform (ILWT) [1]-[4]. We denote this analysis/synthesis system by SP-FLWT-ILWT.

In lossy reconstruction, the reconstruction error of the SP-FLWT-ILWT is much larger than that of the system using forward and inverse wavelet transforms (FWT-IWT) at high bit rates other than near lossless bit rates because of the rounding operation in lifting steps. This results in low lossy compression performance of the PLEWIC. Chokchaitam et al. have derived a non-separable 2D (NSP-) FLWT-ILWT from the SP-FLWT-ILWT which consists of two-step lifting scheme [5]. (The SP-FLWT-ILWT which consists of n-step ($n \geq 3$) lifting scheme generally has low lossy compression performance, since the number of rounding operation is large.) They showed that the reconstruction error of the NSP-FLWT-ILWT is smaller than that of the SP-FLWT-ILWT.

In this paper, we investigate a system which does not use the ILWT but the IWT, that is, FLWT-IWT. We also investigate a mixed-type system which is obtained by applying repeatedly a 4-band NSP-FLWT for the lowest band.

2. NON-SEPARABLE 2D FLWT-ILWT

The NSP-FLWT-ILWT, SP-FLWT-ILWT and FWT-IWT are equivalent, if the rounding in the lifting scheme is eliminated. The SP-FLWT-ILWT can be decomposed into 2-band one-dimensional (1D) FLWT-ILWT. However, the NSP-FLWT-ILWT can not be decomposed into other FLWT-ILWTs such as 4-band NSP-FLWT-ILWT or 2-band 1D FLWT-ILWT. The NSP-FLWT-ILWT is shown in Fig.1, where $c'_0(m,n) = 0$, and expressed by the following equations.

$$\begin{aligned} f_i(m,n) &= c_0(2^{i-1}(2m-1), 2^{i-1}(2n-1)) \\ &+ \lfloor c'_{i-1}(2m-1, 2n-1) \\ &+ \sum_{j=-\infty}^{\infty} p(j)\{c_{i-1}(2m-2j, 2n-1) \\ &+ c_{i-1}(2m-1, 2n-2j) \\ &+ \sum_{k=-\infty}^{\infty} p(k)c_{i-1}(2m-2j, 2n-2k)\}\rfloor \end{aligned} \quad (1)$$

$$\begin{aligned} d_i(m,n) &= c_0(2^i m, 2^{i-1}(2n-1)) + \lfloor c'_{i-1}(2m, 2n-1) \\ &+ \sum_{j=-\infty}^{\infty} \{p(j)c_{i-1}(2m, 2n-2j) \\ &+ u(j)f_i(m-j, n)\}\rfloor \end{aligned} \quad (2)$$

$$\begin{aligned} e_i(m,n) &= c_0(2^{i-1}(2m-1), 2^i n) + \lfloor c'_{i-1}(2m-1, 2n) \\ &+ \sum_{j=-\infty}^{\infty} \{p(j)c_{i-1}(2m-2j, 2n) \\ &+ u(j)f_i(m, n-j)\}\rfloor \end{aligned} \quad (3)$$

$$c_N(m,n) = c_0(2^N m, 2^N n) + \lfloor c'_N(m,n) \rfloor \quad (4)$$

where

$$c_i(m,n) = c_0(2^i m, 2^i n) + c'_i(m,n) \quad (5)$$

$$c'_i(m,n) = \sum_{j=-\infty}^{\infty} \sum_{k=1}^{i} u(j)\{d_k(2^{k-1}m, 2^{k-1}n - j) + e_k(2^{k-1}m - j, 2^{k-1}n) - \sum_{s=-\infty}^{\infty} u(s) f_k(2^{k-1}m - j, 2^{k-1}n - s)\} \quad (6)$$

In [5], the NSP-FLWT-ILWT given by the following equations had best performance in terms of lossy compression efficiency.

$$P(z) = \sum_{n=-\infty}^{\infty} p(n) z^{-n} = -(1+z^{-1})/2 \quad (7)$$

$$U(z) = \sum_{n=-\infty}^{\infty} u(n) z^{-n} = (1+z)/4 \quad (8)$$

Therefore, in this paper, we use Eq.(7) and (8). The SP-FLWT-ILWT using Eq.(7) and (8) was proposed by our group and Calderbank et al.[6],[7], and is used in JPEG2000 [3].

