Image Interpolation based on inter-scale dependency in wavelet domain

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ABSTRACT

Image interpolation in the wavelet domain can be considered as the estimation of wavelet coefficients in the highest frequency subband. In this paper, a novel image interpolation method based on inter-scale dependency in the wavelet domain is proposed. In our method, the Gaussian Mixture Model (GMM) is used to estimate the magnitude of the wavelet coefficient, and the parameters of the GMM are derived from subbands with no training. The sign of the estimated wavelet coefficient is also obtained by using the inter-scale dependency of wavelet subbands. In the simulation results, the proposed method shows an improved PSNR and subjective quality compared with conventional bicubic method and the statistical method [1] that exploits the hidden Markov tree (HMT) model with training.

1. INTRODUCTION

Image interpolation for digital images has become more important because various digital image acquisition equipment such as digital cameras and camcorders has spread widely. The main issue of research on this application is to recover high frequency components that are lost by aliasing. The nonlinear method by Allebach [2] recovers sharpness in edge region in the view of subjective quality but it causes many artifacts compared with the original image. The new edge-directed interpolation (NEDI) by Li [3] exploits the correlation of the neighborhood pixels of low-resolution images to determine weights for interpolation. It shows good performance for long edges where the correlation of low-resolution and high-resolution image matches very well. However, in complex edges such as texture, it causes many artifacts and it suffers from a high computational load.

Recently, various image interpolation methods in the wavelet domain have approximates the property of exponential decay of wavelet coefficients been proposed. The method by Carey [4] through wavelet subbands by the least square method. It enhances the edges of images. However, while the estimation in this method is accurate in isolated and strong edge, it is inaccurate in complex edges. Kinebuchi [1] proposed an image interpolation method using Hidden Markov Tree (HMT) model in the wavelet domain. This method makes images clearer, but it requires a training procedure and includes a complex sign detection method that has shown to be disadvantageous.

In this paper, a statistical interpolation method based on the inter-scale dependency of the wavelet domain is proposed. In our algorithm, the Gaussian Mixture Model (GMM) is used for each wavelet coefficient as a probability density function (pdf). The parameters of the GMM are derived from the coefficients in lower frequency subbands. The variances of the GMM are estimated by using the property that the variance of a wavelet coefficient decays exponentially through subbands with different scales. The weighting factors of the GMM are determined from the statistical method without training. In the simulation results, the proposed method showed improved PSNR results and subjective quality compared with the conventional bicubic method and Kinebuchi's algorithm [1].

This paper is organized as follows. In Section 2, the interpretation of image interpolation in the wavelet domain is introduced. Section 3 discusses the proposed method for the estimation of coefficients. The simulation results are shown in Section 4. We conclude in Section 5.

2. IMAGE INTERPOLATION USING HMT MODEL IN WAVELET DOMAIN

Image interpolation in the wavelet domain can be considered as the estimation of coefficients in the high frequency band from coefficients in lower frequency subbands [1]. Input image is a down-sampled version of the high-resolution image after low-pass filtering. The objective in this application is to estimate the magnitude and sign of wavelet coefficients for high frequency subband from the input image or the training image.

The compaction property of wavelet transform is the fact that the transform of a typical signal consists of a small number of large coefficients and a large number of

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small coefficients. Most wavelet coefficients have small values and contain very little signal information. However, a few wavelet coefficients have large values that represent significant signal information. This property allows a simple model for an individual wavelet coefficient. Each coefficient is modeled to one of two states: "1" corresponding to a wavelet component containing significant information, or "0" representing coefficients with little signal energy. From this property for an individual wavelet coefficient, the GMM is used for the pdf of each wavelet coefficient. The pdf for a given wavelet coefficient at spatial location i, w_i is

$$p(w_i) = \alpha \cdot g(w_i;0,\sigma_{0;i}^2) + (1-\alpha) \cdot g(w_i;0,\sigma_{1;i}^2) \quad (1)$$

where $g(w;\mu,\sigma^2)$ is the marginal Gaussian pdf with mean μ and variance σ^2, and α is the weighting parameter for the GMM that is composed of the convex sum of the marginal Gaussian pdf. In Equation (1), $\sigma_{0,i}^2$ and $\sigma_{1,i}^2$ are variances for state "0" and state "1", respectively. Generally, the mean of the wavelet coefficient can be set to zero.

It has been shown that larger or smaller values of wavelet coefficients tend to propagate through the scale of the quad-tree [7]. For a given wavelet coefficient, the coefficient in the subband with lower frequency band is called the parent coefficient, and the coefficient in the subband with higher frequency band is called the child coefficient.

This property is well represented by the HMT model in the wavelet domain [5-6]. The structure of the HMT model between a given wavelet coefficient and its parent coefficients is shown in Figure 1. In the Figure, black circle indicates the wavelet coefficient, and the white circle with the Arabic numeral indicates the state of the coefficient. The "0" state indicates the non-significant coefficient that has a small magnitude, and the "1" state indicates a significant coefficient that has a large magnitude. $w_{\rho(i)}$ is the parent coefficient of w_i, and $\varepsilon_{i,\rho(i)}^{00}, \varepsilon_{i,\rho(i)}^{01}, \varepsilon_{i,\rho(i)}^{10}, \varepsilon_{i,\rho(i)}^{11}$ are the state transition probabilities between states in the given coefficient and states in its parent. They represent inter-scale dependency between coefficients in different subband dependency across subband.

Let $S_i(0)$ and $S_i(1)$ represent state for state "0" and state "1", respectively, and let $P(S_i(0))$ and $P(S_i(1))$ represent the probability that the wavelet coefficient belongs to $S_i(0)$ and $S_i(1)$, respectively. With this structure, Equation (1) can be expressed as

$$\begin{aligned} p(w_i) &= P(w_i \mid S_i(0))P(S_i(0)) + P(w_i \mid S_i(1))P(S_i(1)) \\ &= g(w_i;0,\sigma_{0;i}^2)P(S_i(0)) + g(w_i;0,\sigma_{1;i}^2)P(S_i(1)) \end{aligned} \quad (2)$$

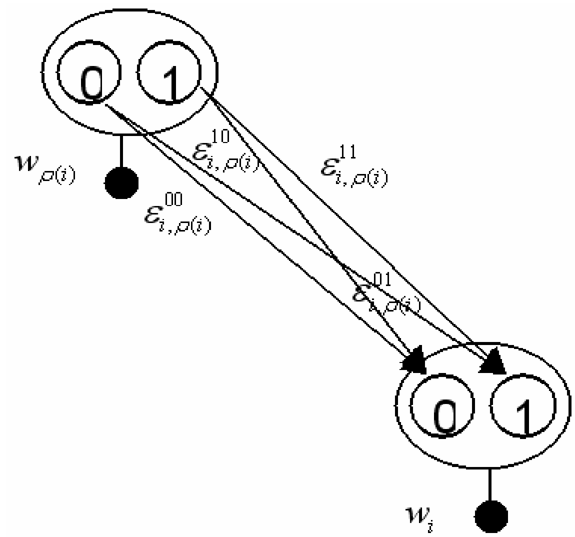

Figure 1. The structure of HMT model between a given coefficient and its parent coefficient.

Comparing Equation (2) with Equation (1), weighting factors α, $1-\alpha$ become state probabilities $P(S_i(0))$ and $P(S_i(1))$. These state probabilities can be expressed in terms of the state probabilities of a parent's coefficient :

$$P(S_i(m)) = \varepsilon_{i,\rho(i)}^{m0} P(S_{\rho(i)}(0)) + \varepsilon_{i,\rho(i)}^{m1} P(S_{\rho(i)}(1)), \ m = 0,1 \quad (3)$$

Combining Equation (3) with Equation (1), the GMM equation combining the HMT model is

$$\begin{aligned} p(w_i) &= g(w_i;0,\sigma_{0;i}^2)\{P(S_{\rho(i)}(0))\varepsilon_{i,\rho(i)}^{00} + P(S_{\rho(i)}(1))\varepsilon_{i,\rho(i)}^{01}\} \\ &+ g(w_i;0,\sigma_{1;i}^2)\{P(S_{\rho(i)}(0))\varepsilon_{i,\rho(i)}^{10} + P(S_{\rho(i)}(1))\varepsilon_{i,\rho(i)}^{11}\} \end{aligned} \quad (4)$$

The parameters in Equation (4) can be obtained by the Expectation-Maximization (EM) algorithm [6], or by the HMT model with no training [5].