3. COMPARISON AMONG NSP-, SP-, AND MIX-FLWT-ILWT

In this section, we calculate lossless and lossy compression efficiency of SP-, NSP- and MIX-FLWT-ILWT, where the MIX-FLWT-ILWT is the mixed-type system defined in section 1. Input images are 512 by 512 and 8bit/pixel grayscale. In our experiments, the 2D 10-band pyramidal decomposition is used.

Fig.2(a) shows the FLWT-ILWT, where (m,n) of $a(m,n)$, $q(m,n)$, etc. are omitted, q is quantization error added by quantizer-dequantizer, f and g are errors added by rounding towards the nearest integer, and i and r mean integer and real number, respectively. We propose an equivalent model of the FLWT-ILWT in order to explain the experimental results. Fig.2(b) shows the system which is equivalent to that of Fig.2(a). Here $r_1 = s - u$ and $r_2 = v - t$ are rounding errors of the FLWT and the ILWT, respectively, where $s = \mathrm{FLWT}(a)$, $u = \mathrm{FWT}(a)$, $v = \mathrm{FWT}(b)$, $t = \mathrm{FLWT}(b)$. The SP-, NSP- and MIX-FLWT-ILWT are equivalent to the FWT-IWT, if the rounding in the lifting scheme is eliminated.

Table 1 shows entropy of transform coefficients, that is, lossless compression performance. It is seen from Table 1 that the differences are very small, but NSP- and SP-FLWT-ILWT have the best and worst performance, respectively.

Table 1. Entropy of transform coefficients (bits/pel)

Images	SP	MIX	NSP
Aerial	5.282	5.278	5.278
Airplane	4.367	4.360	4.358
Baboon	6.328	6.328	6.327
Couple	4.624	4.620	4.619
Lenna	4.729	4.726	4.724
Peppers	4.906	4.905	4.904
[Average]	5.039	5.036	5.035

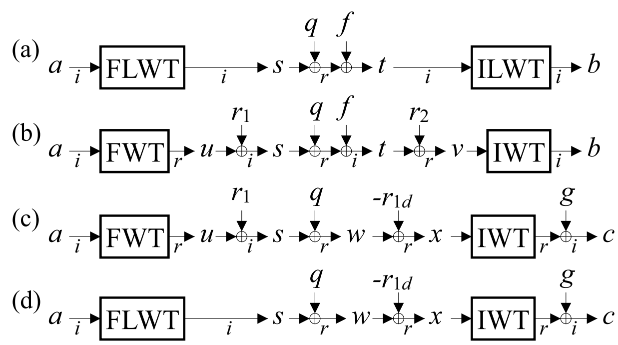

Fig. 2. Lossy reconstruction systems: (a)FLWT-ILWT, (b)Equivalent of (a), (c)Equivalent of (d), (d)FLWT-IWT.

This means that the variances of r_{1a} of the NSP-FLWT-ILWT and the SP-FLWT-ILWT are the smallest and largest, respectively, where r_{1a} is AC component of r_1.

Fig.3 and Fig.4 show the relations (P-E curves) between PSNR (Peak Signal To Noise Ratio) of reconstructed image and entropy of quantized transform coefficients, where a midtread uniform quantizer is used. Input image is Lenna. (Similar results were also obtained for other images.) Descriptions of the P-E curves of the FLWT-IWT will be given later. It is seen from Fig.3 and Fig.4 that the NSP- and SP-FLWT-ILWT have the best and worst lossy compression performance, respectively, at entropies below 3.8b/p, but the NSP-FLWT-ILWT has the worst performance at entropies above 4.0b/p. This means that, when the variances of q is large, r_1 and r_2 do not cancel each other out and the variances of r_1+r_2 of the NSP-FLWT-ILWT is smaller than that of the SP-FLWT-ILWT and that, when the variances of q is small, r_1 and r_2 partially cancel each other out and the variances of r_1+r_2 of the NSP-FLWT-ILWT is larger than that of the SP-FLWT-ILWT. Note that the performance of the MIX-FLWT-ILWT is almost the same as that of the SP-FLWT-ILWT at near lossless bit rates (NLBR).