From the estimated parameters, the lost wavelet coefficients in the highest frequency subband are obtained. Using the pdf of the given wavelet coefficient established by the parameters, the magnitude of the coefficient is generated randomly [1].

3. THE PROPOSED METHOD

In the application of image interpolation, there is no information from the input low-resolution image in the subband with highest frequency wavelet components. In [1], this information is obtained from the input and training images. However, this method requires significant training time and the information from the training image does not match effectively the input image. The training procedure is not unnecessary in our method. All parameters come from the input image. This makes the model more adaptive to input image.

In this paper, to determine the state of parent coefficients, the coefficients in the subband are divided into two sets– one for significant and the other for non-significant coefficients. Next, state probabilities are estimated by counting the significant or non-significant coefficients within the local window. Using these estimated state probabilities, the state of parent coefficient is determined by the expectation value of the states $E[S_{\rho(i)}]$. That is,

$$E[S_{\rho(i)}] = \sum_{m=0}^{1} mP(S_{\rho(i)}(m)) \tag{5}$$

If $E[S_{\rho(i)}] \geq 0.5$, then the parent coefficient belongs to state "1", that is, $P(S_{\rho(i)}(0)) = 0$, and $P(S_{\rho(i)}(1)) = 1$. Therefore, Equation (4) becomes

$$p(w_i) = g(w_i;0,\sigma_{0;i}^2)\varepsilon_{i,\rho(i)}^{01} + g(w_i;0,\sigma_{1;i}^2)\varepsilon_{i,\rho(i)}^{11} \tag{6}$$

But, if $E[S_{\rho(i)}] < 0.5$, then state "0" is selected, and therefore, $P(S_{\rho(i)}(0)) = 1$, and $P(S_{\rho(i)}(1)) = 0$. Equation (4) is now

$$p(w_i) = g(w_i;0,\sigma_{0;i}^2)\varepsilon_{i,\rho(i)}^{00} + g(w_i;0,\sigma_{1;i}^2)\varepsilon_{i,\rho(i)}^{10} \tag{7}$$

To obtain pdf for a given wavelet coefficient w_i, the state transition probabilities $\varepsilon_{i,\rho(i)}^{00}, \varepsilon_{i,\rho(i)}^{01}, \varepsilon_{i,\rho(i)}^{10}, \varepsilon_{i,\rho(i)}^{11}$ should be determined. In our study, we used the state transition matrix A according to the assumption of the universal HMT [5].

$$A = \begin{bmatrix} \varepsilon_{i,\rho(i)}^{00} & \varepsilon_{i,\rho(i)}^{01} \\ \varepsilon_{i,\rho(i)}^{10} & \varepsilon_{i,\rho(i)}^{11} \end{bmatrix} = \begin{bmatrix} 1 & 1/2 \\ 0 & 1/2 \end{bmatrix} \tag{8}$$

Finally, the only parameter to be obtained is the variance for each marginal Gaussian distribution. There is the property of exponential decay through the scale of wavelet subbands, as presented in [4]. Especially, in the edge region, this property appears ideally. A similar inter-scale property is observed for the variance [5].

The method of variance estimation based on this property is proposed in this paper. Let $W_{0,}^k$ and W_1^k of wavelet scale $k (k = 1,2,\cdots,K)$ be the set of wavelet coefficients in state "0" and "1", respectively. $k = 1$ indicates the lowest wavelet subband, and $k = K$ stands for the highest frequency subband. First, the wavelet coefficients in each scale are divided into two states. That is,

$$w_i^k \in \begin{cases} W_1^k & if \left| w_i^k \right| \geq \beta \cdot \sigma^k \\ W_0^k & otherwise \end{cases} \tag{9}$$

where σ^k is the standard deviation of scale k, and β is the weighting constant. After the classification, from the coefficients within each class, the variances for state m $(\sigma_m^k)^2$ are estimated according to

$$(\sigma_m^k)^2 = \frac{1}{N_m^k} \sum_{w_i^k \in W_m^k} (w_i^k)^2 \tag{10}$$

where N_m^k is the number of wavelet coefficients in scale k and state m. With the estimated variances, the variance for each marginal Gaussian distribution in the highest frequency subband can be obtained using the property of exponential decay, as shown in Figure 2. That is,

$$(\sigma_m^K)^2 = \frac{(\sigma_m^{K-1})^2}{(\sigma_m^{K-2})^2} \cdot (\sigma_m^{K-1})^2 \tag{11}$$

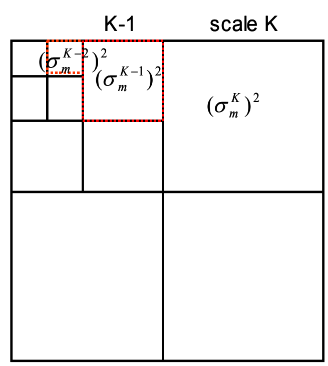

Figure 2. Two subbands used to estimate variance of the highest frequency subband.

Finally, the wavelet coefficients in the highest subband are generated randomly according to the pdf of Equation (6) or (7). The sign of parent coefficients is used as the sign of the wavelet coefficients.

4. SIMULATION RESULT

In the performance evaluation of the proposed algorithm, the PSNR results for the proposed and other algorithms are obtained, and the Lena, Woman, Boat, Peppers, and Barbara images were used for our test. The LL image of original high-resolution image is used as the input low-resolution image. Daubechies's biorthogonal 9/7 filter, popular in image processing is used in the wavelet transform. Actually, in this application, Daubechies's filter achieved a better performance than the other filters. Table 1 shows the PSNR results. As shown in the Table, the proposed algorithm showed a better performance than the conventional bicubic method and Kinebuchi's method [1] that uses the HMT model with triaging.

Figure 3 shows a subjective quality comparison between the bicubic method and the proposed algorithm. As shown in the Figure, the proposed algorithm produced a clearer extended image than the bicubic method. In Figure 4, the comparison between Kinebuchi's method and the proposed algorithm is shown. The Figure shows that whereas the image by the method in [1] had many artifacts such as noise, that by the proposed algorithm was relatively less noisy.

Table 1. The PSNR result for proposed and other algorithms

	Bicubic	HMT in [1]	Proposed
Lena	30.29	32.20	33.97
Woman	36.02	36.69	39.00
Boat	27.40	29.12	31.05
Peppers	30.82	30.92	33.49
Barbara	23.92	23.78	25.07

5. CONCLUSIONS

In this paper, we proposed a novel statistical image interpolation method using inter-scale dependency in the wavelet domain. In our method, all of the parameters for the HMT were obtained with no training. The variances for the GMM were estimated by the property of the exponential decay of the wavelet coefficients through the scales. As shown in the simulation results, the proposed algorithm performed better than the conventional bicubic method and the method based on the HMT with training.

6. REFERENCES

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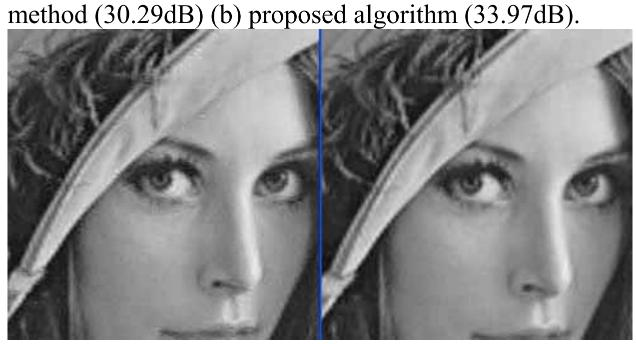
(a) (b)

Figure 3. The subjective quality comparison between bicubic method and proposed algorithm (a) bicubic method (30.29dB) (b) proposed algorithm (33.97dB).



(a) (b)

Figure 4. The subjective quality comparison between method in [1] and proposed algorithm (a) method in [1] (32.20dB) (b) proposed algorithm (33.97dB).