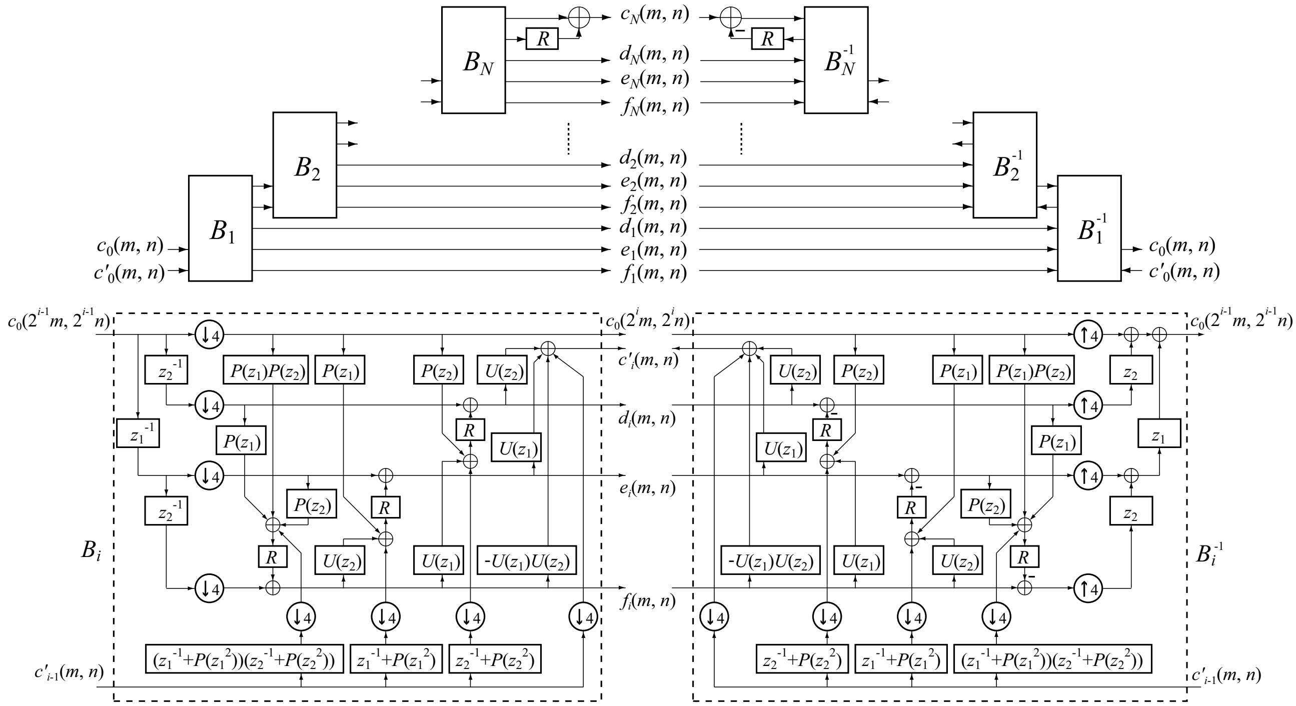

Fig. 1. Non-separable 2D FLWT-ILWT

4. LOSSY RECONSTRUCTION SYSTEM USING IWT

Fig.2(d) shows the FLWT-IWT and is equivalent to Fig.2(c). In these systems, correction of the DC component of r_1, r_{1d}, is done for each band before the IWT is applied and f is deleted, since input of the IWT does not need to be integer. When the variances of q is large, the error other than q is mainly $r_1 - r_{1d} = r_{1a}$ for the FLWT-IWT and $r_1 + r_2$ for the FLWT-ILWT, and r_{1a} and r_{2a} do not cancel each other out. Therefore, the FLWT-IWT is thought to be superior to the FLWT-ILWT, when the variances of q is large. The r_{1d} of the NSP-FLWT are shown in Table 2. The r_{1d} are almost the same for different images. Accordingly, we can use the same values of r_{1d} for different images.

Fig.3 and Fig.4 also show the P-E curves of the SP-, NSP- and MIX-FLWT-IWT. It is seen from Fig.3 and Fig.4 that the lossy compression performances of the FLWT-IWTs are superior to those of the FLWT-ILWTs at bit rates other than NLBR. The NSP- and SP-FLWT-IWT have the best and worst lossy compression performance, respectively. This means that the variances of r_{1a} of the NSP- and SP-FLWT-IWT are the smallest and largest, respectively. Note that the PSNR of the NSP-FLWT-IWT is 2.7dB higher than that of the SP-FLWT-ILWT at 3.8b/p. At low bit rates, the differences between the FLWT-ILWTs and the FLWT-IWTs are small, since q is much larger than rounding error. At NLBR, the lossy compression performances of the FLWT-IWTs are inferior to those of the FLWT-ILWTs. This means that the variances of r_{1a} of the FLWT-IWTs are larger than the variances of $r_1 + r_2$ of the FLWT-ILWTs, since r_1 and r_2 partially cancel each other out.

From the above results we can see that the recommended analysis/synthesis systems for the PLEWIC are the following two methods.

(i) The NSP-FLWT is used in the encoder. In the decoder, the NSP-ILWT is used at NLBR and the IWT is used at bit rates other than NLBR. We need to transmit the means of inverse transforms as side information.

(ii) MIX- is used in place of NSP- in (i).

The method (i) is superior at bit rates other than NLBR and the method (ii) is superior at NLBR.

5. CONCLUSION

In this paper, we proposed a system which does not use the ILWT but the IWT. We must correct the DC component of rounding errors of each band in this system. We also investigated a mixed-type system obtained by applying repeatedly a 4-band NSP-FLWT. The simulation showed that the PSNR of the NSP-FLWT-IWT is 2.7dB higher than that of the SP-FLWT-ILWT at a bit rate and that the mixed-type system switching inverse transforms depending on the bit rate has good performance at all bit rates.

Table 2. DC component of r_1 for each band

	Aerial	Airplane	Baboon	Couple	Lenna	Peppers	[Average]
c_3	-1.209	-1.215	-1.211	-1.210	-1.215	-1.228	-1.215
d_3	-0.727	-0.729	-0.734	-0.732	-0.741	-0.737	-0.733
e_3	-0.718	-0.735	-0.728	-0.717	-0.725	-0.736	-0.726
f_3	-0.492	-0.493	-0.495	-0.489	-0.507	-0.499	-0.496
d_2	-0.727	-0.730	-0.731	-0.731	-0.731	-0.728	-0.730
e_2	-0.729	-0.728	-0.728	-0.725	-0.731	-0.730	-0.728
f_2	-0.487	-0.491	-0.492	-0.492	-0.494	-0.491	-0.491
d_1	0.562	0.562	0.564	0.558	0.561	0.550	0.559
e_1	0.562	0.558	0.561	0.560	0.558	0.551	0.558
f_1	0.373	0.373	0.375	0.373	0.371	0.367	0.372

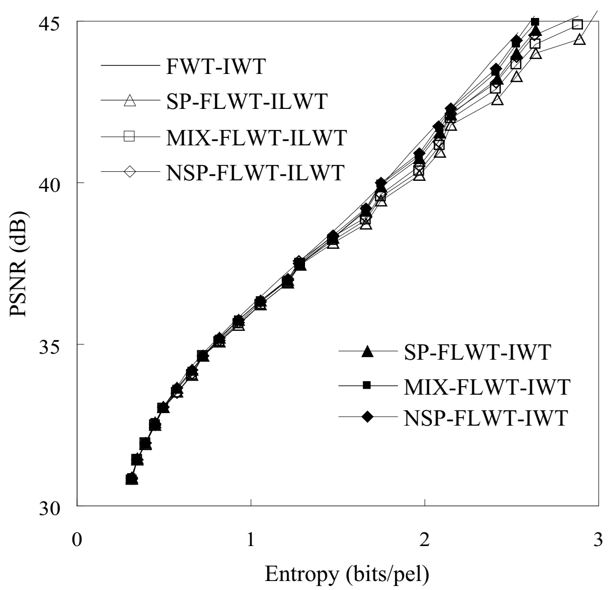

Fig. 3. Lossy compression performance at low entropy.

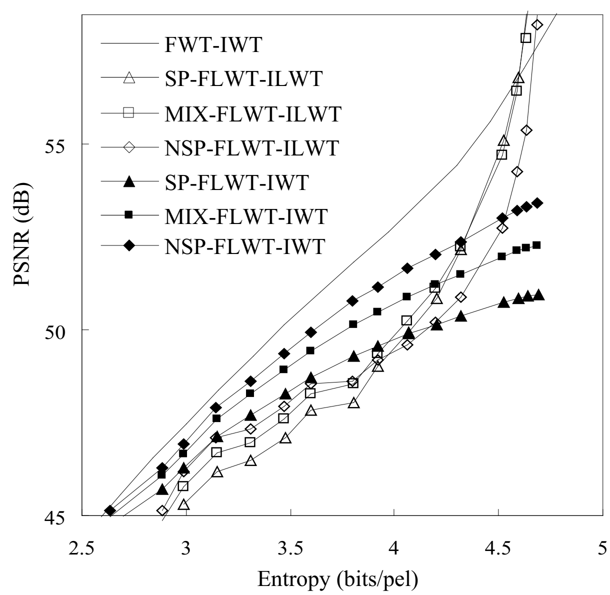

Fig. 4. Lossy compression performance at high entropy.

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